Nuclear asymmetry enthalpy

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The asymmetry enthalpy

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Recent work has sought to extract the asymmetry energy at very low density from observables in heavy-ion collisions. The logic employed starts from the assumption that the fragment yields are determined by a minimization of the Helmholtz free energy. As volume is in reality unconstrained, nor can a single freeze-out volume be expected, the physical relevance of the Helmholtz free energy must be questioned. If, for example, the identical logic were used, but the Gibbs free energy was the more relevant quantity to minimize, it would be the asymmetry enthalpy that would be extracted. The purpose of this report is to provide one measure of the difference between the asymmetry energy and enthalpy.

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The asymmetry energy not only determines (via its battle with the Coulomb term) the course of the valley of stability of real nuclei with increasing mass (a battle that it initially wins and ultimately loses) but through its density dependence, the structure of neutron stars [1]. Recently there has been an attempt to extract the asymmetry energy, E_{asy}, at low density from observables in heavy-ion reactions [2, 3]. The logic of this extraction depends on the validity of the mental image of fragment production via an equilibrium mechanism at constant volume. That is, it is the minimization of the Helmholtz free energy F that determines fragment yields. While one might argue that this extremalization represents reality better than the extremalization of any of the other standard macroscopic thermodynamic state functions, the true production scheme undoubtedly requires a kinetic treatment.

The purpose of this report is simply to provide one uncertainty benchmark for extracted values of E_{asy}. This is done by calculating the difference between E_{asy} and H_{asy}, the asymmetry enthalpy, both calculated within a consistent thermodynamic model. The logic employed in references [2, 3] would lead to the latter quantity if fragment equilibration occurred under constant pressure conditions [4]. We find that below the liquid-gas phase transition, that $H_{asy} \sim E_{asy}$ (as one would expect as H $\sim$ E) but above the phase-transition, the PV product that one can ascribe to the asymmetry of the matter becomes finite and enhances H_{asy} relative to E_{asy}, a difference that increases with the temperature. We stress that this result is model dependent and thus this work only provides one measure of the uncertainty of extracted values of E_{asy}.

The canonical thermodynamics model of das Gupta and collaborators [5] is used to calculate the thermodynamic state functions. The state variables are {A,T,V} in a 1D model or {Z,N,T,V} in the 2D version. (All calculations are done in three spacial dimensions.) The number of nucleons is A = N + Z, with asymmetry $\delta \equiv (N-Z)/A$, which exist in volume V at temperature T (in energy units). The volume can be expressed as an average number density $\rho = A/V$, or as a reduced density $\rho_r = (\rho/\rho_0)$, with $\rho_0 \sim 0.16$ (fm^{-3}) being the saturation density. The parameterization of the model is identical to that presented in detail in [5]. We only outline the physics of the model here. The partition function for a system composed of Z protons and N neutrons (in 2D) in volume V is given in terms of the unit partition functions $\omega_{i,j}$ (with i protons and j neutrons) that are calculated with the standard form for the translational partition function (i.e. the actual volume to the thermal box size) and an internal partition function $q_{i,j}$ for each unit. The latter, for $A > 4$, is given by a liquid-drop binding energy augmented by a $-TS$ term where the entropy (S) is of the Fermi-gas form,

$$\omega_{i,j} = \frac{V}{h^3}(2\pi m T)^{3/2} a^{3/2} q_{i,j}, \tag{1}$$

$$q_{i,j} = \exp\{-\frac{1}{T}f_{i,j}\} = \exp\{-\frac{1}{T}(E_{i,j} - TS_{i,j})\}, \tag{2}$$

$$f_{i,j} = -[W_o a - \sigma(T) a^{2/3} - \kappa \frac{i^2}{a^{1/3}} - s\frac{(i-j)^2}{a}] - \frac{T^2 a}{\varepsilon_0}. \tag{3}$$

Here $f_{i,j}$ is the Helmholtz free-energy contribution from clusters with i protons and j neutrons, the sum of these indices being a. The volume energy is $W_o = 15.8$ (MeV), the surface energy has a critical form with temperature $\sigma(T) = 18[(T_c^2 - T^2)/(T_c^2 + T^2)]^{5/4}$ (MeV), the Coulomb parameter (nominally $\kappa = 0.72$ (MeV)) is set to zero in this work, the asymmetry energy (averaged over density) $s = 23.5$ (MeV) (when employed) and the Fermi-gas constant $\varepsilon_0 = 16$ (MeV). The unit free energy of the Z = 1, 2 fragments is given by their bindings energies without an entropy term, i.e. they are taken to have no excited states.

