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# Neutron charge radius and the neutron electric form factor

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For nearly forty years, the Galster parametrization has been employed to fit existing data for the neutron electric form factor,  $G_E^n$ , vs. the square of the four-momentum transfer,  $Q^2$ . Typically this parametrization is constrained to be consistent with experimental data for the neutron charge radius. However, we find that the Galster form does not have sufficient freedom to accommodate reasonable values of the radius without constraining or compromising the fit. In addition, the  $G_E^n$  data are now at sufficient precision to motivate a two-parameter fit (or three parameters if we include thermal neutron data). Here we present a modified form of a two-dipole parametrization that allows this freedom and fits both  $G_E^n$  (including recent data at both low and high four-momentum transfer) and the charge radius well with simple, well-defined parameters. Analysis reveals that the Galster form is essentially a two-parameter approximation to the two-dipole form, but becomes degenerate if we try to extend it naturally to three parameters.

## I. INTRODUCTION

The electromagnetic form factors of nucleons provide critical information about the distribution of electric charge and magnetization within these fundamental particles of nuclear physics [1, 2]. For the specific case of the neutron, the non-uniform charge distribution leads to a finite value for the mean squared charge radius,  $\langle r_n^2 \rangle$ , which corresponds to the second moment of the Breit-frame distribution in position space,  $\rho_B(r)$ . The charge radius has been measured via the scattering of low energy (0.1 eV to 1000 eV) neutrons from high-Z, diamagnetic atoms [3, 4]. It can also be determined from the scattering of high energy electrons (0.1 GeV to 3 GeV) from effective neutron targets. In the former case, the charge radius is determined from measurements of the neutron-electron scattering length, whereas in the latter case it is determined from the slope of the neutron electric form factor,  $G_E^n$ , in the limit of zero four-momentum transfer,  $Q^2$ . “Double-polarization” experiments, which employ both polarized electrons and polarized targets and/or recoil polarimeters, have substantially improved our knowledge of  $G_E^n(Q^2)$ . These results provide more detailed information about the spatial extent of the positive and negative charge [5–7]. The shape of  $G_E^n(Q^2)$  is compared to theoretical models of the nucleon, but is also parametrized for use in other investigations [1, 8]. Our discussion here focuses on an issue in the relationship of the charge radius determined from neutron-electron scattering with the long-standing but nevertheless phenomenological Galster parametrization.

In 1971, the Galster parametrization [9] was introduced to fit data for  $G_E^n$  vs.  $Q^2$ . After extensive new experimental results with increased accuracy and range in  $Q^2$ , this form has continued to be employed. For example, in 2004 Kelly used the Galster form, written as

$$G_E^n(Q^2) = \frac{A\tau}{(1+B\tau)} G_D(Q^2) \quad (1)$$

where  $\tau = Q^2/(4m_p^2c^2)$  and  $m_p=0.9383$  GeV/ $c^2$ . The

dipole form factor is  $G_D(Q^2) = (1 + Q^2/\Lambda^2)^{-2}$  with  $\Lambda^2 = 0.71$  (GeV/ $c$ )<sup>2</sup>, and  $A$  and  $B$  are fitted parameters [8]. We will refer to this as the Galster form, but note that it is the approach employed by Kelly, with two free parameters. The parameter  $A$  can be related to  $\langle r_n^2 \rangle$  with the relationship

$$\langle r_n^2 \rangle = -6 \left. \frac{dG_E^n}{dQ^2} \right|_{Q^2=0} = \frac{-3A}{2m_p^2c^2}. \quad (2)$$

In Fig. 1 we show the world’s data for  $G_E^n$  from double-polarization experiments, including recent experiments at low[10] and high momentum transfer[11]. Our goal here will be to fit these data and compare the results to independent determinations of the charge radius from neutron-electron scattering. Towards this end, we will consider both fits in which the slope of  $G_E^n$  is allowed to vary freely and fits in which an experimental value for the slope is included as a datum in the fit. In the Appendix, we list the specific  $G_E^n$  values and uncertainties employed in the plots and fits below, along with the references.

