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Reexamination of the astrophysical S factor for the $\alpha + d \to {}^6\mathrm{Li} + \gamma$ reaction.

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Recently a new measurement of the ^{6}Li(A 150 MeV) dissociation in the field of ^{208}Pb has been reported in [F. Hammache *et al.* Phys. Rev **C82**, 065803 (2010)] to study the radiative capture $\alpha + d \to {}^6\mathrm{Li} + \gamma$ process. However, the dominance of the nuclear breakup over the Coulomb one prevented from obtaining the information about the $\alpha + d \to {}^6\mathrm{Li} + \gamma$ process from the breakup data. The astrophysical $S_{24}(E)$ factor has been calculated within the $\alpha - d$ two-body potential model with potentials determined from the fits to the $\alpha - d$ elastic scattering phase shifts. However, the scattering phase shift itself, according to the theorem of the inverse scattering problem, doesn't provide a unique $\alpha - d$ bound state potential, which is the most crucial input when calculating the $S_{24}(E)$ astrophysical factor at astrophysical energies. In this work we emphasize an important role of the asymptotic normalization coefficient (ANC) for ${}^6\mathrm{Li} \to \alpha + d$, which controls the overall normalization of the peripheral $\alpha + d \to {}^6\mathrm{Li} + \gamma$ process and is determined by the adopted $\alpha - d$ bound state potential. Since the potential determined from the elastic scattering data fit is not unique, the same is true for the ANC generated by the adopted potential. However, a unique ANC can be found directly from the elastic scattering phase shift, without invoking intermediate potential, by extrapolation the scattering phase shift to the bound state pole [Blokhintsev *et. al* Phys. Rev. **C 48**, 2390 (1993)]. We demonstrate that the ANC previously determined from the $\alpha - d$ elastic scattering s-wave phase shift in [Blokhintsev *et. al* Phys. Rev. **C 48**, 2390 (1993)] and confirmed by the Faddeev equations calculations, gives $S_{24}(E)$, which is at low energies about 38% lower than the one reported in [F. Hammache *et al.* Phys. Rev **C82**, 065803 (2010)]. We recalculate also the reaction rates, which are also lower than those obtained in [F. Hammache *et al.* Phys. Rev **C82**, 065803 (2010)].

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I. INTRODUCTION

The question of using low-energy scattering data to describe bound states of composite systems turns out to be quite important in different areas of contemporary quantum physics: nuclear astrophysics, halo nuclei, effective field theory, generalized Faddeev equations, condensed matter, ultra-cold atoms etc. Elastic scattering data are usually accessible for stable and even for exotic nuclei (in inverse kinematics). In a simplified approach these data are analyzed within a two-body potential model. This procedure consists in determination of the two-body potential, which fits the elastic scattering phase shift in the specific partial wave. A priori, the two-body potential found from the fitting the elastic scattering data can be complex and energy dependent. However, at low enough energies where only the elastic channel is open, i.e. below all inelastic thresholds, the obtained phenomenological potential becomes real. Using this potential one can calculate the binding energy of the bound state, which generates a pole of the elastic scattering S-matrix element on the positive imaginary semi-axis in the momentum plane (negative energy) and the bound state wave function of two interacting objects as though that they are structureless. However, an obstacle in such an approach is the Gel'fand-Levitan and Marchenko theorems of the inverse scattering problem [1]: there is infinite number of the phase-equivalent potentials, which provide different binding energy and asymptotic normalization coefficient (ANC). To find a unique two-body bound state potential from elastic scattering data one has to add into analysis these two parameters.

Another way of obtaining the information about the bound state and two-body potential has been used in [2]. It is based on analyticity of the scattering S matrix, what allows one to extrapolate the experimental scattering phase shift directly to the bound-state pole without using an intermediate two-body potential. In this procedure the elastic scattering phase shift is analytically approximated in the physical region with the subsequent analytical continuation to the bound state pole. This analytical approximation in the physical region can be done using different approaches: Pade approximation [2], effective range expansion [3–5]. There is also a possibility to solve N/D equations, which take into account the right unitary cut and the left dynamical singularities. Using these equations one can fit the experimental scattering phase shift at given partial wave and find the bound state pole and its residue [6, 7]. We note that effective field theory can also be applied to obtain binding energy (if necessary) and the ANC [8–10]. In principle, if experimental data are quite accurate, such an extrapolation can provide the location of the bound state pole (binding energy) and its residue, which is expressed in terms of the ANC [11–13]. If the binding energy is known,

then it is preferable to include this information into extrapolation procedure. In this case the extrapolation should provide only one missing parameter - the ANC. This procedure has been realized in [2] for the $\alpha - d$ elastic scattering and the ANC has been found for the virtual decay $^6\mathrm{Li} \to \alpha + d$ in the s-wave channel. Once we know both parameters of the bound state, binding energy and the ANC, using the experimental data we can find, according to the inverse scattering theorems, a unique two-body potential. This potential will provide the bound state wave function with the binding energy which should coincide with the value used in the extrapolation procedure. Besides, the ANC obtained from the calculated bound state wave function and from the residue of the elastic scattering S-matrix in the bound state pole must coincide [14].

