Anomaly of the moment of inertia of shape transitional nuclei

J. B. Gupta and J. H. Hamilton

Phys. Rev. C **83**, 064312 — Published 13 June 2011

DOI: 10.1103/PhysRevC.83.064312

Anomaly of the moment of inertia of shape transitional nuclei

J. B. Gupta[1], J. H. Hamilton[2]

[1]*Ramjas College, University of Delhi, Delhi-110 007 India*

[2]*Physics Department, Vanderbilt University, Nashville, TN 37235, USA*

The change in the structure of the collective levels with spin angular momentum in atomic nuclei is often expressed in terms of the classical concepts of the kinematic and the dynamic moments of inertia varying with spin. For the well deformed even-even nuclei, the kinematic moment of inertia increases with spin up to 10-20%, at say $I^{\pi}=12^{+}$. However, for the shape transitional nuclei, or almost spherical nuclei, it increases with spin much faster. The pitfalls of using the rotor model form of kinematic moment of inertia in such cases are pointed out here. Alternative methods of extracting the nuclear structure information are explored. The important role of the ground state deformation is illustrated. The use of the power index formula for evaluating the effective moment of inertia, free from the assumption of the rotor model, is described.

Key words: [Nuclear Structure, Collective motion, moment of inertia anomalies].

PACS: Number(s): 21.10.Re; 21.60.Ev

Coll. structure, coll. Model

I. INTRODUCTION

In the geometric approach of the Bohr collective Hamiltonian, nuclei are classified as spherical vibrators or axially symmetric deformed rotors [1]. Small deviations from the two limiting symmetries are treated by perturbation methods. However, there are many nuclei that lie far from these two symmetry limits with $2.0<R_{4/2}<10/3$, for which a perturbation treatment is not valid. Various microscopic methods have evolved to treat these nuclei [2-4]. Also algebraic models such as various versions of the Interacting Boson Model [5] have been developed to understand the nuclear structure of these nuclei.

The work with heavy ion beams enabled the excitation of the medium mass nuclei to very high angular momentum. With the advent of large gamma-detector arrays in fission work [6], detailed studies of the high spin states of medium-mass, neutron-rich nuclei have become possible. The level energies of the ground band of even-even deformed nuclei are expressed through the quantum mechanical rotor formula.

$$E(I)= \hbar^2 I(I+1)/2J, \qquad (1)$$

I=0, 2, 4, 6, … and J is the moment of inertia (MoI) of the nucleus. If the deformation of the nucleus stays constant with spin (for brevity here we use spin to mean spin angular momentum), one would obtain a horizontal line for J versus spin I. The MoIs as obtained from Eq. (1) for the ground band of even-even nuclei, are smaller by a factor of 2 to 3 as compared to the rigid body value [1]

$$J_{rigid} = (2/5)\ AMR^2\ (1 + (1/3)\beta). \qquad (2)$$

Here M is the nucleon mass, A is the mass number, R is the nuclear radius, and β is the quadrupole deformation (see Fig 4.12 p. 74 [1]). The reduction in the value of J is due to the pairing correlations among the identical nucleons, especially near the Fermi surface. Using the cranking formalism with pairing included, one can reproduce the experimental MoIs of the ground states [1].

For β-soft nuclei, there is some increase in the deformation with spin due to centrifugal stretching (CS). The coriolis anti-pairing (CAP) effect also increases the MoI with spin. For a good rotor, at $I^{\pi}=10^{+}$ there may be a 10-15% change in β for a nucleus of A = 160 [7].

This deviation from the rotor formula (1) may be expressed by I(I+1) term expansion:

$$E(I) = AI(I+1) + B[I(I+1)]^2. \qquad (3)$$

Even in a well-deformed nucleus like ^{172}Hf, a failure of the two-term expression for $I^{\pi} > 10^{+}$ was noted (see Fig.4.11 in Ref. [1]). In Fig. 4.11(b), a plot of 2J versus $(\hbar\omega)^2$, a fast increase in the MoI beyond 10^{+} was observed [1].

II. VARIATION OF MoI AT HIGH SPIN

A. The kinematic and dynamic moments of inertia

For the ground band of well-deformed even-even nuclei, one can express the MoI, also called the kinematic (or kinetic) MoI, directly in terms of E_{γ} (I→I-2):

$$J^{(1)} = \hbar\ (2I-1)/E\gamma\ (I\rightarrow I-2). \qquad (4)$$

In the γ-spectrum, one expects to observe a series of equi-spaced lines that form a rotational spectrum. Slight deviations from this rule commonly occur which is due to the slight compression of the energy levels. In the spectrum of cascade γ-rays, the spacing between the consecutive γ-ray peaks, is used to define the dynamic moment of inertia $J^{(2)}$ through the expression

$J^{(2)} = 4/(E_{\gamma 1} - E_{\gamma 2})$. (5)

Here $E_{\gamma 1}$=E(I)-E(I-2), and $E_{\gamma 2}$=E(I-2)-E(I-4). Thus the kinetic and dynamic moments of inertia are used to study the dependence of nuclear structure on the spin I or the rotational frequency ω or ω^2.

In some nuclei, at high spin a break in the smooth curve of J versus spin I was observed. This was ascribed to the occurrence of a band crossing [8] of the ground-band by a S-band, which had a different moment of inertia J [8]. When the MoI was plotted against the rotational frequency ω, or ω^2, where $\hbar\omega=[E(I\rightarrow I\text{-}2)]/2$, a back bending was exhibited at this point. This may also be due to the collapse of pairing [9]. The phenomenon of rotation alignment (RAL) may also occur, giving rise to the back bending [10].