## II. FITTING $G_E^n$ WITH THE GALSTER FORM

If we fit to Eq. (1) and allow both parameters to vary freely, ie. without constraint from experimental charge radius determinations, we obtain  $\langle r_n^2 \rangle = -0.0935(48)$  fm<sup>2</sup> with a reduced  $\chi^2$  of 0.90. The difference between the fitted value and the experimental value of Ref. [3] for  $\langle r_n^2 \rangle$  ( $-0.1149(35)$  fm<sup>2</sup>) is  $0.0214(64)$  fm<sup>2</sup>, which is 3.3 times its uncertainty. As discussed in Ref. [4] and references therein, there are two groups of experimental charge radius determinations that differ by more than their respective uncertainties. For the Dubna group of charge radius determinations  $\langle r_n^2 \rangle = -0.134(3)$  fm<sup>2</sup> [4], which yields a difference from the fitted value of 6.6 times the uncertainty in the difference. If instead we include a datum for  $\langle r_n^2 \rangle = -0.1149$  fm<sup>2</sup> or  $\langle r_n^2 \rangle = -0.134$  fm<sup>2</sup> in the fit, we obtain reduced  $\chi^2$  values of 1.27 or 2.13, respectively. Hence there is disagreement between the neutron charge

radius extracted from the Galster form and both the experimental results of Ref. [4] and Ref. [3]. The simplest conclusion from this disagreement between the charge radius extracted from electron and thermal neutron scattering is that there is an issue with the shape of the fit function at low  $Q^2$ , where  $G_E^n$  is sensitive to the charge radius. This issue indicates that a better phenomenological form is required that provides a parametrization for  $G_E^n$  with the freedom to accommodate the charge radius determined from thermal neutron scattering.

There was already some evidence for an issue even before the addition of new data from Refs. [10] and [11]. Fitting to the Galster form with two free parameters, but without including these new recent data, yields  $\langle r_n^2 \rangle = -0.095(8) \text{ fm}^2$  with a reduced  $\chi^2 = 0.96$ . If we include a datum for  $\langle r_n^2 \rangle = -0.1149 \text{ fm}^2$  or  $\langle r_n^2 \rangle = -0.134 \text{ fm}^2$  in the fit, we obtain reduced  $\chi^2$  values of 1.13 or 1.61, respectively. Even without these new data the magnitude of  $G_E^n$  at its peak was already fairly well established, and the essence of the issue with the Galster fit is that this magnitude over-constrains the slope at the origin.

The primary results of fitting with the two-parameter Galster form are listed in Table I. Next we will consider alternative forms. First we consider the physically motivated, Galster-like parametrization discussed in Ref. [12]. The parameters in this fit are written slightly differently than in Eq. (1), but for our purposes this fit corresponds to fixing the parameter  $B$  at 6.65. However,  $\Lambda^2$  is now allowed to vary. Including a datum from Ref. [3] we obtain  $A=1.670(54)$ , which yields  $\langle r_n^2 \rangle = -0.1107(32) \text{ fm}^2$ , and  $\Lambda^2 = 1.03(5) (\text{GeV}/c)^2$ , with a reduced  $\chi^2 = 1.20$ , slightly better than the corresponding Galster fit. If the parameter  $B$  is allowed to vary, the fit is not stable and converges to either  $A=1.730(52)$ ,  $B = 13.0(2.0)$  and  $\Lambda^2 = 1.76(27) (\text{GeV}/c)^2$  or  $A=1.725(52)$ ,  $B = 0.67(36)$  and  $\Lambda^2 = 0.468(37) (\text{GeV}/c)^2$ , with reduced  $\chi^2$  values of 0.88 or 0.90, respectively. Either value of  $B$  is far from the value determined in Ref. [12], hence the fit to the data do not support the original physics that motivated it. Furthermore, despite the improved  $\chi^2$  the instability of the fit is undesirable. The origin of this issue is discussed in Sec. IV.