However, there is important point to discuss. The binding energy and the ANC are the on-shell characteristics of the bound states in the same sense as the scattering phase shifts are the on-shell characteristics of continuum. And these on-shell characteristics reflect the many-body character of the scattered nuclei. The invoking of the two-body potential to analyze the elastic scattering of composite particles, definitely, is an approximation, which, nevertheless, is supposed to reproduce the on-shell bound state parameters, the binding energy and the ANC. It means that, using the two-body approach to reproduce the elastic scattering data, one effectively takes into account many-body effects. However, the off-shell properties of the bound states, are not necessarily correctly reproduced. Consider, for example, the s-wave elastic scattering of composite nuclei $\alpha + d$. The ANC for the virtual decay of the ^{6}Li bound state $^6\mathrm{Li} \to \alpha + d$ is the amplitude of the tail of the overlap function

$$I^{^6\mathrm{Li}}_{\alpha\, d\,(01)}(\mathbf{r}) = < \varphi_\alpha\, \varphi_d | \varphi_{^6\mathrm{Li}} > . \tag{1}$$

Here, φ_i is the bound-state wave function of nucleus i, $l = 0$ and $J = 1$ are the relative orbital angular momentum of α and d in the bound sate and the total angular momentum. The integration in (1) is taken over the internal coordinates of α and d, so that the overlap function depends on the radius $\mathbf{r}$ connecting their centers-of-mass. The overlap function is essentially a many-body object, while the bound state wave function calculated using the two-body potential is not. In particular, the overlap function is not normalized to unity (square of its norm is the spectroscopic factor). The behavior of the radial part of the overlap function and the two-body bound-state wave function, a priori, is different. However, the tail of both functions has the same radial shape, i.e. they decay as Whittaker function, but the amplitudes of the tail, i.e. the ANCs, a priori, are different. The condition that the two-body potential, which fits the elastic scattering data, must provide a correct ANC means that the amplitudes of the tails of the radial overlap function and bound state wave function coincide. The ANC of the bound state wave function is called the single-particle ANC, although it is more appropriate to call it the two-body ANC what we are going to use here. In the two-body approach the ANC and the two-body ANC are related as (applying for the ^{6}Li case)

$$C^{^6\mathrm{Li}}_{\alpha\, d\,(01)} = S^{1/2}_{\alpha\, d\,(101)}\, b_{\alpha\, d\,(101)}. \tag{2}$$

Here, $C^{^6\mathrm{Li}}_{\alpha\, d\,(01)}$ and $b_{\alpha\, d\,(101)}$ are the ANC and the two-body ANC and $S_{\alpha\, d\,(101)}$ is the spectroscopic factor of the configuration $(\alpha\, d)_{101}$ in the ground state of ^{6}Li. Note that the right-hand-side depends on the additional quantum number $n = 1$, which is the principal quantum number showing the number of nodes of the adopted radial $\alpha - d$ bound-state wave function at $r > 0$. This dependence appears due to use of the two-body potential model. The condition $b_{\alpha\, d\,(101)} = C^{^6\mathrm{Li}}_{\alpha\, d\,(01)}$ requires the spectroscopic factor $S_{\alpha\, d\,(01)} = 1$.

The ANC plays a very important role in nuclear astrophysics because many astrophysical radiative capture processes are peripheral and their overall normalization is governed by the tail of the overlap function, i.e. by the ANC [15–18]. Among the important astrophysical processes, where the ANC plays a crucial role, is the primordial radiative capture process $\alpha + d \to {}^6\mathrm{Li} + \gamma$, which is the only process that produces ^{6}Li in the big bang model. A special interest to this reaction has been triggered by almost three order disagreement between the observational ratio $^6\mathrm{Li}/^7\mathrm{Li}$ and the calculated one [19]. Direct measurements of the $\alpha + d \to {}^6\mathrm{Li} + \gamma$ radiative capture are very difficult at astrophysically relevant $\alpha - d$ relative kinetic energies $E \leq 300$ keV due to extremely low cross section. Nevertheless, the first direct measurements of the $\alpha + d \to {}^6\mathrm{Li} + \gamma$ reaction cross section at low energies are currently underway at the LUNA underground accelerator facility at the Gran Sasso laboratory.