B. Moment of inertia anomaly

In the present work we point out the possible anomalies, if such rotational expressions of $J^{(1)}$ and $J^{(2)}$ are used for the shape transitional nuclei. For example, take the extreme case of a nearly spherical nucleus. Its energy levels in the ground band would be almost equi-spaced. For the intra-band transition γ-rays, this would yield the E_γ (I→I-2) of almost equal value and ΔE_γ would be almost zero.

Then with increasing spin, the numerator in Eq. (4) would go on increasing with spin I, to give an ever-increasing moment of inertia $J^{(1)}$. Obviously with a uniform level structure, such increasing moment of inertia has no physical sense. Also this would yield a very large dynamic moment of inertia, which again leads us away from a realistic understanding.

A similar problem arises with the use of rotational frequency, which is ≈ half of E_γ, while for vibrational levels it is just equal to E_γ itself. Obviously, this situation arises purely because of the use of a pure rotor expression for the transitional nuclei.

In general, *the anomaly in the use of the kinematic MoI expression* (4) *is that the numerator is from the pure rotor expression while the denominator comes from the experiment* which may increase slowly for the transitional nuclei, so *that $J^{(1)}$ increases continuously, even when there is no change in the structure of the nucleus***.** The increase in $J^{(1)}$ is merely arithmetic. The slope of $J^{(1)}$ expresses the sphericity of the nuclear core, and so also the magnitude of the dynamic moment of inertia. The slope is zero for a pure rotor and 90° for a vibrator. No other parameter, such as quadrupole deformation β or absolute E2 matrix elements, exhibit such an increase with spin of this magnitude.

The problem is really complex, since one does not have a sharp dividing line between the two limiting symmetries, the spherical harmonic vibration and the pure rotor. The value of $E(2^+)$ varies continuously from a certain maximum to a certain minimum of 70-

80 keV for rare-earths. The energy ratio $R_{4/2}$=E(4)/E(2) also varies from 2.0 to 3.33 continuously. Casten et al. [11] have noted that the ratio 0.70 x B(E2, 4-2)/B(E2,2-0) varies between 1.0 for the Rotor Model to 1.40 in the Vibration Model and it varies smoothly over the rigid to the soft rotor path.

On the spherical-deformed path, for $R_{4/2} \geq 2.2$ the nuclear structure may change from the particle like character to the collective anharmonic vibrator. Recently, Iachello [12] suggested an X(5) critical symmetry point on this path which has an analytical solution and for which $R_{4/2} \cong 3.0$. Above this, the nucleus may be considered as a good rotor and below this as a shape transitional soft rotor. Based on this division, we shall illustrate that the validity of expressions (4) and (5) is reasonable only above the X(5) limit. But for the shape transitional nuclei below X(5), the herein stated anomaly arises. Here we discuss only the axially symmetric deformed nuclei.

Regan et al. [13] also recognized the pitfalls (as discussed above) in the use of Eq. 4. They suggested the use of an alternative formula, called E-GOS defined as E_γ (I→I-2)/I and illustrated its use to distinguish between a vibrational and a rotational band. This expression was claimed to be free from the assumption of a shape model. But in effect it is essentially just the inverse of the expression in (4). Here we suggest two different approaches to resolve this anomaly in Sec. III and IV

.

III. ROLE OF THE SHAPE OF THE GROUND STATE

A. The Mallmann plots

Mallmann [14] noted that the ratios $R_{I/2}$=E(I)/E(2) are related to $R_{4/2}$, so that if one knows $R_{4/2}$ for a given nucleus, one can determine other energy ratios $R_{I/2}$ of higher levels. If one writes a composite expression of rotor energy and vibrational energy:

$$E(I) = a\, I(I+1) + b\, I \qquad (6)$$

for the total energy, it yields a linear relation of $R_{I/2}$ versus $R_{4/2}$.:

$$R_{I/2}=(R_{4/2})\, I(I-2)/8 - I(I-4)/4. \qquad (7)$$

In Fig. 1, the ratio of the two sides of the two similar triangles also yields this. The Eq. 7 can be used [15] to derive the kinematic MoI for each I from the given $R_{4/2}$. We may call it the linear kinetic MoI. Based on the linear Mallmann plot, the variation of E(I), E_γ, $J_I^{(1)}$ and $J_I^{(2)}$, with $R_{4/2}$ can be evaluated. For a few $R_{4/2}$ values, these values are illustrated in Table I.

Here we note that the slope of variation of the kinetic moment of inertia $J_I^{(1)}$ versus spin I based on the linear Mallmann plot is dependent on the energy ratio $R_{4/2}$. *With increasing spherical content and the decreasing $R_{4/2}$ the slope of the kinetic MoI versus* *spin increases. For* $R_{4/2}$=3.0, 2.5 and 2.25, the linear MoI rises from 3.0 to 4, 7 and 12 respectively at 18^+ (see Fig. 2).

TABLE I. The values of $R_{I/2}$ or E(I), relative to E(2), given by Eq. 7.