### III. FITTING $G_E^n$ WITH TWO-DIPOLE FORMS

To investigate a two-dipole approach, we will employ an early parametrization for  $G_E^n(Q^2)$  [13]. In the notation of this work, this form (which we refer to as the Bertozzi form) was written

$$G_E^n(Q^2) = \frac{1}{(1 + Q^2 r_1^2/12)^2} - \frac{1}{(1 + Q^2 r_2^2/12)^2} \quad (3)$$

where  $Q^2$  is in units of  $\text{fm}^{-2}$ . Each dipole form was meant to represent the Fourier transform of a exponential charge distribution, where the total charge in the positive (negative) distribution was equal to the electronic

TABLE I: Results of fitting  $G_E^n$ . The column labelled " $\langle r_n^2 \rangle^d$ " lists the reference for the  $\langle r_n^2 \rangle$  datum included in the fit. For the Galster form, the parameters  $A$  and  $B$  are listed, along with the resulting value for  $\langle r_n^2 \rangle$ . For the Bertozzi and mod-Ber (modified Bertozzi) forms, the parameters  $\langle r_n^2 \rangle$ ,  $r_{av}$ , and  $a$  are listed (for the Bertozzi form the normalization parameter  $a$  is fixed at unity).  $\chi_{red}^2$  is the reduced  $\chi^2$  for the fit. "dof" refers to the number of degrees of freedom for each fit.

| form    | Eq. | $\langle r_n^2 \rangle^d$ | A         | B        | $\langle r_n^2 \rangle [\text{fm}^2]$ | $\chi_{red}^2$ | dof |
|---------|-----|---------------------------|-----------|----------|---------------------------------------|----------------|-----|
| Galster | (1) | —                         | 1.409(82) | 2.09(39) | -0.0935(54)                           | 0.90           | 20  |
| Galster | (1) | [3]                       | 1.664(47) | 3.27(32) | -0.1104(31)                           | 1.27           | 21  |
| Galster | (1) | [4]                       | 1.950(43) | 4.82(36) | -0.1293(29)                           | 2.13           | 21  |

| form     | Eq. | $\langle r_n^2 \rangle^d$ | $r_{av} [\text{fm}]$ | $a$       | $\langle r_n^2 \rangle [\text{fm}^2]$ | $\chi_{red}^2$ | dof |
|----------|-----|---------------------------|----------------------|-----------|---------------------------------------|----------------|-----|
| Bertozzi | (3) | —                         | 0.709(19)            | 1         | -0.0906(64)                           | 0.94           | 20  |
| Bertozzi | (3) | [3]                       | 0.763(11)            | 1         | -0.1107(32)                           | 1.33           | 21  |
| Bertozzi | (3) | [4]                       | 0.809(10)            | 1         | -0.1295(29)                           | 2.14           | 21  |
| mod-Ber  | (4) | [3]                       | 0.856(32)            | 0.115(20) | -0.1147(35)                           | 0.91           | 20  |
| mod-Ber  | (4) | [4]                       | 0.950(30)            | 0.095(11) | -0.1337(30)                           | 0.96           | 20  |

charge  $e$  ( $-e$ ). The two radii  $r_1$  and  $r_2$  were rewritten as  $r_1^2 = r_{av}^2 + \frac{1}{2}\langle r_n^2 \rangle$  and  $r_2^2 = r_{av}^2 - \frac{1}{2}\langle r_n^2 \rangle$ , where  $r_{av}^2$  is the average of the squared radii for the two distributions and  $\langle r_n^2 \rangle$  is the mean squared charge radius. By constraining  $\langle r_n^2 \rangle$  to an experimental value,  $G_E^n$  vs.  $Q^2$  was parametrized using the single parameter  $r_{av}$ . Allowing the charge radius to vary (two parameters), we find that this approach yields results similar to the Galster form. If the charge radius datum from Ref. [3] is included, we obtain  $\langle r_n^2 \rangle = -0.1107(32) \text{ fm}^2$  and  $r_{av}=0.763(11) \text{ fm}$ . The reduced  $\chi^2$  is 1.33, slightly higher than that obtained for the Galster fit for the same conditions. The values of  $r_1$  and  $r_2$  are 0.726(11) fm and 0.800(11) fm, respectively. A charge radius of 0.0906(64)  $\text{fm}^2$  with a reduced  $\chi^2$  of 0.94 is obtained if the charge radius constraint is removed, and a reduced  $\chi^2$  value of 2.14 is obtained if the experimental charge radius result from Ref. [4] is included as a datum. Hence for our current purposes the Bertozzi form has both the same capabilities and limitations as the Galster form. The results for fitting with the Bertozzi form are summarized in Table I.