The first indirect information about the astrophysical factor $S_{24}(E)$ for the $\alpha + d \to {}^6\mathrm{Li} + \gamma$ process has been obtained in [20] from the Coulomb breakup process $^{208}\mathrm{Pb}(^6\mathrm{Li}\,(26\mathrm{A\,MeV}),\, \alpha\, d)^{208}\mathrm{Pb}$. The analysis of the data was performed under assumption that the reaction mechanism is contributed purely by the Coulomb breakup. However, the extracted data, which can be considered as an upper limit of the astrophysical factor, at low energies turned out to be constant what contradicted to all the calculations showing significant drop [17]. Recently in [21] a new attempt has been done to get the astrophysical factor $S_{24}(E)$ at astrophysically relevant energies from the $^{208}\mathrm{Pb}(^6\mathrm{Li}\,(150\mathrm{A\,MeV}),\, \alpha\, d)^{208}\mathrm{Pb}$ breakup reaction. However, analysis has shown significant dominance of the nuclear breakup over the Coulomb one making impossible to determine the needed information about $S_{24}(E)$. Nevertheless, in [21] the $S_{24}(E)$ has been calculated using the $\alpha - d$ two-body potential model. The potentials, which are required to make such calculations,

were obtained from the fitting the $\alpha - d$ elastic scattering phase shift for the s, p and d waves. The approach used to calculate the astrophysical factor is not related with the studied ^{6}Li breakup process. The common information in the analysis of the breakup data and calculation of the astrophysical factor were the same bound state and scattering $\alpha - d$ potentials used to generate the corresponding $\alpha - d$ bound and scattering wave functions. In the potential approach used in [21] the bound state potential is the most crucial part of the input, which affects the overall normalization of the astrophysical factor. Only one out of infinite number of the phase equivalent potentials, namely the Woods-Saxon potential with the standard geometry, has been used in [21] to describe the $\alpha - d$ bound state. However, the dominance of the nuclear breakup and dependence of the breakup data analysis on the optical potentials requires a check of the sensitivity of the breakup calculations to the variation of the bound state potential along with the optical potentials. Such a test, which has not been done in [21], would allow one to determine the uncertainty in the astrophysical factor and reaction rates, because, as we can see below, the adopted bound state potential is the most important information required to calculate the astrophysical factor $S_{24}(E)$ at astrophysical energies.

Taking into account the critical importance of the ANC in the calculation of the astrophysical factor $S_{24}(E)$ for the $\alpha + d \to {}^6\mathrm{Li} + \gamma$ radiative capture and connection between the potential model, elastic scattering data and ANC discussed above, we critically reexamine the procedure used in [21] and what should be done to improve our knowledge about the astrophysical factor $S_{24}(E)$ within the potential two-body problem.

II. ASTROPHYSICAL FACTOR FOR THE $\alpha + d \to {}^6\mathrm{Li} + \gamma$ RADIATIVE CAPTURE IN THE $\alpha - d$ POTENTIAL MODEL

It has been long ago recognized that the $\alpha + d \to {}^6\mathrm{Li} + \gamma$ process at astrophysical energies is entirely peripheral reaction in the two-body potential model [17]. Evidently, the potential model itself is a limitation and it would be nice to check peripherality of this reaction within a many-body ab initio approach similar to what has been done recently for the $^3\mathrm{He} + {}^4\mathrm{He} \to {}^7\mathrm{Be} + \gamma$ in [22]. However, an important issue in a many-body approach remains to be solved is reproduction of the experimental binding energy, because the calculated astrophysical factor is sensitive to its value. Since such ab initio many-body calculations are not yet available we have to live with a more simple two-body potential model. The matrix element for the $\alpha + d \to {}^6\mathrm{Li} + \gamma$ direct radiative capture in the long-wave approximation is given by [17]

$$M^{\lambda}_{01\,l_i J_i} = A_\lambda \int_{R_c}^{\infty} dr\, I^{^6\mathrm{Li}}_{\alpha\, d\,(01)}(r)\, r^{\lambda+2}\, \psi_{l_i\, J_i}(r). \tag{3}$$

Here, $I^{^6\mathrm{Li}}_{\alpha\, d\,(01)}(r)$ is the radial overlap function of the bound-state wave functions of ^{6}Li, α and d, $l = 0$ and $J = 1$ are the relative orbital and total angular momentum of the $\alpha - d$ configuration in the ground state of ^{6}Li, $\psi_{l_i\, J_i}(r)$ is the $\alpha - d$ scattering wave function in the initial state of the radiative capture in the partial wave l_i with the total angular momentum J_i, $\lambda = 1,\, 2$ is the multipolarity of the transition, A_λ is the kinematical factor. The cut-off radius R_c is introduced to reflect a peripheral character of the process. It has been shown in [17] that the matrix element shows a very small sensitivity for $R_c \leq 4.5$ fm, i.e. until distances, which exceed the ^{6}Li radius. It makes possible to approximate the radial overlap function by the $C_{\alpha d(01)}\, W_{-\eta,\,1/2}(2\,\kappa_{\alpha d}\, r)/r$, where $W_{-\eta,\,1/2}(2\,\kappa_{\alpha d}\, r)$ is the Whittaker function determining the radial shape of the overlap function beyond of the $\alpha - d$ nuclear interaction region, η is the Coulomb $\alpha - d$ bound state parameter, $\kappa_{\alpha\, d} = \sqrt{2\,\mu_{a\, d}\, \varepsilon_{\alpha\, d}}$ is the $\alpha - d$ bound state wave number, $\mu_{\alpha\, d}$ and $\varepsilon_{\alpha\, d}$ are the reduced mass and binding energy of α and d. As we can see, there are two inputs needed to calculate the matrix element in the $\alpha - d$. One is the $\alpha - d$ potential describing the continuum in the partial waves $l_i = 1,\, 2$. This potential has been found in [21] by fitting the elastic scattering phase shifts in these partial waves and it is a legitimate procedure. Note that at very low energies, say around the most effective energy of 70 keV, one can use a pure Coulomb scattering wave function in the initial state of the reaction. However, the role of nuclear interaction becomes very important with energy increase and it is responsible for reproduction of the resonance at $E_R = 0.711$ MeV. In the potential approach used in [21, 24] the overlap function is replaced by the