$R_{4/2}$	Spin I	2	4	6	8	10	12	14	16	18
3	E(I)	1	3	6	10	15	21	28	36	45
	E_γ	1	2	3	4	5	6	7	8	9
	E_γ^{I+2}-E_γ^{I}		1	1	1	1	1	1	1	1
	$J_I^{(1)}$	3	3.5	11/3	15/4	19/5	23/6	27/7	31/8	35/9
	$J_I^{(2)}$	4	4	4	4	4	4	4	4	4
2.5	E(I)	1	2.5	4.5	7	10	13.5	17.5	22	27
	E_γ	1	1.5	2	2.5	3	3.5	4	4.5	5
	E_γ^{I+2}-E_γ^{I}		0.5		constant					
	$J_I^{(1)}$	3	14/3	11/2	6	19/3	46/7	54/8	62/9	7
	$J_I^{(2)}$	8			constant					
2.25	E(I)	1	2.25	3.75	5.5	7.5	9.75	12.25	15.0	18
	E_γ	1	1.25	1.50	1.75	2	2.25	2.5	2.75	3
	E_γ^{I+2}-E_γ^{I}		0.25		constant					
	$J_I^{(1)}$	3	5.6	7.3	8.6	9.5	10.2	10.8	11.3	11.7
	$J_I^{(2)}$		16		constant					

Since the actual data points deviate from the linear Mallmann plot, the linear kinematic moment of inertia values would change slightly. The difference between $J^{(1)}_{expt}$

and $J^{(1)}_{lin}$ represents the effect of the rotation-vibration interaction, which increases with spin. Only these differences represent the structural changes with spin. The slope of the linear MoI curve is a measure of the sphericity of the nuclear core in the ground state itself (see Fig. 2**)**, and it overlaps the slope of $J^{(1)}_{expt}$ at the lower spin.

For the linear relation assumed in Eq. (7), the dynamic moment of inertia $J_I^{(2)}$ stays constant for all spins I, but this constant value is dependent on $R_{4/2}$ and increases with decreasing $R_{4/2}$. For $R_{4/2}$=3.0, 2.5 and 2.25 the dynamic MoI assumes a constant value of 4, 8 and 16 respectively (energies expressed in units of $E(2^+)$) (Table I). The minimum value of the dynamic moment of inertia is 3 units for the pure rotor.

B. Illustration of real cases

Zhu et al. [16] extended the yrast spectra of Ba, Ce and Nd isotopes (N>82) to high spin ($I^\pi \leq 18^+$) and studied the variations in their MoIs with spin. In Table II we present these data for three Nd isotopes and ^{150}Sm with $R_{4/2}$ ranging from 3.27 to 2.32. The level energies and kinematic MoI $J^{(1)}_{expt}$ as obtained from Eq. (4), using E_γ, are listed in the first two rows. The third row gives the linear kinematic MoI $J^{(1)}_{lin}$ as obtained through Eq. (7). The differences in the two are given in the 4th row.

TABLE II. The kinematic moment of inertia, using E_γ (expt.) and from $R_{I/2}$.(MeV^{-1}).

Isotope	R4/2	2^+	4^+	6^+	8^+	10^+	12^+	14^+	16^+
^{152}Nd ($R_{4/2}$=3.27)									
E_I		72.4	236.2	483.6	805.3	1194.8	1646.8	2157.7	2720.3
$J^{(1)}$.		41.44	42.74	44.46	46.63	48.78	50.88	52.85	55.18
lin MoI		41.44	42.74	43.10	43.28	43.38	43.45	43.49	43.53
Diff.		0	0	1.36	3.35	5.40	7.44	9.36	11.57
^{150}Nd (2.94)									
E_I		129.7	380.9	719.5	1128.4	1597.2	2117.3	2679.5	3277.5
$J^{(1)}$.		23.13	27.87	32.49	36.68	40.53	44.18	48.07	51.84

Lin MoI.	23.13	27.87	29.51	30.35	30.86	31.20	31.64	31.3
Diff.	0	0	2.97	6.33	9.67	12.98	15.63	20.21
^{148}Nd (2.50)								
E(keV)	301.4	752.0	1279.8	1855.6	2471.6	3107.4		
$J^{(1)}$.	9.95	15.53	20.84	26.05	30.84	36.16		
Lin MoI	9.95	15.53	18.34	20.03	21.15	21.96		
Diff.	0	0	2.5	6.02	9.69	14.20		
^{150}Sm (2.316)								
E(keV)	333.9	773.4	1278.9	1837.0	2432.0	3048.0	3646.0	4306.0
$J^{(1)}$.	9.0	15.93	21.76	26.88	31.93	37.34	45.15	46.97
Lin MoI	9.0	15.93	20.2	23.1	25.1	26.7	27.9	28.9
Diff.	0	0	1.6	3.8	6.8	10.7	17.2	18.1

In the well-deformed nucleus, ^{152}Nd, $J^{(1)}_{expt}$ increases by 22% at 12^+. Of this about 5% is accounted for $J^{(1)}_{lin}$ based on $R_{4/2}$, the remaining 17% is related to the structural change. In ^{150}Nd (an X(5) nucleus) with $R_{4/2}$=2.94, $J^{(1)}_{expt}$ increases by about 85% at 12^+, of which 35% is accounted for by the core shape of the ground state. For ^{148}Nd, the $J^{(1)}_{expt}$ increases a factor of 3.5!, out of which 2/3rd is accounted for by the core deformation itself and 1/3rd of the increase is related to a possible shape change. In ^{150}Sm, with still smaller $R_{4/2}$=2.32, the kinematic MoI increases from 9.0 to 45.1 at 14^+, i.e. 5-fold increase, out of which a 3-fold increase is because of its core sphericity! **(**see Fig. 3**).** The smaller is the value of $R_{4/2}$, the greater is the increase in $J^{(1)}_{expt}$, most of which can be accounted for by the ground state shape itself!

Adding a rotation-vibration term in Eq. 6 yields

$$E(I) = a\, I(I+1) + b\, I + c\, I^2(I+1). \qquad (8)$$

Interpreting the first term as the rotational energy (Rote) and the second term as the vibrational energy (Vibe) (see [17]), the Rote/E is only 26% in ^{150}Sm at $I^{\pi}=2^{+}$. This supports the above findings.