The ensemble partition function is given by

$$Q_{Z,N} = \frac{1}{Z}\sum_{i,j}^{N,Z} i\omega_{i,j} Q_{Z-i,N-j}, \tag{4}$$

which is evaluated recursively starting from $Q_{0,0} = 1$ with $i + j \geq 1$.

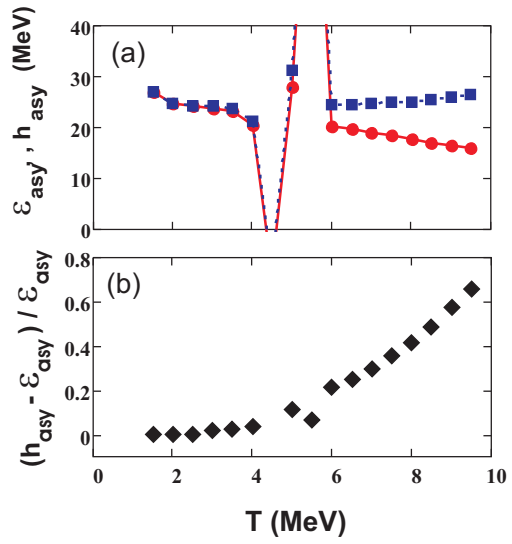

FIG. 1: (color online) The asymmetry enthalpy $h_{asy}(T)$ (squares) and energy $\varepsilon_{asy}(T)$ (circles) per nucleon (a) and the relative difference (b) evaluated for a system with Z = 80, N = 120 evaluated at $\rho_r = 0.27$.

The ensemble Helmholtz free energy, internal energy per particle and pressure are calculated with the usual expressions: $f = F/A = -\frac{T}{A}\ln(Q)$, $\varepsilon = E/A = \frac{T^2}{A}(\frac{\partial \ln Q}{\partial T})_v$ and $P = (\frac{T}{V})\Sigma_{i,j=1}^{Z,N} < n_{i,j} >$, where the average number of particles with {i,j} is given by $< n_{i,j} >= \omega_{i,j}\frac{Q_{Z-i,N-j}}{Q_{Z,N}}$. The evaluation of the pressure is consistent with this model as one of an ideal gas (no interfragment interactions) composed of all possible clusters. The construction of the above quantities is identical to that presented in [5] and the evaluation is done in simple MATHEMATICA[6] worksheets, one each for 1D and 2D models.

The enthalpy is given by $H \equiv E + PV$, and the asymmetry energy and enthalpy are determined from the scaled *difference* between the extracted values of these quantities (H and E) calculated with, and without, the asymmetry term, i.e. $E_{asy} = [E(s = 23.4) - E(s = 0)]/\delta^2$ and $H_{asy} = [H(s = 23.4) - H(s = 0)]/\delta^2$.

The heat capacities C_v and C_p, extracted from dE/dT and dH/dT, are everywhere positive with the former (per particle) going to 3/2 at high T and the latter being larger than the former (this by construction) by a value that approaches 1 (per nucleon) at high T and low density. The (liquid-gas) phase transition is seen clearly in both 1D and 2D models.

Shown in Fig. 1a are $\varepsilon_{asy}(T) = E_{asy}(T)/A$ and $h_{asy}(T) = H_{asy}(T)/A$ for a system with Z = 80, N = 120 and $\rho_r = 0.27$ (a value consistent with a chemical freeze-out logic). The statistical mechanics returns an asymmetry energy $\varepsilon_{asy} \sim s$ below the phase transition, the region dominated by large fragments. In this temperature region, h_{asy} is indistinguishable from ε_{asy}, as one would expect for any condensed phase. Above the phase transition, h_{asy} diverges from ε_{asy}. The relative deviation, plotted in Fig. 1b, exceeds 60% by 10 MeV at this typical freeze-out density. This divergence between h_{asy} and ε_{asy} is partially due to a shift in the temperature of the phase transition when the asymmetry term is removed. However correcting for this shift will lead to a qualitatively similar result. The model employed here [5] is not the most suitable for the ultra-low densities studied in the work by Natowitz at el [2, 3]. At such low densities the thermodynamics is better approached by the method of Horowitz and Schwenk [7], which considers nucleons and helium clusters and, via the virial expansion, all the interactions between these units. In this case it is known that ε_{asy} will be greatly reduced from matter values (to values near those found in [2, 3]) however extraction of the h_{asy} has yet to be done.

This work uses a simple model, with straightforward physics, to evaluate what should be called the asymmetry enthalpy. (We are not aware of this quantity ever being calculated before.) This quantity has no intrinsic value that we know of, other than as an uncertainty gauge for extraction of the asymmetry energy from models for which fragment production is assumed to be produced in a freeze-out volume while reality, though perhaps closer to this limit than any other simple prescription, is undoubtedly more complex.

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