$$I^{^6\mathrm{Li}}_{\alpha\, d\,(01)} = S^{1/2}_{\alpha\, d\,(101)}\, \varphi_{\alpha\, d(101)}(r), \tag{4}$$

Here, $\varphi_{\alpha\, d(101)}(r)$ is the radial $\alpha - d$ bound-state wave function in the field generated by the Woods-Saxon nuclear plus Coulomb potentials. The spectroscopic factor reflects the fact that the overlap function is not an eigenfunction of any Hamiltonian and, hence, is not normalized to unity in contrast to the bound state wave function. The potential, which is used to calculate the bound state wave function, has been determined in [21] from the fitting to the $\alpha - d$ elastic scattering phase shift in the channel $l = 0$, $J = 1$. Since the experimental elastic scattering phase shift includes the many-body effects of the scattered nuclei, the same is true for the two-body potential which fits the elastic scattering

data. Hence, the spectroscopic factor in Eq. (4) should be set to $S_{\alpha\, d\,(01)} = 1$. It is exactly what has been done in [21]. Since the reaction under consideration is peripheral at astrophysical energies $E \leq 300$ keV, functions in Eq. (4) can be replaced by their tails, i.e.

$$I^{^6\mathrm{Li}}_{\alpha\, d\,(01)} \overset{r>R_c}{\approx} C_{\alpha d(01)}\, \frac{W_{-\eta,\,1/2}(2\,\kappa_{\alpha d}\, r)}{r} = b_{\alpha d(101)}\, \frac{W_{-\eta,\,1/2}(2\,\kappa_{\alpha d}\, r)}{r}, \tag{5}$$

i.e. in the two-body potential model the amplitude of the tail of the overlap function is the two-body ANC $b_{\alpha d(101)}$, i.e. the amplitude of the tail of the radial bound state wave function. The value of the single-particle ANC depends on the adopted bound-state potential. From the parameters of the bound state potential given in [21] we find that $b_{\alpha d(101)} = 2.7$ fm$^{-1/2}$. Thus from the Woods-Saxon potential given in [21] we get the ANC, which is about 17% larger than the ANC $C_{\alpha d(101)} = 2.3 \pm 0.12$ fm$^{-1/2}$ obtained in [2] and used in [17].

An independent confirmation of this ANC follows from just published paper [23], where the first *ab initio* calculations of the $\alpha - d$ scattering and ^{6}Li bound and unbound states are presented. From the calculated $\alpha - d$ bound state wave function, see Fig. 5 [23], we find the ANC $C_{\alpha d(101)} = 2.94$ fm$^{-1/2}$. But this value is obtained for the $\alpha - d$ binding energy 1.83 MeV. Extrapolating this value to the experimental binding energy we obtain the ANC $C_{\alpha d(101)} = 2.39$ fm$^{-1/2}$, what is in an excellent agreement with the ANC obtained in [2].

Since the astrophysical factor at astrophysical energies is proportional to the square of the ANC the usage of $C^2_{\alpha d(01)} = 7.29$ fm^{-1} rather than $C^2_{\alpha d(01)} = 5.29$ fm^{-1} [2] leads to the increase of the astrophysical factor compared to the one obtained in [17] by almost 38%. It can be seen from Fig. 9 [21], where both astrophysical factors are shown in the logarithmic scale. We came to the main point of this paper, the ANC, which is the most crucial input in the calculation of the $S_{24}(E)$.

