γ*γ*→π^{0} form factor from anti–de Sitter-space/QCD correspondence

Alexander Stoffers and Ismail Zahed

Phys. Rev. C **84**, 025202 — Published 18 August 2011

DOI: 10.1103/PhysRevC.84.025202

$\gamma^*\gamma^* \to \pi^0$ Form Factor from AdS/QCD

Alexander Stoffers and Ismail Zahed

Department of Physics and Astronomy, Stony Brook University, Stony Brook NY 11794

June 14, 2011

Abstract

The recently measured $\gamma\gamma^* \to \pi^0$ anomalous form factor is analyzed using the $D4/D8\overline{D8}$ holographic approach to QCD. The half-on-shell transition form factor is vector meson dominated and is shown to exactly tie to the charged pion form factor. The holographic result compares well with the data for the lowest vector resonance.

1. Recently the BaBar collaboration has extended the measurement of the half-on-shell $\gamma\gamma^* \to \pi^0$ transition form factor up to $Q^2 \approx 40\,\mathrm{GeV}^2$ photon virtualities [1, 2]. The reported measurements are considerably above the predicted values using factorization and pQCD [3, 4, 5]. Although seen as a key benchmark for pQCD, this exclusive process is tied with the flavor triangle anomaly in QCD and maybe more subtle. Similar difficulties were reported earlier by the JLAB collaboration for fixed angle Compton scattering $\gamma p \to \gamma p$ [6].

A number of analyses have been put forward to try to reconcile the BaBar data with pQCD factorization through a modification of the pion distribution amplitude [8, 9, 7], whereby the pion distribution amplitude is argued to be flatter. However, there are difficulties in reconciling these modifications with the data at lower Q^2 which are seen to demand a vanishing pion distribution amplitude at the edges [10].

In this letter, we will put aside the idea of factorization and analyze the BaBar data using holographic QCD, a fully non-perturbative framework. Our analysis will be based on the top-down dual construction [11, 12], in contrast to the bottom-up constructions recently discussed in [13, 14, 15]. In the bottom-up approach [13] with a hard-wall the pion wave function needs an additional boundary term. As pointed out in [14], the model studied here can be view as a hard-wall model albeit no changes to the pion wave function are necessary. While differences between these two approaches will show up in the IR of the boundary theory, we expect similarities in the UV. An interesting analysis within the context of large-N_c Regge models is given in [16].

2. The $\pi^0\gamma^*\gamma^*$ form factor can be assessed in holographic QCD using the $D4/D8\overline{D8}$ embedding formulated by Sakai and Sugimoto [11, 12] which supports vector meson dominance. Specifically ($k = q_1 + q_2$ and $Q^2_{1,2} = -q^2_{1,2}$))

$$\int d^4x e^{-iq_1x} \langle \pi^0(k)| T\left(J^\mu_{em}(x) J^\nu_{em}(0)\right)|0\rangle = \epsilon^{\mu\nu\alpha\beta} q_{1\alpha} q_{2\beta} F_{\gamma^*\gamma^*\pi^0}\left(Q_1^2, Q_2^2\right) \;, \tag{1}$$

is saturated at tree level by vector meson resonances

$$F_{\gamma^*\gamma^*\pi^0}\left(Q_1^2, Q_2^2\right) = \frac{N_c}{12\pi^2 f_\pi} \sum_{m,n} a_m a_n c_{mn} \frac{1}{1+\frac{Q_1^2}{m_m^2}} \frac{1}{1+\frac{Q_2^2}{m_n^2}} \;, \tag{2}$$

where the a_n characterize the vector couplings to the external EM current and the c_{mn} the anomalous π^0 coupling to the vectors (see [11, 12] and Appendix for details). In particular, the vector couplings obey the sum rule

$$\sum_{mn} a_m a_n c_{mn} = 1 \ , \tag{3}$$

which shows that at the photon point (2) is fixed by the Abelian anomaly

$$F_{\gamma\gamma\pi^0}\left(0,0\right) = \frac{N_c}{12\pi^2 f_\pi} \ . \tag{4}$$