To obtain greater freedom for this two-dipole form, we consider a modified version of the Bertozzi form, which is similar to the BLAST form recently employed[10]. It is also similar to that presented by Friedrich and Walcher[14] for the smooth part of  $G_E^n$ , but their fitting included additional terms to address a possible bump in  $G_E^n$ . The modified form we employ is given by

$$G_E^n(Q^2) = \frac{a}{(1 + Q^2 r_1^2/12)^2} - \frac{a}{(1 + Q^2 r_2^2/12)^2}. \quad (4)$$

The two rms radii  $r_1$  and  $r_2$  are now given by  $r_1^2 = r_{av}^2 + \langle r_n^2 \rangle/2a$  and  $r_2^2 = r_{av}^2 - \langle r_n^2 \rangle/2a$ . The data are fit with three parameters: the charge radius  $\langle r_n^2 \rangle$ , the av-

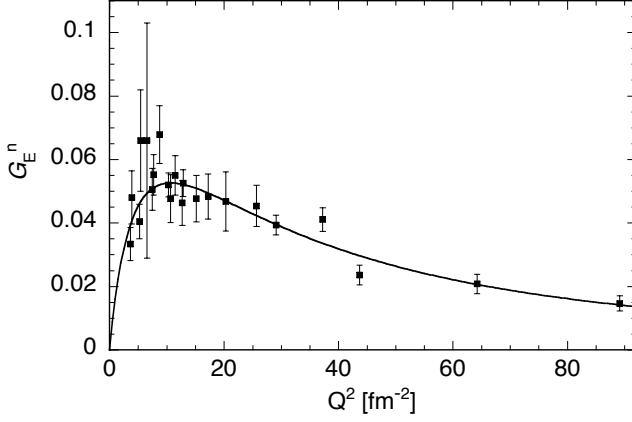


FIG. 1: The world's double-polarization data for  $G_E^n$  including the two new experimental results in Ref. [10] and Ref. [11], along with a fit to Eq. (4) that includes the experimental charge radius of Ref. [3] as a datum. The form of the fit is the modified version of the Bertozzi fit discussed in the text. The values of the fitted parameters are  $\langle r_n^2 \rangle = -0.1147(35) \text{ fm}^2$ ,  $r_{av} = 0.856(32) \text{ fm}$ , and  $a = 0.115(20)$ .

average rms radius  $r_{av}$ , and the normalization parameter  $a$ . Note that the charge radius has units  $e \cdot \text{fm}^2$ , but is typically written in units where  $e = 1$ . The original Bertozzi form represented two dipoles of unit charge  $e$ . For this modified fit, the total charge for each dipole is  $q = ae$ , hence in the denominator the charge radius is normalized by  $a$  to keep the correct slope at  $Q^2 = 0$ . On the other hand,  $r_{1,2}^2 = \int dq r^2 / \int dq$  is truly a distance squared and is already normalized by charge. This is formed for each dipole separately and for the average of the two,  $r_{av}^2 = (r_1^2 + r_2^2)/2$ , but not for the neutron charge radius  $\langle r_n^2 \rangle = ae(r_1^2 - r_2^2)$ , which cannot be normalized since  $\int dq = 0$ . In the modified Bertozzi form the average spatial extent and separations of the positive and negative distributions are given by  $r_{av}$  and  $(\langle r_n^2 \rangle / a)^{1/2}$ , respectively.