III. INVERSE SCATTERING PROBLEM, BOUND-STATE POTENTIAL AND ANC

In [21], as in [24], the s-wave scattering phase shift has been used to determine the bound-state potential, which generates the bound state wave function and, correspondingly, the ANC. However, it is well known from the inverse scattering problem [1] (see also discussion in Introduction) that there is infinite number of the phase-equivalent energy-independent potentials and to single out a unique potential one has to add two parameters (if only one bound state is present in the given partial wave): the binding energy and the ANC. In [21] the adopted potential reproduces experimental $\alpha - d$ binding energy but the ANC remained unfixed and its value was determined by the parameters of the adopted potential, which fits the s-wave $\alpha - d$ elastic scattering phase shift. Hence, the potential found in [21] is one out of an infinite set of the phase equivalent potentials and the ANC, which it generates, is not necessarily a correct one. However, there is another way of using the elastic scattering data which has been realized in [2], which has been discussed in Introduction. In this approach the statistical Padé approximation was used to interpolate the elastic scattering S-matrix element in the physical region and then directly continue it to the unphysical region in the energy (momentum) plane to find the pole corresponding to the ground state of ^{6}Li and to determine the residue in this pole through which the ANC is expressed. The obtained poles, whose locations depend on the order of the Pade approximants, were very close to the physical value of -1.476 MeV. It was found that the ANC values in the proximity of the experimental binding energy of the $\alpha - d$ system linearly depend on the binding energy. Using this linear extrapolation to the experimental binding energy the ANC $C_{\alpha d(01)} = 2.28$ fm$^{-1/2}$ was obtained [2], which is lower than $C_{\alpha d(01)} = 2.7$ fm$^{-1/2}$ following from the potential used in [21]. Once the ANC has been determined, according to the inverse scattering problem theorem [1], a unique bound-state potential can be found [12] from the elastic scattering data. Note that, if the energy independent potential satisfies the condition (assuming, for simplicity, that there is only one bound state in the given partial wave)

$$\lim_{r\to\infty} V^N(r)\, e^{2\kappa r} \to 0, \tag{6}$$

where κ is the bound state wave number, then such a potential is unique and its scattering amplitude can be analytically continued into the bound state pole [25]. Thus the ambiguity in the two-body potential calls for more thorough selection of the potential because eventually the adopted potential for the bound state will determine the overall normalization of the astrophysical factor for peripheral direct radiative capture processes.

If we would confine ourselves to the astrophysical energies then it would be enough to replace in the matrix element (3) the overlap function by its asymptotic term (5) and, evidently, the astrophysical factor will be proportional to $C^2_{\alpha d(01)}$. However, if we want to extend our calculations to higher energies including the resonance region and above we need to carry more accurate calculations. To compare our results with the $S_{24}(E)$ factor presented in [21] we adopt the same $\alpha - d$ scattering potential as in [21]. For the $\alpha - d$ bound state potential it would be logically reasonable to

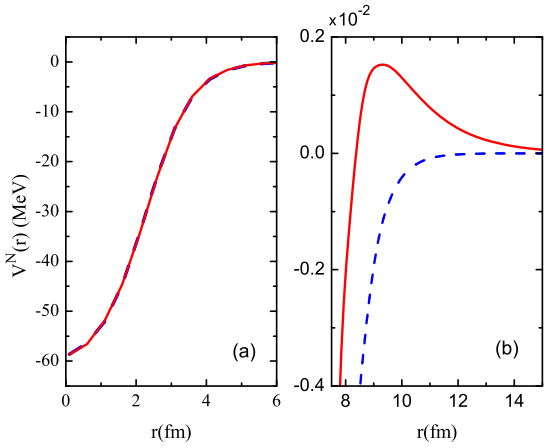

FIG. 1: (Color online) The red solid line is $\alpha - d$ bound state nuclear potential $V^N(r)$ [21] and the blue dashed line is $V_1^N(r)$.

use the bound state potential from [2, 26]. However, since it has quite complicated form, we simply modify the bound state potential used in [21] using the theorem of the inverse problem in scattering theory [1]. This theorem allows one to recover a phase equivalent potential to the Woods-Saxon potential used in [21] with arbitrary ANC. Assume that we adopt a nuclear potential $V^N(r)$, which together with the added Coulomb potential $V^C(r)$ fits the elastic $\alpha - d$ scattering phase shift in the $l = 0$ partial wave and the bound state wave function calculated with this potential has the ANC C. Then the phase-equivalent potential is given by

$$V_1^N(r) = V^N(r) - 2\,\frac{\mathrm{d}^2K(r)}{\mathrm{d}r^2}, \tag{7}$$

$$K(r) = \mathrm{Log}[1 + (\tau - 1)\int_0^r \mathrm{d}x\, x^2\, \varphi^2(x)]. \tag{8}$$

The bound state wave function in the potential $V_1(r) = V_1^N(r) + V^C(r)$ can be expressed in terms of the bound state wave function $\varphi(r)$ in the potential $V(r) = V^N(r) + V^C(r)$:

$$\varphi_1(r) = \tau^{1/2}\,\frac{\varphi(r)}{1 + (\tau - 1)\int_0^r \mathrm{d}x\, x^2\, \varphi^2(x)}. \tag{9}$$