3. For one photon on-mass shell, the transitional pion form factor is

$$K\left(0,Q^2\right) \equiv \frac{12\pi^2 f_\pi}{N_c} F_{\gamma\gamma^*\pi^0}\left(0,Q^2\right) = \sum_n a_n g_{n\pi\pi} \frac{1}{1+\frac{Q^2}{m_n^2}}, \tag{5}$$

where we have used [11, 12]

$$\sum_m a_m c_{mn} = g_{n\pi\pi} \ . \tag{6}$$

For $n=1$ we have $g_1 = g_{\rho\pi\pi} \approx 6$, the standard rho-pi-pi coupling. In Fig. 1 we show

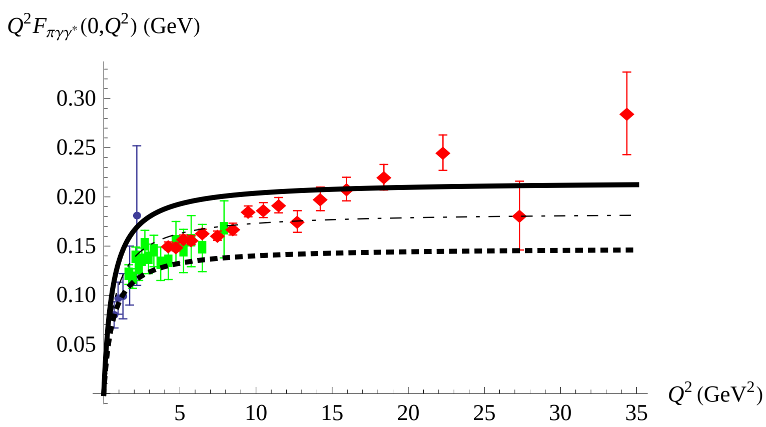

Figure 1: "(Color online)" Transitional pion form factor ($n=1$) vs. data. See text.

the transitional pion form factor (for $n=1$) versus the data from Cello [25] (blue, circle),

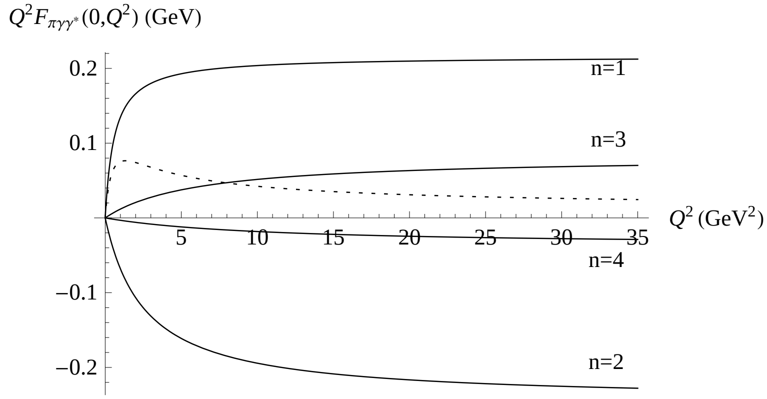

Figure 2: "(Color online)" Contributions to the transitional pion form factor. See text.

Cleo [26] (green, square) and BaBar [1] (red, diamonds). We use $f_\pi = 0.0924$ GeV and $N_c = 3$. The contribution from $n = 1$ is shown with one photon on-shell (solid line) and one photon at $Q_1^2 = 0.18\mathrm{GeV}^2$ (dashed line). The dashed-dotted line is the pQCD interpolation [5]

$$Q^2 F^{\rm BL}_{\gamma\gamma^*\pi^0}(0,Q^2) = \frac{Q^2}{4\pi^2 f_\pi}\left(1+\frac{Q^2}{8\pi^2 f_\pi^2}\right)^{-1} \simeq \frac{Q^2}{4\pi^2 f_\pi}\left(1+\frac{Q^2}{m_\rho^2}\right)^{-1} \quad . \tag{7}$$