Since this interpretation still suffers from the anomaly of the use of the rotor model formula for the numerator in Eq. (4), we consider an alternative method of expressing the level energies versus spin.

IV. ENERGY IN THE FORM OF A SINGLE TERM FORMULA

A. Variation of energy with spin I

A new way of expressing the level energies, free from the assumption of a nuclear shape was proposed earlier in [18]. Here one adopts the geometric-mean approach instead of the arithmetic-mean approach adopted in Eq. 6:

$$E(I) = a\, I^{b} \,. \qquad (9)$$

A plot of log E(I) versus log I yields 'b' as the slope and 'log a' as the **i**ntercept [18]. The power index 'b' varies between 1 and 2 in going from the vibration to the rotational limit. It is a direct measure of the degree of deformation of the nuclear core. For example, the index *'b'* is 1.73, 1.56, 1.31 and 1.23 for ^{152}Nd, ^{150}Nd, ^{148}Nd and ^{150}Sm, with $R_{4/2}$=3.27, 2.94, 2.50 and 2.32 respectively. The coefficient '1/a' is related to the MoI.

TABLE III. The variation of power index b_I and the coefficient a_I with spin I.

Isotope	$R_{4/2}$	2^+	4^+	6^+	8^+	10^+	12^+	14^+	16^+
^{152}Nd ($R_{4/2}$=3.27)									
E_I		72.4	236.2	483.6	805.3	1194.8	1646.8	2157.7	2720.3
b_I' (1.732)[a]			1.706	1.729	1.738	1.742	1.744	1.744	1.744
a_I' (21.75)			22.17	21.85	21.71	21.65	21.62	21.61	21.62
$E_{calc.}$ (16.8)		72.4	241.1	487.1	802.5	1181.9	1621.8	2119.1	2671.7

^{150}Nd ($R_{4/2}$=2.94)								
E_I	129.7	380.9	719.5	1128.4	1597.2	2117.3	2679.5	3277.5
b_I' (1.557)		1.554	1.560	1.561	1.560	1.559	1.556	1.553
a_I' keV (44.07)		44.16	44.00	43.97	43.99	44.03	44.11	44.19
$E_{calc.}$ (2.0)	129.7	381.8	717.9	1123.6	1590.6	2112.9	2686.3	3307.7
^{148}Nd ($R_{4/2}$=2.50)								
E(keV)	301.4	752.0	1279.8	1855.6	2471.6	3107.4		
b_I' (1.311)		1.319	1.316	1.311	1.307	1.302		
a_I' keV (121.5)		120.8	121.04	121.47	121.78	122.22		
$E_{calc.}$ (21.8)	301.4	747.9	1272.7	1855.9	2486.7	3158.2		
^{150}Sm ($R_{4/2}$=2.32)								
	333.9	773.4	1278.9	1837.0	2432.0	3048.0	3646.0	4306.0
b_I (1.227)		1.212	1.222	1.230	1.234	1.234	1.228	1.230
a_I (142.7)		144.2	143..2	142.4	142.0	142.0	142.6	142.4
$E_{calc.}$ (19.8)	334.0	781.8	1285.7	1830.0	2406.3	3009.6	3636.2	4283.5

---

a. The values in parenthesis are the average values. For the calculated energies, it represents the standard deviation in keV for up to $I^{\pi} = 12^{+}$.

Here one may determine the values of the parameters b_I and a_I from the consecutive level energies or from E(I) and E(2). Then one can study the variation of these parameters with spin I. In [18], the near constancy of the power index b_I with spin I up to 12^+ was illustrated for many rare-earth nuclei. Further, the slight variation in b_I may reflect a small change in the nuclear structure (or MoI) with spin

In Table III we illustrate the values of b_I and a_I in $^{148\text{-}152}$Nd and ^{150}Sm. A constancy of these values within 1% is seen. The expression (9) clearly represents the variation of level energy with spin, and the constancy with spin of a. and b. indicates that there is almost no change in the nuclear structure! This is not surprising. The index is 1.9 for a well-deformed nucleus and 1.0 for a spherical vibrator. An intermediate value represents the nuclear core sphericity.

One criticism of this method may be that the 'b' being an index, the variation is **less visible**. To examine this aspect, we recalculated the level energies using the average 'b'. The standard deviation of the calculated energies up to spin 12^+ in the cited cases in (Table III), covering the full range spherical to deformed, (see the first number within parenthesis) is rather small (within 1-2%) for a mean energy, say at 10^+. Thus the expression (9) reproduces the dependence of level energies on spin I very well. In fact at higher spin, a change in 'b' reflects the change in the nuclear structure.

In the cases cited here, one sees a pattern of variation in b_I. For instance, in ^{152}Nd b_I increases with increasing spin I. In ^{150}Nd, it increases upto 8^+, then decreases. In ^{148}Nd, it decreases monotonically with increasing spin I. In fact, the same pattern of variation in the slope of the $J^{(1)}$ with spin I occurs for the three isotopes of Nd, as seen in Fig. 6 of [16]. Thus the small change in the slope of the kinetic MoI corresponds to the change in the magnitude of the index b_I, which is easier to realize. Further, it provides a new perspective to the MoI plots, not so well realized before. The average slope of the kinetic MoI plot corresponds to the average value of 'b' and the variation in its slope with spin corresponds to the variation in b_I (in the opposite sense). We have observed the same trend in Ba and Ce isotopes. The curvature in the slope of $J^{(1)}$ curves (see [16]) corresponds to the curvature in the b_I curves, in the opposite sense**.** Also, a sharp change in b_I reflects a crossing of the two bands or rotation alignment (see below). This is an additional advantage of this method.