Fitting  $G_E^n(Q^2)$  with the datum from Ref. [3] for the charge radius yields  $\langle r_n^2 \rangle = -0.1147(35) \text{ fm}^2$ ,  $a = 0.115(20)$  and  $r_{av} = 0.856(32) \text{ fm}$ , with a reduced  $\chi^2$  of 0.91. The fitted value of the charge radius is essentially identical to the experimental datum that was included, with a similar uncertainty, and the extracted value for the average radius for the neutron is well-defined. The reduced  $\chi^2$  is improved over the two-parameter fits, with well-defined parameters that have reasonable uncertainties. This fit is shown in Fig. 1. The dramatic reduction in the normalization parameter from unity to 0.115 allows for a much greater difference between the two radii; the values of  $r_1$  and  $r_2$  are 0.48 fm and 1.11 fm, respectively. Results for fitting with the modified Bertozzi form are listed in Table I.

In Fig. 2 we show the fits for  $G_E^n$ , for both the original and modified Bertozzi fitting forms along with the form factors for the individual positive and negative compo-

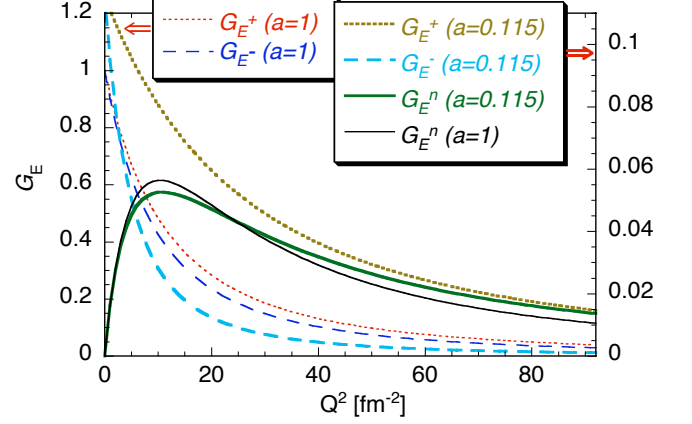


FIG. 2: (Color online) Fits for  $G_E^n$  for both the original and modified Bertozzi fitting forms, along with the form factors for the individual positive and negative components. For the positive and negative components of the original form, the y-axis scale is on the left. For the positive and negative components of the modified form, as well as  $G_E^n$  for both forms, the y-axis scale is on the right.

nents. (For these plots and those of the Fourier transforms below we show the fits for which the charge radius datum from Ref. [3] was included.)

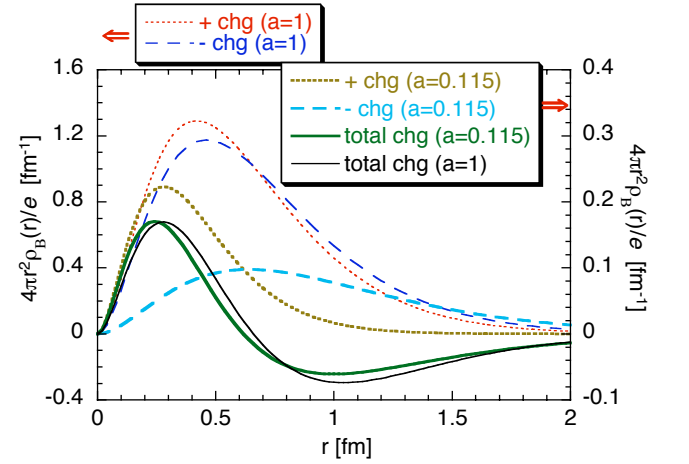


FIG. 3: (Color online) Breit-frame Fourier transforms  $4\pi r^2 \rho_B(r)/e$  for the original and modified Bertozzi fits. As discussed in the text, the positive, negative, and total transforms are shown for each fit. For the positive and negative components of the original form, the y-axis scale is on the left. For the positive and negative components of the modified form, as well as for the total transform for both forms, the y-axis scale is on the right.