The higher contributions from the holographic vectors are shown as the solid line contribution in Fig. 2. These vectors contribute with alternating sign to the transitional form factor and add up to zero asymptotically. The dotted line in Fig. 2 shows the result for the transitional form factor including the first 8 resonances. Indeed [1]

$$\lim_{Q^2\to\infty} Q^2 K\left(0,Q^2\right) \simeq \sum_n a_n g_{n\pi\pi} m_n^2 = 0 \ . \tag{8}$$

As shown in [17] the transitional form factor in a vector-meson-dominance model is sensitive to small Q_1^2. Here, the nature of the couplings dictated by the wave functions in the holographic direction yields a vanishing result for $Q^2 F_{\gamma\gamma^*\pi^0}$ at large Q^2 (independent of a non-vanishing Q_1^2), when the infinite tower of vector resonances is included. We recall that the top-down holographic approach effectively describes the QCD degrees of freedom for flavor excitations below $M_{KK} \approx 1\,\mathrm{GeV}$. When only the $n = 1$ or rho resonance is retained,

[1] This can be checked by expanding the result in (5) and using (14) as well as the completeness relation for the functions ψ_{2n-1}, see Appendix.

the large Q^2 asymptotic is

$$\lim_{Q^2\to\infty} Q^2 F_{\gamma\gamma^*\pi^0}(0,Q^2)\Big|_{n=1} \simeq a_1\, g_{1\pi\pi}\frac{m_1^2}{4\pi^2 f_\pi} = 1.31\frac{m_1^2}{4\pi^2 f_\pi}\ , \tag{9}$$

with $m_1 = m_\rho$ and $a_1 g_{1\pi\pi} \approx 1.31$ [12]. This asymptotics, is in a better agreement with the data in the range $10 < Q^2 < 35$ GeV^2 [1]. We recall that the pQCD result does not vanish asymptotically [4]

$$\lim_{Q^2\to\infty} Q^2 F^{BL}_{\gamma\gamma^*\pi^0} = 2f_\pi \simeq \frac{m_\rho^2}{4\pi^2 f_\pi}\ , \tag{10}$$

where the last relation follows from the second KSRF relation $m_\rho^2 = 2g^2_{\rho\pi\pi} f_\pi^2$ with $g^2_{\rho\pi\pi} \approx 4\pi^2$. The pQCD asymptotic (dashed-dotted line) is 30% lower than the holographic asymptotic (dashed line) with $n = 1$ (rho meson only) as is explicit in Fig. 1.

4. The charged pion form factor is studied in various holographic QCD models, see e.g. [18, 19, 20, 21]. An analysis within large-N_c Regge models is given in [22]; see also [23]. The model used here shows a rather unexpected result: For one photon on-mass shell, the transitional pion form factor is directly related to the charged pion form factor $F_\pi(Q^2)$ in holographic QCD:

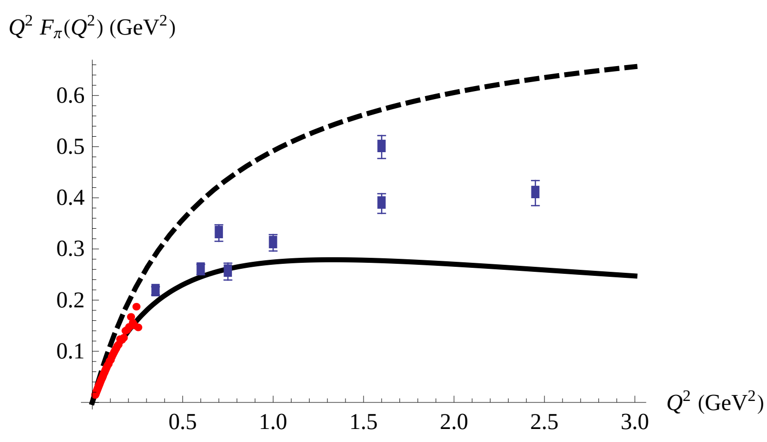

Figure 3: "(Color online)" Charged pion form factor from (11) with $n = 1, ..., 8$. See text.