TABLE IV. The effective MoI from the power index formula Eq. 9 and 10*.

Isotope	$R_{4/2}$	2^+	4^+	6^+	8^+	10^+	12^+	14^+	16^+
^{152}Nd	(3.27)								
E(keV)		72.4	236.2	483.6	805.3	1194.8	1646.8	2157.7	2720.3
Num			7.4	11.2	14.6	17.8	20.8	23.6	26.1
1/a (45.58)		45.06	45.06	45.08	45.37	45.64	45.90	46.09	46.47
^{150}Nd	(2.94)								
E (keV)		129.7	380.9	719.5	1128.4	1597.2	2117.3	2679.5	3277.5

Num		5.69	7.66	9.28	10.67	11.90	12.96	13.89
1/a (22.82)	22.64	22.64	22.63	22.70	22.77	22.86	23.07	23,23
^{148}Nd (2.50)								
E(keV)	301.4	752.0	1279.8	1855.6	2471.6	3107.4		
Num		3.73	4.37	4.80	5.14	5.37		
1/a (8.32)	8.28	8.28	8.28	8.34	8.34	8.45		
^{150}Sm (2.316)								
E(keV)	333.9	773.4	1278.9	1837.0	2432.0	3048.0	3646.0	4306
Num		3.05	3.49	3.85	4.12	4.33	4.41	4.58
1/a (6.99)	6.94	6.94	6.91	6.89	6.93	7.02	7.38	6.94

*Numbers in parenthesis following 1/a are the average values of 1/a.

.

Since the anomaly of a steep increase of $J^{(1)}$ is due to the use of a rotor model value in the numerator, we evaluated the same through the Eq.10

$$1/a_I = (I^b-(I-2)^b)/E_\gamma. \quad (10)$$

(see the Num(erator) row in Table IV). For example, at $I^\pi=12^+$, the numerator in Eq 4 is equal to 23 for all nuclei. But here it is ~(21, 12, 5.4) for Nd (A=152,150 and 148) with $R_{4/2}$ = 3.27, 2.94 and 2.50 respectively. See Fig. 4 for the numerator in Eq.10 versus spin I plot. A plot of the numerator versus E_γ illustrates the almost linear relation of the two, which yields an almost constant $1/a_I$. (Fig. 5).

The inverse of the coefficient 'a' corresponds to the MoI, in a much more realistic way than the kinematic MoI in Eq. (4), since the former is free from any shape model and the latter is essentially a rotor model concept. Because of the appropriate numerator, dependent on the spherical content of the nuclear core (expressed through b_I), one gets an almost constant effective MoI in $1/a_I$ (Fig. 6). A plot against ω^2 is also shown in Fig 7.

The information conveyed from the slope of the kinematic MoI is given here by the magnitude of the index b_I and that of the effective MoI $1/a_I$, which are almost constant with respect to spin I.

There is no a priori reason to express level energies in integer power of spin. If a constant non-integer power index can express the dependence of level energy on spin, there is no reason to exclude such an index. As stated above, the index is equal to 1.0 for a spherical vibrator and nearly 2 for a rotor, which serve as benchmarks of collective structure. Hence for the intermediate nuclei, the index has to be a non-integer in a single term expression.

The breaking point in the value of b_I or a_I is better exhibited on a (1/a) versus $(\hbar\omega)^2$ plot. It indicates the back bending at $I^{\pi}=8^{+}$ for ^{116}Pd and a back bending at 12^{+} for ^{158}Er (Fig. 8). Similar features can be obtained for all other back benders. Thus the power index 'b' and (1/a) can be used as realistic indicators of the constancy or a measure of variation of the nuclear structure.

Eq. (9) not only represents a different approach of expressing level energies, it enables a model independent *new definition* of the MoI. Applying to E(2), E(4), it yields the value of 'b' independent of 'a'. The value of b_I is simply equal to logR(I/2)/log(I/2). So b_I is a function of $R_{I/2}$ and thereby it incorporates in itself the deformation of the nucleus at each nuclear state. Eq. (9) is just another way of expressing the linear relation of all $R_{I/2}$ to $R_{4/2}$, as noted by Mallmann [14]. In other words, Eq. (9) is based on Eq. (7) in Sec. III.B, expressed in a different (and simple) mathematical form. The method is equally well applicable to β- and γ-bands and to the S-band or RAL band.

Thus, this work is an attempt to point out the anomaly in the too sharp variation of the kinematic moment of inertia, numerically exceeding the rigid body MoI in cases of near-vibrational nuclei, as illustrated for ^{148}Nd and ^{150}Sm, wherein at 12^{+} we noted a 400% and 500% increase respectively in the kinematic MoI. In a positive sense, this work points to the importance of the variation in the slope of the kinetic MoI plots, rather than of the actual increase in MoI with spin.

B. Correspondence of power index 'b' with IBM-1 control parameter ξ

In the IBM-1, a schematic Hamiltonian is often used to study the variation of the nuclear structure along the transition class-A from the U(5) limit to the SU(3) limit:

$$H_{IBM} = \varepsilon\, n_d + k\, Q\cdot Q. \qquad (11)$$

Note the similarity with energy expression (6). This can be rewritten in the form

$$H_{IBM} = a\,[(1-\xi)\,n_d + \xi\,Q\cdot Q]. \qquad (12)$$

For ξ=0, it represents the U(5) limit and for ξ=1 the SU(3) limit. Thus ξ varies 0 to 1. In a study of the nuclear structure of ^{150}Sm ($R_{4/2}$=2.316), Gupta et al. [19] obtained the ε/k ratio $\cong 39=(1-\xi)/\xi$, equivalent to ξ= 0.025, close to the U(5) limit. This has a good correspondence with the value of (b-1) = x = 0.227 (Table III) on a scale of zero to 0.9, closer to the vibrational model limit of zero. Thus (b-1)=x or the index 'b' plays the role of the control parameter to reproduce the variation of nuclear structure as represented by the K-bands in the nucleus. Corresponding to the use of the energy scale parameter defined in Eq. 11 or 12, the coefficient 'a' in Eq. 9 acts as the energy scale parameter.