In Fig. 3 we show the Breit-frame Fourier transforms for the original Bertozzi fit ( $a=1$  and  $r_{av}=0.763 \text{ fm}$ ) and the modified Bertozzi fit ( $a=0.115 \text{ fm}$  and  $r_{av}=0.856 \text{ fm}$ ). Similar transforms are shown in a recent study of the

role of mesons in the electromagnetic form factors of the nucleon [15].

#### IV. DISCUSSION OF FITTING FORMS

The form of the Galster fit is actually closely related to the two-dipole fit in the same way a dipole comes from two oppositely charged monopoles. The form for two oppositely charged dipoles, given by

$$G_{2\text{-dipole}} = \frac{a}{(1 + b_1\tau)^2} - \frac{a}{(1 + b_2\tau)^2} \quad (5)$$

with comparable parameters  $b_1$  and  $b_2$  can be approximated by

$$\frac{d}{db} \frac{a}{(1 + b\tau)^2} \Delta b = \frac{2a \Delta b \tau}{(1 + \bar{b}\tau)^3}. \quad (6)$$

The dimensionless parameters  $\bar{b}$  and  $\Delta b$  are given by

$$\Delta b = (b_2 - b_1) = \frac{-\langle r_n^2 \rangle}{3\lambda_p^2 a} \quad \text{and} \quad (7)$$

$$\bar{b} = \frac{1}{2}(b_2 + b_1) = \frac{r_{av}^2}{3\lambda_p^2}, \quad (8)$$

where  $\lambda_p = \hbar/m_p c = 0.2103$  fm is the reduced Compton wavelength of the proton. This is essentially the Galster form

$$G_{\text{Galster}} = \frac{A\tau}{1 + B\tau} G_D = \frac{A\tau}{(1 + B\tau)} \frac{1}{(1 + D\tau)^2} \quad (9)$$

if  $B \approx D \approx \bar{b}$ . The dimensionless parameters are

$$A = 2a\Delta b = \frac{-2\langle r_n^2 \rangle}{3\lambda_p^2} \quad \text{and} \quad (10)$$

$$D = \frac{4m_p^2 c^2}{\Lambda^2} = \frac{r_D^2}{3\lambda_p^2}, \quad (11)$$

where  $G_D = (1 + D\tau)^{-2}$ . The dipole radius  $r_D = 0.81$  fm (corresponding to  $\Lambda^2 = 0.71$  (GeV/c)<sup>2</sup>) yields  $D = 4.96$ , which is roughly comparable to the values of  $B$  listed in Table I, as well as the original Galster parameter  $B = 5.6$ . This has the correct asymptotic dependence as  $G \approx A\tau$  at low  $Q^2$  and  $G \approx G_D$  at high  $Q^2$ . Not relying on the dipole form factor or two dipoles, the general expansion would have been  $A\tau/(1 + B\tau + C\tau^2 + D\tau^3)$  which has more parameters than can be fit from the data [8]. We can fit to this general expansion by dropping the term in  $D$ , and obtain  $A=1.723(52)$ ,  $B=13.6(1.8)$ ,  $C=90(8)$  with a reduced  $\chi^2=1.03$  (for the datum in Ref. [3]). Although this fit can accomodate the charge radius and yields well-defined fit parameters, it does not have the proper  $Q^{-4}$  dependence at high  $Q^2$  [8] and does not have the simple form of the modified Bertozzi fit.