$$F_\pi\left(Q^2\right) = \sum_n a_n g_{n\pi\pi}\frac{1}{1+\frac{Q^2}{m_n^2}} = K\left(0,Q^2\right)\ . \tag{11}$$

Note that the top-down model yields the same couplings for the charged and neutral pions.

In Fig. 3 we show the behavior of the charged pion form factor following from (11) by using the first eight resonances ($n = 1, ..., 8$). The data are from [27] (red dots, error bars omitted for clarity) and from [29] (black squares). At small virtualities,

$$K(0, Q^2) \approx 1 - Q^2/m_1^2 \approx 1 - a_\pi\, Q^2/m_\pi^2 \ , \tag{12}$$

where $a_\pi \approx 0.039$ can be tied to the pion charge radius by isospin $a_\pi \equiv m_\pi^2 \left\langle r^2 \right\rangle^\pi /6$. The measured value is $a_\pi = 0.026 \pm 0.024 \pm 0.0048$ [24]. The holographic relation (11) between

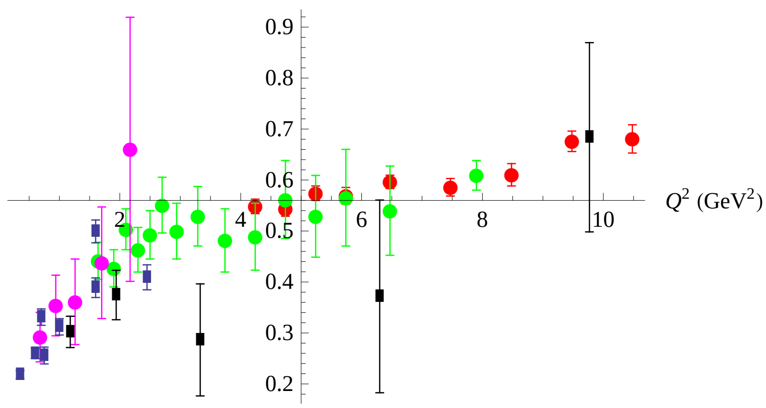

Figure 4: "(Color online)" Transitional form factor (circles) versus pion form factor (squares). See text.

the pion form factor and the transitional form factor implies a Ward-identity like relation at strong coupling. The consistency of this relation is checked in Fig. 4 where we have plotted the transitional form factor $Q^2 4\pi^2 f_\pi F_{\gamma\gamma^*\pi^0}\left(Q^2\right)$ from Cello ([25], magenta circles), Cleo ([26], green circles) and BaBar [1] (red, circles) versus the measured pion form factor $Q^2 F_\pi\left(Q^2\right)$ from [28] (black squares) and [29] (blue squares) with $f_\pi = 0.0924$ GeV. The latter data are only up to 10 GeV^2. The identity is held rather well at low Q^2 and within the error bars at large Q^2.

5. We have used the $D4/D8\overline{D8}$ holographic construction to analyze the pion transitional form factor. The transitional form factor at large N_c and strong coupling is entirely dominated by vector resonances while its on-shell intercept is still fixed exactly by the Abelian anomaly. A comparison to the existing BaBar data implies that only the $n = 1$ or ρ resonance should be retained to accomodate the measured data up to $Q^2 = 40$ GeV^2. This is consistent with the expectation that the $D4/D8\overline{D8}$ holographic model with vector excitations works at or below the $M_{KK} \approx 1\,\mathrm{GeV}$ scale. The holographic construction ties the transitional pion form factor to the charged pion form factor. This Ward-like identity is found to be well obeyed by the existing data for both form factors, including the recent BaBar data up to $Q^2 \approx 10\ \mathrm{GeV}^2$.