From Eq. (9) one obtains that at $r = 0$ $\varphi_1(0)/\varphi(0) = \tau^{1/2}$, while $\lim\limits_{r\to\infty} \varphi_1(r)/\varphi(r) = \tau^{-1/2}$, which is the ratio of the corresponding ANCs. Let $V(r) = V^N(r) + V_C(r)$ be the nuclear Woods-Saxon plus Coulomb bound-state $\alpha - d$ potential adopted in [21], which generates the bound-state wave function with $C_{\alpha d(01)} = 2.7$ fm$^{-1/2}$. Then for $\tau = 1.378$ we obtain the wave function in the potential $V_1(r)$, which has the same ANC as obtained in [2] and used in [17]. In Fig. 1 both nuclear potentials are shown. Since the difference between the potentials is very small, panel (a), in panel (b) we show the tails of both potentials in the scale allowing to see the difference. As one can see in Fig. 2(a), the wave function $\varphi_1(r)$ with the smaller ANC has smaller tail but is higher than $\varphi(r)$ in the nuclear interior so that the total norm is conserved. In 2(b) we show the ratio $\varphi_1(r)/\varphi(r)$. Because both bound-state wave functions have one node at $r > 0$, there is also forbidden nodeless bound state for each potential with the binding energy 29.7 MeV.

Note that the existence of the potential $V_1^N(r)$, which is phase equivalent to the bound state potential found in [2, 26] with the same ANC, doesn't contradict to the inverse scattering theorem because the addition $-\frac{\mathrm{d}^2K(r)}{\mathrm{d}r^2}$ to the potential $V^N(r)$ asymptotically decays as $\exp(2\,\kappa_{\alpha\,d}\,r)$ violating condition (6). It doesn't allow one to use potential $V_1^N(r)$ for analytical extrapolation of the scattering amplitude generated by this potential to the bound state pole but we need this potential here only to generate the bound state wave function with correct amplitude of the tail, i.e. the ANC. Note that formally we can obtain the overlap function with the same ANC as in [2] by multiplying the bound-state wave function generated by the potential $V(r)$ on the square root of the proper spectroscopic factor. However, this procedure is not legitimate, as it has been discussed above, because potential $V(r)$ fits the elastic scattering

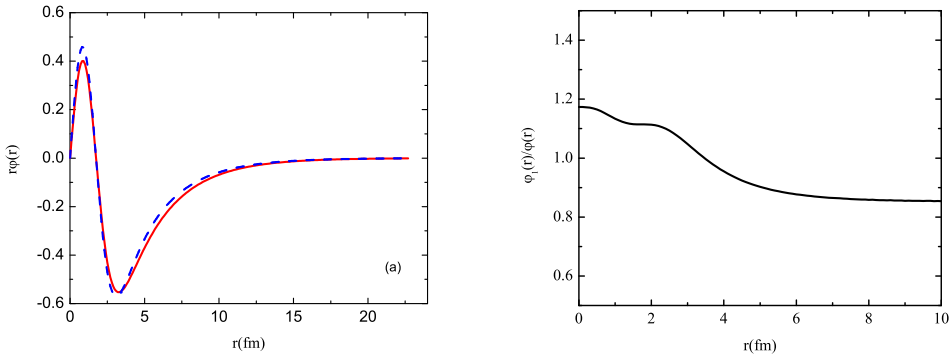

FIG. 2: (a) (Color online) The red solid line is $\alpha - d$ bound state wave function $\varphi(r)$ generated by the potential $V(r) = V^N(r) + V^C(r)$ and the blue dashed line is the bound state wave function $\varphi_1(r)$ generated by $V_1(r) = V_1^N(r) + V^C(r)$. The explanation see in the text. (b) The ratio of the bound-wave functions $\varphi_1(r)$ and $\varphi(r)$.

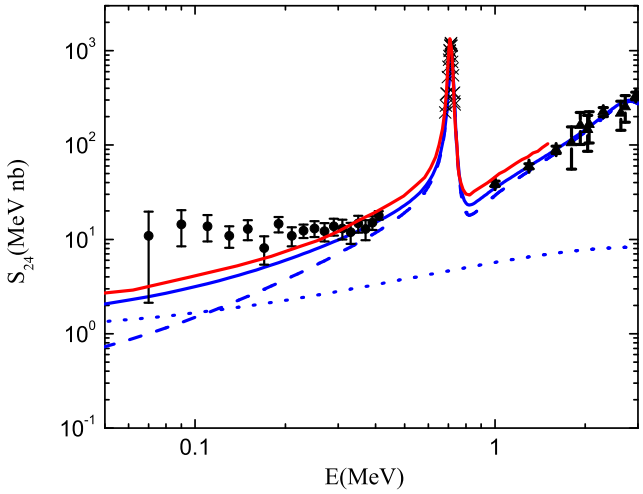

FIG. 3: (Color online) The astrophysical factors $S_{24}(E)$ for the radiative capture $\alpha + d \to {}^6\mathrm{Li} + \gamma$. Black dots are data from [20]; black crosses are data from [24]; black triangles are data from [27]. The red solid line is the $S_{24}(E)$ factor from [21]. The blue dotted line is $S_{24}(E1)$ factor, the blue dashed line is $S_{24}(E2)$ and the blue solid line is our total astrophysical factor $S_{24}(E)$.

data, i.e. it takes effectively into account many-body effects, and, hence, any additional spectroscopic factor would be superfluous. Incorrect ANC should be changed by changing the potential and not by multiplying the bound-state wave function by the square root of the spectroscopic factor.