In a recent study of Eq. (7), its universal validity to predict the energy ratio $R_{I/2}$, (for I≤10), the same as, in the three limiting symmetries of U(5), SU(3) and O(6) as well as in the two critical point symmetries of E(5) and X(5) (approximately) has been illustrated [20[.

V. DISCUSSION AND CONCLUSION

In this work, the anomaly in the use of the terms kinetic and dynamic moments of inertia, essentially rotor model concepts, for the non-rotational nuclei is pointed out. The relation of R(I) to R(4) as expressed in Eq. (7) is used to determine the contribution of the sphericity in these quantities. This partly explains the observed increasing slopes of $J^{(1)}_{expt}$ against spin I or ω (or ω^2). The difference in $J^{(1)}_{expt}$ and $J^{(1)}_{linear}$ is a better measure of the variation of nuclear structure, but both still suffer from the use of a numerator derived from the rotor formula.

In a wholly different approach, we use a power index formula to express the energy, which is free from the shape model. The validity of the formula is demonstrated by the near constancy of its parameters against spin, and the reproduction of level energies using the mean value within 1-2%. The parameters b_I and a_I or ($1/a_I$) reflect the variation in the nuclear structure with spin, including the back-bending. The average 'b' and 'a' (measure of the sphericity of the core) vary with Z and N. The information available in the slope of the kinetic MoI curves from Eq. 4 is now available in the magnitude of the index b_I and $1/a_I$, as illustrated for the three isotopes of Nd. The range of validity of Eq. 9 (and 10) is in the full collective space, rotor to spherical-vibrator, the two benchmarks. Further it yields an expression for the MoI using a numerator derived from the power index formula, and the E_γ (experiment). For the rotor it reduces to the usual formula. For the spherical vibrator, the numerator is 2 instead of 3 for $I^\pi=2^+$, as it should be. For higher spins, appropriate values are obtained, depending upon the rotational content of the nuclear core. Thus the use of the power index formula for E(I) solves the anomaly of the too large increase of kinetic MoI with spin in shape transitional nuclei. We have also pointed out the correpondence of the two parameters 'b' and 'a' with the control parameter ξ, and the energy scaler in the 2-parameter form of the IBM-1 Hamiltonian.

Further, it is known that there is something constant in a rotational band, gsb or a band based on an excited state, its band head. This is true for the even-even nuclei as well as for the regular (smoothly spaced) bands in odd-A nuclei. For a stable structure, the increase in level energies in a rotational band arises from the dynamics and which is a function of spin I=F(I). If the parameters involved in the function F(I) are constant, i.e. non-varying with spin, except the spin itself, such a function should have deeper meaning. In this sense, the index 'b' or x=(b-1) represents a useful form of this function. In fact this is the essence of the empirical observation of Mallmann. Compared to the other variants of this function such as the BM expansion in powers of I(I+1) or the various forms of anharmonic vibrator expressions, in powers of I itself or more complex expression like the ab formula [21] or the pq formula [22], the power index expression is the simplest with a clear meaning of the power index 'b' or x=(b-1), which are directly related to the deformation of the nuclear core. One can even correlate the CBS model [23] with this expression. Once one fixes the square well potential parameters, viz. the β_{min} and β_{max}, the two ends of the square well and the depth, which are constant for a given K-band, one gets the energies of the band. Thus 'x=b-1' is related to the parameters of the well, on the $\hat{\upsilon}$-axis. The x=0 corresponds to the β_{min}=0, and x>0 but <1 corresponds to the β_{min} greater than zero. In conclusion, we have presented a model independent expression for the level energies in all regular collective K-bands.

ACKNOWLEDGEMENTS

Association with the College is gratefully acknowledged. The work at Vanderbilt University is supported by the U.S. Department of Energy under Grant No.DE-FG05-88ER40407.

REFERENCES

[1] A. Bohr and B. R. Mottelson, *Nuclear Structure*, Vol. II (Benjamin, New York, 1975).

[2] K. Kumar and M. Baranger, Nucl. Phys. A **92**, 608 (1967).

[3] P. Ring and P. Schuk, *The Nuclear Many Body Problems* (Springer, Berlin, 1967).

[4] A. L. Goodman, *Advances in Nuclear Physics,* J. Negel and E. Vogt, eds., (Plenum, New York, 1979) **11,** 263.

[5] F. Iachello and A. Arima, *The Interacting Boson Model* (Cambridge University Press, 1987).

[6] J. H. Hamilton *et al*., Prog. Part. Nucl. Phys. **35**, 635 (1995).

[7] F. S. Stephens, Rev. Mod. Phys. **47,** 43 (1975).

[8] A. Johnson, H. Ryde, and J. Sztarkier, Phys. Lett. **B34**, 605 (1971).

[9] B. R. Mottelson and J. G. Valatin, Phys. Rev. Lett. **5**, 511 (1960).

[10] F. S. Stephens and R. S. Simon, Nucl. Phys. A **183** 267 (1972).

[11] R. F. Casten *et al*., Nucl. Phys. A **693** 105 (2001).

[12] F. Iachello, Phys. Rev. Lett. **87**, 052502 (2001).