Acknowledgements This work was supported in part by the US Department of Energy under contract No. DE-FG-88ER40388.

Appendix: Holographic Summary.

In this Appendix we briefly note some of the holographic conventions and results of the $D4/D8\overline{D8}$ construction of relevance to our analysis in the text. We refer to [11, 12] for further details. The effective Lagrangian (DBI plus Chern-Simons) contributions below the M_{KK} scale are

$$\begin{aligned}\mathcal{L}_{\rm D8}^{\rm DBI}+\mathcal{L}_{\rm D8}^{\rm CS} \approx\ & \frac{1}{2}{\rm tr}\,\left(\partial_\mu v_\nu^n-\partial_\nu v_\mu^n\right)^2+a_n{\rm tr}\,\left(\partial^\mu\mathcal{V}^\nu-\partial^\nu\mathcal{V}^\mu\right)\left(\partial_\mu v_\nu^n-\partial_\nu v_\mu^n\right)+m_n^2{\rm tr}\,\left(v_\mu^n\right)^2\\ & -\frac{iN_c}{4\pi^2 f_\pi}\epsilon^{\alpha\beta\gamma\delta}{\rm tr}\,\left(\Pi\partial_\alpha v_\beta^n\partial_\gamma v_\delta^m\right)c_{nm}\end{aligned} \tag{13}$$

with $U(N_f)$ valued pion (Π), photon ($\mathcal{V}_\mu$) and vector (v_μ^n) fields. The vectors $v_\mu^n=iT^a v_\mu^{na}$ are $U(N_f)$ valued with the normalization tr $\left(T^aT^b\right)=\delta^{ab}/2$. Here and in the text the sum over the vector modes $m,n=1,2,3,....$ is implied. All the vector couplings in (13) are fixed by the behavior of the holographic wave functions. Specifically,

$$a_n=\kappa\int dz\;K^{-1/3}\psi_{2n-1}\;,\quad c_{nm}=\frac{1}{\pi}\int dz\;K^{-1}\psi_{2n-1}\psi_{2m-1}\;.$$

with $K=1+z^2$. $\kappa=\lambda N_c/216\pi^3\simeq 0.00745$ is fixed by the pion decay constant. The holographic wave functions ψ_{2n-1} and the masses for the vector modes satisfy the equation

$$-K^{-\frac{1}{3}}\partial_z\left(K\partial_z\psi_{2n-1}\right)=\lambda_n\psi_{2n-1}\;,\quad \lim_{z\to\pm\infty}\psi_{2n-1}\to 0\;,\quad \partial_z\psi_{2n-1}(0)=0\;,\quad m_n^2=\lambda_n M_{\rm KK}^2\;. \tag{14}$$

They are normalized by

$$\kappa\int dz\;K^{-\frac{1}{3}}\psi_{2n-1}\psi_{2m-1}=\delta_{nm}\;. \tag{15}$$

The scale of the vector masses is set by $M_{KK}\approx 1\,{\rm GeV}$.

References

[1] B. Aubert *et al.* [The BABAR Collaboration], Phys. Rev. D **80**, 052002 (2009)

[2] V. Druzhinin, "Recent results on two-photon physics at BABAR", [arXiv:hep-ex/0909.3148].

[3] G. P. Lepage and S. J. Brodsky, "Exclusive Processes In Quantum Chromodynamics: Evolution Equations For Hadronic Wave Functions And The Form-Factors Of Mesons," Phys. Lett. B **87**, 359 (1979).

[4] G. P. Lepage and S. J. Brodsky, "Exclusive Processes In Perturbative Quantum Chromodynamics," Phys. Rev. D **22**, 2157 (1980).