IV. ASTROPHYSICAL $S_{24}(E)$ FACTOR AND REACTION RATES

In Fig. 3 we present two calculated $S_{24}(E)$ factors for the $\alpha + d \to {}^6\mathrm{Li} + \gamma$ radiative capture compared with the experimental data. One is the astrophysical factor from [21]. To calculate it we use the $\alpha - d$ bound-state wave function generated by the bound-state potential $V(r) = V^N(r) + V^C(r)$ and the scattering potential in the partial waves $l_i = 1, 2$ adopted in [21]. The second astrophysical factor is ours. To calculate it we use the bound-state potential $V_1(r) = V_1^N(r) + V^C(r)$ keeping the same scattering potential as in [21]. Thus, the only difference between

our $S_{24}(E)$ and the one from [21] is in the $\alpha - d$ bound-state wave functions. At astrophysically relevant energies, $E \leq 300$ keV, at which the $\alpha + d \to {}^6\mathrm{Li} + \gamma$ radiative capture is totally peripheral, the $S_{24}(E)$ is proportional to the square of the ANC $C^2_{\alpha d(01)}$ for the virtual decay ${}^6\mathrm{Li} \to \alpha + d$. Since the square of the ANC in [21] is higher than our one by 38%, correspondingly, the astrophysical factor from [21] at astrophysicall energies exceeds our $S_{24}(E)$ by $\sim 38\%$.

There is another important point to discuss. As one can see, our calculations, definitely, agree well with the experimental data [27] at energies larger than the resonance energy, where the calculations from [21] clearly overestimate the data. We note that usually with energy increase the radiative capture reactions become less peripheral, i.e. the nuclear interior becomes more important and the overall normalization of the astrophysical factors is determined not only by the ANC. However, the $\alpha + d \to {}^6\mathrm{Li} + \gamma$ process has quite specific features. At energies above the resonance, say $E \sim 1$ MeV, the $E2$ capture dominates over $E1$. For the former the transition is $l_i = 2 \to l = 0$. The Coulomb plus centrifugal barrier in the initial state of the reaction is dominated by the centrifugal part due to the angular momentum $l_i = 2$ in the entry channel of the reaction. For example, at $E = 1$ MeV and $r = 4$ fm the total barrier is 6.57 MeV with the centrifugal part 5.85 MeV. Such a high barrier compared to the relative $\alpha - d$ energy $E \sim 1$ MeV explains why the reaction is still peripheral at such high energies. For comparison we would like to mention that at astrophysically most effective energy $E \approx 70$ keV the $E1$ capture with $l_i = 1$ dominates. For this transition at $E = 70$ keV and $r = 4$ fm the total barrier is 2.67 MeV with the centrifugal barrier 1.95 MeV. This significant difference in the barriers at higher and low energies explains why the reaction under consideration remains peripheral even at energies $E \sim 1$ MeV. At energy $E = 1$ MeV our calculated $S_{24}(E)$ factor is about 30% lower than the one in [21] and, as we have mentioned, at energies $E \geq 1$ MeV perfectly reproduces higher energy data from [27] what is another compelling evidence that the ANC in [21] was overestimated. At resonance energies both calculations reproduce the data [24] very well. Finally, in Table I we present the $\alpha + d \to {}^6\mathrm{Li} + \gamma$ reaction rates, which are also systematically lower than those presented in [21].

V. SUMMARY

In this work we have demonstrated that a crucial quantity, which is necessary to pinpoint the $S_{24}(E)$ astrophysical factor at astrophysical energies, is the ANC for the virtual decay ${}^6\mathrm{Li} \to \alpha + d$. Due to the peripheral character of the $\alpha + d \to {}^6\mathrm{Li} + \gamma$ direct radiative capture, this ANC determines the overall normalizatiion of the astrophysical factor at astrophysically relevant energies. From our calculations and Fig. 3 we can see that at low energies the contribution from the isospin forbidden $E1$ transition dominates over the allowed $E2$ transition. For example, at $E = 70$ keV, which is the most effective energy, the contribution from the $E1$ transition to the total $S_{24}(E)$ astrophysical factor is about 60%. Even at $E = 100$ keV the $E1$ transition contributes about 52% to the total astrophysical factor. Meantime, even if the Coulomb breakup of ${}^6\mathrm{Li}$ would dominate, at $E = 70$ keV the $E1$ transition will be suppressed compared to the $E2$ by a factor of 60. It can hardly make possible to determine the total astrophysical factor from the ${}^6\mathrm{Li}$ experiment. Since the ANC is the only crucial information needed to calculate the $S_{24}(E)$ astrophysical factor at astrophysical energies, we call for more accurate measurements of the s-wave $\alpha - d$ elastic scattering phase shift at lower energies. It will help to extrapolate more accurately the data to the bound state pole to get more ANC for ${}^6\mathrm{Li} \to \alpha + d$. We note that the problem of determination of the two-body bound state potential from the elastic scattering phase shift is quite important in different applications of nuclear reaction theory, in particular, in Faddeev approach for reactions with composite particles.