[13] P. H. Regan *et al*., Phys. Rev. Lett. **90,** 15 (2003).

[14] C. A. Mallmann, Phys. Rev. **2**, 507 (1959).

[15] J. B. Gupta, *Third International Conference on Fission and Properties of Neutron-Rich Nuclei*, J. H. Hamilton, A. V. Ramayya and H. K. Carter, eds., (Singapore: World Scientific, 2003) p. 320.

[16] S. J. Zhu *et al*., J. Phys. G **21**, L75 (1995).

[17] J. B. Gupta and A. K. Kavathekar, Phys. Scripta **56**, 576 (1997).

[18] J. B. Gupta, A. K. Kavathekar, and R. Sharma, Phys. Scripta **51**, 316 (1995).

[19] J. B. Gupta, K. Kumar, and J. H. Hamilton, Int. J. Mod. Phys E **19**, 1491 (2010).

[20] J.B. Gupta, Nucl. Phys. Symp. (India) **55,** 84 (2010).

[21] P. Holmberg and P. O. Lipas, Nucl. Phys. A **117**, 552 (1968).

[22] G. M. Zeng, W. Liu, and E. G. Zhao, Phys. Rev. C **52**, 1864 (1995).

[23] N. Pietralla and O. M. Gorbachenko, Phys. Rev. C **70**, 011308 (2004).

Figure captions

1. The linear relation of R_I with $R_{4/2}$, illustrated for $R_{6/2}$. The ratio of the two sides in the two similar triangles yields the Eq. (7).

2. The dependence of the linear kinetic moment of inertia on the energy ratio $R_{4/2}$ for three different values. Energy for $J^{(I)}$ expressed in units of $E(2^+)$.

3. The kinetic MoI using the rotor formula and the linear relation of $R_{I/2}$ with $R_{4/2}$ in the Mallmann plot, for ^{152}Nd and ^{150}Sm. A large part of the increase in ^{150}Sm is given by the latter.

4. The values of the numerator $I^b - (I-2)^b$ in Eq. 10 versus spin I compared with rotor model values. Note the decreasing slope for decreasing rotational content in the nucleus.

5. Same as Figure 3, but for the plot against E_γ. Note the almost linear relation of the numerator in Eq. 10 and the experimental E_γ . At $I^\pi=2^+$, the numerator reduces to $R_{4/2}$ in Eq. (10), exceeding rotor value of 3.0, if $R_{4/2}$ exceeds it.

6. The kinetic MoI $J^{(1)}$ and $1/a_I$ versus spin for $^{148-152}$Nd. At $I^\pi=2^+$, the numerator in Eq. 10, exceeding 3.0 for ^{152}Nd yields $1/a_I$ exceeding $J^{(1)}$ in the rotor model .

7. The same as in Figure 5, but the plot is against rotational frequency ω^2. The $1/a_I$ plots of effective MoI show an almost constant value.

8. The back bending of kinetic MoI is also given in the 1/a plot for ^{158}Er.at I=10.

Fig.1 JBGupta

Fig. 2 JBGupta

Fig. 3 JBGupta

Fig. 4 JBGupta

Fig. 5 JBGupta

Fig. 6 JBGupta

Fig. 7 JBGupta

Fig. 8 JBGupta

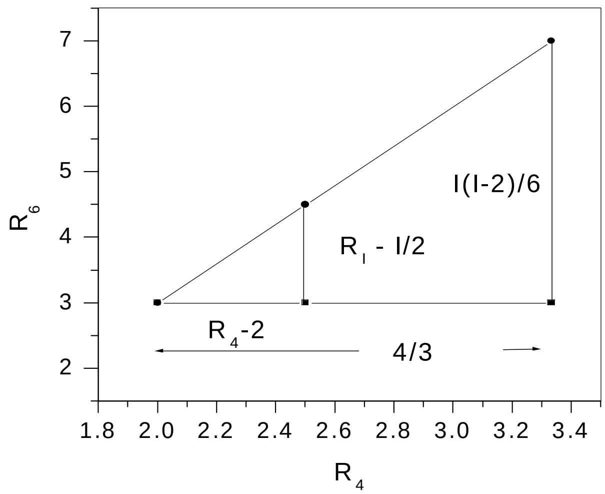


Fig.1. CC10230/Gupta

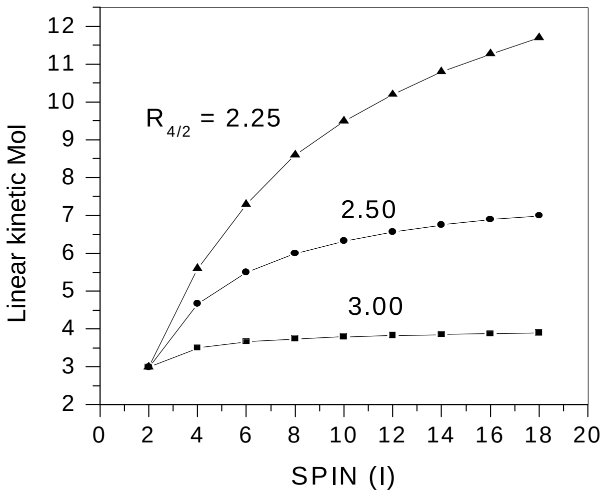
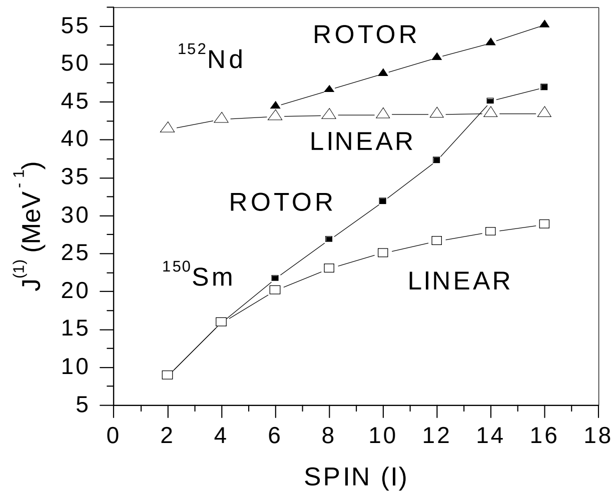