TABLE II: Comparison of four values for  $A$  obtained from use of the three relationships in Eq. (14), based on the fits for which the experimental value for  $\langle r_n^2 \rangle$  from Ref. [3] was included. For the line labelled “2-par”, the values were determined from the fitted values of  $A$  and  $B$  (Galster) and by using Eq. (8) and (7) to obtain  $\bar{b}$  and  $\Delta b$  from  $\langle r_n^2 \rangle$  and  $r_{av}$  (Bertozzi). For the line labelled “3-par”, we used Eqs. (10) and (11) to obtain  $D$  from  $\Lambda$  (Galster-like, Sec. II) and also the fitted value of  $a$  (modified Bertozzi, Table I).

|          | $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|----------|-------|-------|-------|-------|
| 2-par    | 1.66  | 1.67  | 2.80  | 2.77  |
| 3-par(+) | 1.72  | 1.73  | 1.13  | 1.29  |
| 3-par(-) | 1.73  | 1.73  | -1.98 | -2.07 |

To account for the difference between  $B$  and  $D$ , one can expand the Galster form in powers of  $\tau/(1 + D\tau)$  using the relation  $(1 + B\tau) = (1 + D\tau)(1 - \frac{D-B}{1+D\tau}\tau)$ :

$$G_{\text{Galster}} = A\tau G_D^{3/2} \times \left( 1 - (D - B)\tau G_D^{1/2} + (D - B)^2 \tau^2 G_D + \dots \right), \quad (12)$$

where  $(D - B)\tau/(1 + D\tau)$  is small for experimentally accessible values of  $\tau$ . This can be compared to the expansion of two dipoles, where we use the approach above to expand  $G_{2\text{-dipole}} = a/(1 + b_1\tau)^2 - a/(1 + b_2\tau)^2$  and combine terms to express it as a function of  $\bar{b}$  and  $\Delta b$ :

$$G_{2\text{-dipole}} = 2a\Delta b\tau G_D^{3/2} \left( 1 + 3(D - \bar{b})\tau G_D^{1/2} + (6(D - \bar{b})^2 + \frac{1}{2}\Delta b^2)\tau^2 G_D + \dots \right). \quad (13)$$

Comparing each term and simplifying we get the three equalities

$$A = 2a\Delta b \quad (14)$$

$$D - B = 3(D - \bar{b}), \quad \text{and} \quad (15)$$

$$3(D - \bar{b})^2 = \frac{1}{2}\Delta b^2. \quad (16)$$

While it may seem reasonable to extend the Galster form to a three parameter fit for  $A$ ,  $B$ , and  $D$ , the third equality is quadratic and has two solutions for  $D(\bar{b}, \Delta b)$ . Thus the Galster fit becomes degenerate as  $(D - B) \rightarrow 0$ , and is not useful as a three-parameter fit. Manipulation of Eq. (14) leads to four different determinations of  $A$ , which should all be equal, up to a sign, to  $-2\langle r_n^2 \rangle / 3\lambda_p^2$  (1.73 for fits that employ the experimental value for  $\langle r_n^2 \rangle$  from Ref. [3]). These four values are given by  $A_1 = A$ ,  $A_2 = 2a\Delta b$ ,  $A_3 = \pm 24^{1/2}a(D - \bar{b})$ , and  $A_4 = \pm (8/3)^{1/2}a(D - B)$ . We compare them in Table II, with results shown for the two-parameter fits listed in Table I and the two three-parameter Galster-like fits discussed at the end of Sec. II. One could say that the Galster fit has done so well over the years because it is the lowest order approximation to a two-dipole fit. The two-dipole fit describes a positive and negative charge distribution, where the average  $\langle r_n^2 \rangle$  of the two is approximately equal to  $r_D^2$  for the dipole distribution  $G_D$ .

## V. CONCLUSION

In summary, neither the Galster nor the Bertozzi two-parameter forms provide the freedom needed to simultaneously fit  $G_E^n(Q^2)$  and the experimental values for the charge radius. The three-parameter, two-dipole form (modified Bertozzi, Eq. 4) is a simple form that is consistent with experimental data for both  $G_E^n(Q^2)$  and  $\langle r_n^2 \rangle$ , yields parameters with reasonable values and low fit uncertainties, and has an improved reduced  $\chi^2$  as compared to the two-parameter Galster or two-dipole (Bertozzi) fits. An experimental program that aims to determine the charge radius using a completely different method should provide new information on the charge radius [16].