Finally, the 40% decrease of the square of the ANC reported in this paper compared to the one adopted in [21] results in $\sim 40\%$ decrease of the $S_{24}(E)$ at astrophysical energies and corresponding decrease of the reaction rates. It makes even more difficult explanation of the discrepancy between the observed ${}^6\mathrm{Li}/{}^7\mathrm{Li}$ ratio and the Big Bang nucleosynthesis calculations invoking the $\alpha + d \to {}^6\mathrm{Li} + \gamma$ reaction [19].

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TABLE I: The rate for $\alpha + d \rightarrow {}^{6}\mathrm{Li} + \gamma$ reaction calculated using our astrophysical S-factor for the temperature range $10^{6}\mathrm{K} \leq \mathrm{T} \leq 10^{10}\mathrm{K}$.

T_9	$N_a\langle\sigma v\rangle$ $(cm^3\, mol^{-1}\, s^{-1})$	T_9	$N_a\langle\sigma v\rangle$ $(cm^3\, mol^{-1}\, s^{-1})$
0.001	6.467×10^{-30}	0.260	6.823×10^{-04}
0.002	1.857×10^{-23}	0.270	7.876×10^{-04}
0.003	2.470×10^{-20}	0.280	9.032×10^{-04}
0.004	2.286×10^{-18}	0.290	1.030×10^{-03}
0.005	5.693×10^{-17}	0.300	1.167×10^{-03}
0.006	6.592×10^{-16}	0.310	1.317×10^{-03}
0.007	4.651×10^{-15}	0.320	1.478×10^{-03}
0.008	2.327×10^{-14}	0.330	1.652×10^{-03}
0.009	9.067×10^{-14}	0.340	1.840×10^{-03}
0.010	2.923×10^{-13}	0.350	2.040×10^{-03}
0.011	8.127×10^{-13}	0.360	2.254×10^{-03}
0.012	2.008×10^{-12}	0.370	2.482×10^{-03}
0.013	4.508×10^{-12}	0.380	2.725×10^{-03}
0.014	9.343×10^{-12}	0.390	2.983×10^{-03}
0.015	1.811×10^{-11}	0.400	3.256×10^{-03}
0.016	3.318×10^{-11}	0.500	6.930×10^{-03}
0.017	5.787×10^{-11}	0.600	1.271×10^{-02}
0.018	9.676×10^{-11}	0.700	2.148×10^{-02}
0.019	1.559×10^{-10}	0.800	3.462×10^{-02}
0.020	2.432×10^{-10}	0.900	5.385×10^{-02}
0.025	1.538×10^{-09}	1.000	8.079×10^{-02}
0.030	6.277×10^{-09}	1.100	1.166×10^{-01}
0.035	1.929×10^{-08}	1.200	1.618×10^{-01}
0.040	4.870×10^{-08}	1.300	2.164×10^{-01}
0.045	1.066×10^{-07}	1.400	2.797×10^{-01}
0.050	2.093×10^{-07}	1.500	3.508×10^{-01}
0.060	6.375×10^{-07}	1.600	4.288×10^{-01}
0.070	1.554×10^{-06}	1.700	5.126×10^{-01}
0.080	3.245×10^{-06}	1.800	6.002×10^{-01}
0.090	6.057×10^{-06}	1.900	6.915×10^{-01}
0.100	1.038×10^{-05}	2.000	7.854×10^{-01}
0.110	1.665×10^{-05}	2.100	8.808×10^{-01}
0.120	2.533×10^{-05}	2.200	9.773×10^{-01}
0.130	3.690×10^{-05}	2.300	$1.074 \times 10^{+00}$
0.140	5.185×10^{-05}	2.400	$1.171 \times 10^{+00}$
0.150	7.071×10^{-05}	2.500	$1.268 \times 10^{+00}$
0.160	9.398×10^{-05}	3.000	$1.745 \times 10^{+00}$
0.170	1.222×10^{-04}	3.500	$2.210 \times 10^{+00}$
0.180	1.559×10^{-04}	4.000	$2.673 \times 10^{+00}$
0.190	1.954×10^{-04}	4.500	$3.145 \times 10^{+00}$
0.200	2.416×10^{-04}	5.000	$3.631 \times 10^{+00}$
0.210	2.946×10^{-04}	6.000	$4.645 \times 10^{+00}$
0.220	3.552×10^{-04}	7.000	$5.689 \times 10^{+00}$
0.230	4.238×10^{-04}	8.000	$6.725 \times 10^{+00}$
0.240	5.008×10^{-04}	9.000	$7.723 \times 10^{+00}$
0.250	5.868×10^{-04}	10.00	$8.664 \times 10^{+00}$

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