[5] S. J. Brodsky and G. P. Lepage, "Large Angle Two Photon Exclusive Channels In Quantum Chromodynamics," Phys. Rev. D **24**, 1808 (1981).

[6] A. Danagoulian *et al.* [Hall A Collaboration], "Compton scattering cross-section on the proton at high momentum transfer," Phys. Rev. Lett. **98**, 152001 (2007).

[7] A. E. Dorokhov, arXiv:1003.4693 [hep-ph].

[8] A. V. Radyushkin, "Shape of pion distribution amplitude", [arXiv:hep-ph/0906.0323].

[9] M. V. Polyakov, "On the pion distribution amplitude", [arXiv:hep-ph/0906.0538].

[10] S. M. Mikhailov and N. G. Stephanis, "Pion transition form factor at the two-loop level vis-a-vis experimental data", [arXiv:hep-ph/0910.3498].

[11] T. Sakai and S. Sugimoto, "Low energy hadron physics in holographic QCD," Prog. Theor. Phys. **113**, 843 (2005)

[12] T. Sakai and S. Sugimoto, "More on a holographic dual of QCD," Prog. Theor. Phys. **114**, 1083 (2005)

[13] H. R. Grigoryan and A. V. Radyushkin, "Anomalous Form Factor of the Neutral Pion in Extended AdS/QCD Model with Chern-Simons Term," Phys. Rev. D **77**, 115024 (2008)

[14] L. Cappiello, O. Cata and G. D'Ambrosio, "The hadronic light by light contribution to the $(g-2)_\mu$ with holographic models of QCD," arXiv:1009.1161 [hep-ph].

[15] S. J. Brodsky, G. F. de Teramond and A. Deur, "The AdS/QCD Correspondence and Exclusive Processes," arXiv:1007.5385 [hep-ph].

[16] E. Ruiz Arriola and W. Broniowski, Phys. Rev. D **74**, 034008 (2006)

[17] P. Lichard, Phys. Rev. **D83**, 037503 (2011).

[18] H. R. Grigoryan, A. V. Radyushkin, Phys. Rev. **D76**, 115007 (2007).

[19] H. J. Kwee and R. F. Lebed, JHEP **0801**, 027 (2008) [arXiv:0708.4054 [hep-ph]].

[20] H. J. Kwee and R. F. Lebed, Phys. Rev. D **77**, 115007 (2008)

[21] S. J. Brodsky and G. F. de Teramond, Phys. Rev. D **77**, 056007 (2008)

[22] E. Ruiz Arriola and W. Broniowski, Phys. Rev. D **78**, 034031 (2008)

[23] M. Gorchtein, P. Guo and A. P. Szczepaniak, arXiv:1102.5558 [nucl-th].

[24] F. Farzanpay *et al.*, "Measurement of the slope of the pi0 electromagnetic form-factor," Phys. Lett. B **278**, 413 (1992).

[25] H. J. Behrend *et al.* [CELLO Collaboration], "A Measurement of the pi0, eta and eta-prime electromagnetic form-factors," Z. Phys. C **49**, 401 (1991).

[26] J. Gronberg *et al.* [CLEO Collaboration], "Measurements of the meson photon transition form factors of light pseudoscalar mesons at large momentum transfer," Phys. Rev. D **57**, 33 (1998)

[27] S. R. Amendolia *et al.* [NA7 Collaboration], "A Measurement Of The Space - Like Pion Electromagnetic Form-Factor," Nucl. Phys. B **277**, 168 (1986).

[28] C. J. Bebek, C. N. Brown, S. D. Holmes *et al.*, "Electroproduction of Single Pions at Low Epsilon and a Measurement of the Pion Form-Factor Up to Q**2 = 10-GeV**2," Phys. Rev. **D17**, 1693 (1978).

[29] G. M. Huber *et al.* [Jefferson Lab Collaboration], of, and results for, the pion form factor," Phys. Rev. C **78**, 045203 (2008)