## ACKNOWLEDGEMENTS

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## Appendix A: $G_E^n(Q^2)$ data

In Table III we list double-polarization data for  $G_E^n(Q^2)$ . The values in the column labelled  $G_E^n$  were used for the fits discussed in this paper. Following Ref. [17], we used the results of Ref. [18] and Ref. [19], which supersede that of Ref. [20] and Ref. [21], respectively. In addition, we have used the result from the analysis in Ref. [22] of the experiment in Ref. [23]. We have also followed Ref. [17] in not including the pioneering results of Refs. [24–26] because they were not corrected for nuclear interaction effects. For weighting the fits, we employed the uncertainties listed, which were obtained by adding the reported statistical and systematic uncertainties in quadrature.

The column labelled  $G_M^n$  lists the source for the  $G_M^n$  value employed to determine  $G_E^n$ , as reported in the corresponding reference. We found that re-extracting  $G_E^n$  values using a specific parameterization[8] for  $G_M^n$  had

a negligible effect on our fitting results. Nevertheless, we provide these re-extracted values in the column labelled  $G_E^n(\text{consistent})$ . Where “ratio” is listed for  $G_M^n$ ,  $G_E^n(\text{consistent})$  was determined by simply multiplying the reported  $G_E^n/G_M^n$  ratio by our chosen  $G_M^n$ . Where a reference or “dipole” is listed,  $G_E^n(\text{consistent})$  was determined by applying a correction to the reported  $G_E^n$ . Where nothing is listed for  $G_M^n$ , it was not clear how to perform the correction from the information available in the original reference.

TABLE III: Double-polarization data for  $G_E^n(Q^2)$ . The contents of the table are discussed in the text.

| $Q^2$ (GeV/c) <sup>2</sup> | $G_E^n$      | reference | $G_M^n$ | $G_E^n(\text{consistent})$ |
|----------------------------|--------------|-----------|---------|----------------------------|
| 0.142                      | 0.0334(52)   | [10]      | ratio   | 0.0337                     |
| 0.15                       | 0.0481(84)   | [19]      | -       | -                          |
| 0.203                      | 0.0405(54)   | [10]      | ratio   | 0.0405                     |
| 0.21                       | 0.066(16)    | [29]      | dipole  | 0.0637                     |
| 0.255                      | 0.066(37)    | [30]      | [27]    | 0.0624                     |
| 0.291                      | 0.0506(66)   | [10]      | ratio   | 0.0502                     |
| 0.30                       | 0.0552(64)   | [31]      | [28]    | 0.0550                     |
| 0.34                       | 0.0679(91)   | [19]      | -       | -                          |
| 0.40                       | 0.0520(38)   | [22]      | -       | -                          |
| 0.415                      | 0.0477(75)   | [10]      | ratio   | 0.0465                     |
| 0.447                      | 0.0550(62)   | [32]      | ratio   | 0.0549                     |
| 0.495                      | 0.04632(704) | [33]      | dipole  | 0.0468                     |
| 0.50                       | 0.0526(42)   | [34]      | [28]    | 0.0528                     |
| 0.59                       | 0.0477(73)   | [31]      | [28]    | 0.0480                     |
| 0.67                       | 0.0484(71)   | [18]      | [28]    | 0.0486                     |
| 0.79                       | 0.0468(93)   | [31]      | [28]    | 0.0469                     |
| 1.00                       | 0.0454(65)   | [34]      | [28]    | 0.0451                     |
| 1.132                      | 0.0394(31)   | [32]      | ratio   | 0.0397                     |
| 1.450                      | 0.0411(37)   | [32]      | ratio   | 0.0417                     |
| 1.72                       | 0.0236(31)   | [11]      | ratio   | 0.0246                     |
| 2.48                       | 0.0208(31)   | [11]      | ratio   | 0.0206                     |
| 3.41                       | 0.0147(24)   | [11]      | ratio   | 0.0141                     |

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