Fig. 2. CC10230/Gupta



F[g. 3. CC10230/Gupta

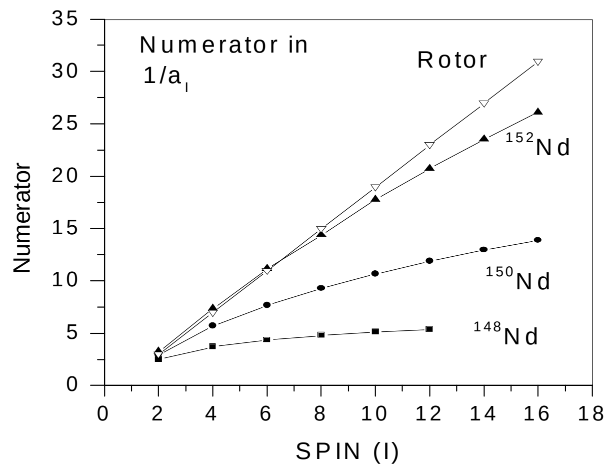


Fig.4. CC10230/Gupta

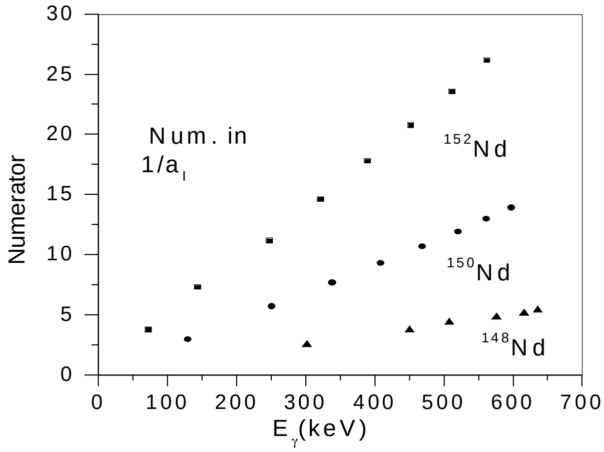


Fig.5. CC10230/Gupta

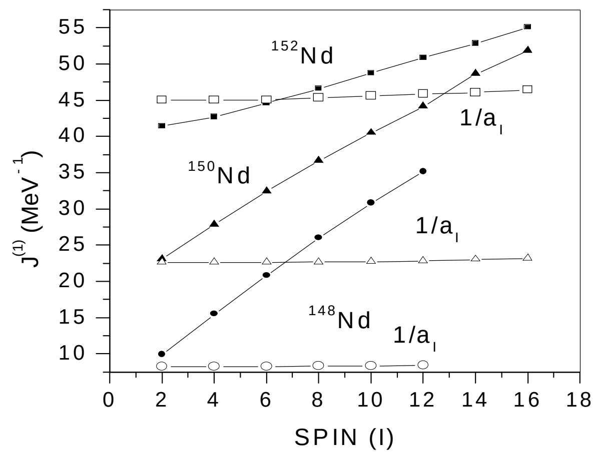


Fig.6. CC10230/Gupta

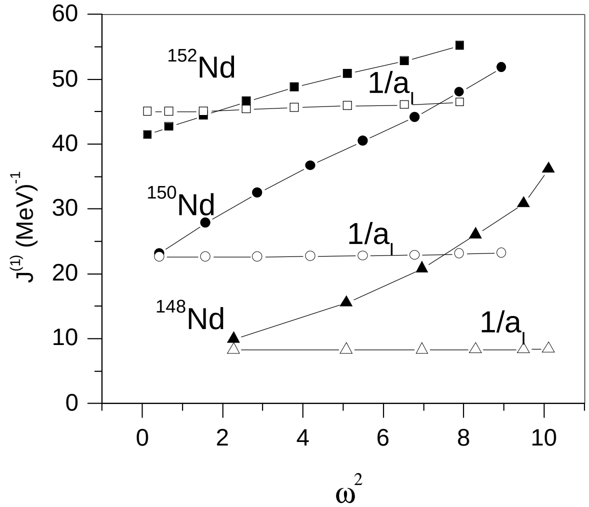


Fig.7. CC10230/Gupta

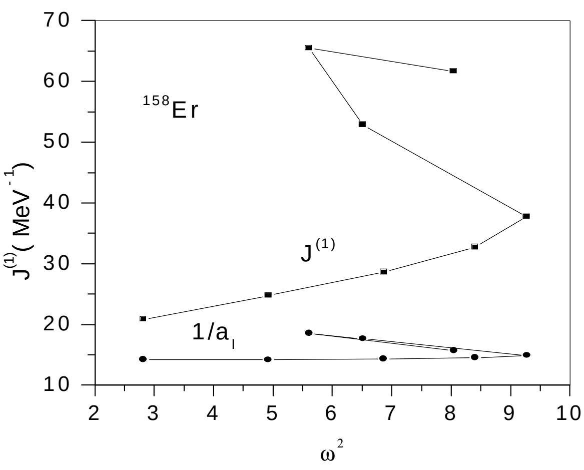


Fig. 8. CC10230/Gupta