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Phys. Rev. C **83**, 054912 — Published 27 May 2011

DOI: 10.1103/PhysRevC.83.054912

Elliptic flow in $\sqrt{s} = 200$ GeV Au+Au collisions and $\sqrt{s} = 2.76$ TeV Pb+Pb collisions: insights from viscous hydrodynamics + hadron cascade hybrid model

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(Dated: April 21, 2011)

Using the newly developed hybrid model `VISHNU` which connects viscous hydrodynamics with a hadron cascade model, we study the differential and integrated elliptic flow v_2 at different centrality bins for 200 A GeV Au+Au collisions and 2.76 A TeV Pb+Pb collisions. We find that the *average* Quark Gluon Plasma (QGP) specific shear viscosity η/s slightly increases from Relativistic Heavy Ion Collider (RHIC) to Large Hadron Collider (LHC) energies. However, a further study assuming different temperature dependencies for $(\eta/s)_{\mathrm{QGP}}$ shows that one cannot uniquely constrain the form of $(\eta/s)_{\mathrm{QGP}}(T)$ by fitting the spectra and v_2 alone. Based on our current understanding, the question whether the QGP fluid is more viscous or more perfect in the temperature regime reached by LHC energies is still open.

PACS numbers: 25.75.-q, 12.38.Mh, 25.75.Ld, 24.10.Nz

The first heavy-ion data from the Large Hadron Collider (LHC) have revealed many phenomena that are very similar to those seen at the Relativistic Heavy-Ion Collider (RHIC) at lower beam energies, such as elliptic flow and jet energy-loss. The question arises whether the increased reach in temperature and energy-density attainable at the LHC allows for the identification of systematic trends, for example in the temperature dependence of the specific shear viscosity η/s of the produced Quark-Gluon Plasma (QGP).

Measurements by the ALICE collaboration for $2.76\,A$ TeV Pb+Pb collisions shows that the charged hadron multiplicity density is about a factor of 2.2 higher than the one from 200 A GeV Au+Au collisions [1, 2]. Assuming a similar QGP thermalization time at RHIC and LHC energies and a linear relationship between the final multiplicity and initial entropy, one finds that the initial temperature of the QGP fireball at lower LHC energies is about 30% larger than the one at top RHIC energies. Meanwhile, the differential elliptic flow $v_2(p_T)$ of charged hadrons measured by the ALICE collaboration as a function of transverse momentum p_T (using the 4-particle cumulant method) is, up to transverse momenta of 3 GeV/c, nearly identical to that measured by the STAR collaboration at RHIC, independent of collision centrality [3]. When integrated over the transverse momentum, on the other hand, the total v_2 from ALICE is about 30% higher than that from STAR, due to an increase of the mean p_T for the LHC spectra [3].

The larger integrated charged hadron elliptic flow at the LHC implies a larger total momentum anisotropy [4, 5] and suggests a higher efficiency of the QGP fluid for converting the initial spatial deformation of the collision fireball into anisotropic collective flow. This raises the following questions: is the QGP fluid still strongly coupled at LHC energies? Does the QGP fluid have a similar specific shear viscosity at RHIC and LHC energies? In this article, we shall address the latter question.

Several groups [6–8] have recently published analyses of the ALICE data [3] based on a purely hydrodynamic approach, and another group offered an assessment of the same data within an ideal hydrodynamic + hadron cascade hybrid approach [9]. In contrast, we here use the newly developed hybrid approach `VISHNU` [10] which correctly describes viscous and other dissipative effects in both the early QGP and late hadronic rescattering stages, including the breakdown of chemical equilibrium in the hadron gas [9], by connecting the (2+1)-dimensional *viscous* hydrodynamic code `VISH2+1` [11] with the microscopic hadronic transport model `UrQMD` [12] (see Refs. [10–12] for details). Comparing to a pure hydrodynamic approach [6–8], the kinetic and chemical freeze-out of the system is naturally described by the microscopic hadronic evolution part of VISHNU through elastic, semi-elastic and inelastic scatterings, which eliminates the corresponding freeze-out parameters used in purely hydrodynamic calculations. The transition from the viscous hydrodynamic to the microscopic evolution parts of the model occurs at the switching temperature, T_{sw}, which is chosen to be 165 MeV, i.e. near the QGP phase transition temperature T_{c} and adjusted to reproduce the chemical freeze-out temperature at RHIC energies [13]. This is the highest T_{sw} possible for a hadronic transport description as well as the lowest T_{sw} possible for hydrodynamics without introducing additional parameters related to hadronic non-chemical equilibrium and dissipative effects in the hydrodynamic part [10]. For the hydrodynamic evolution above T_{sw}, we use the equation of state (EOS) `s95p-PCE` [14] which is based on recent lattice results [16].

In this manuscript, we will focus on the specific shear viscosity of the QGP at RHIC and LHC energies, ne-

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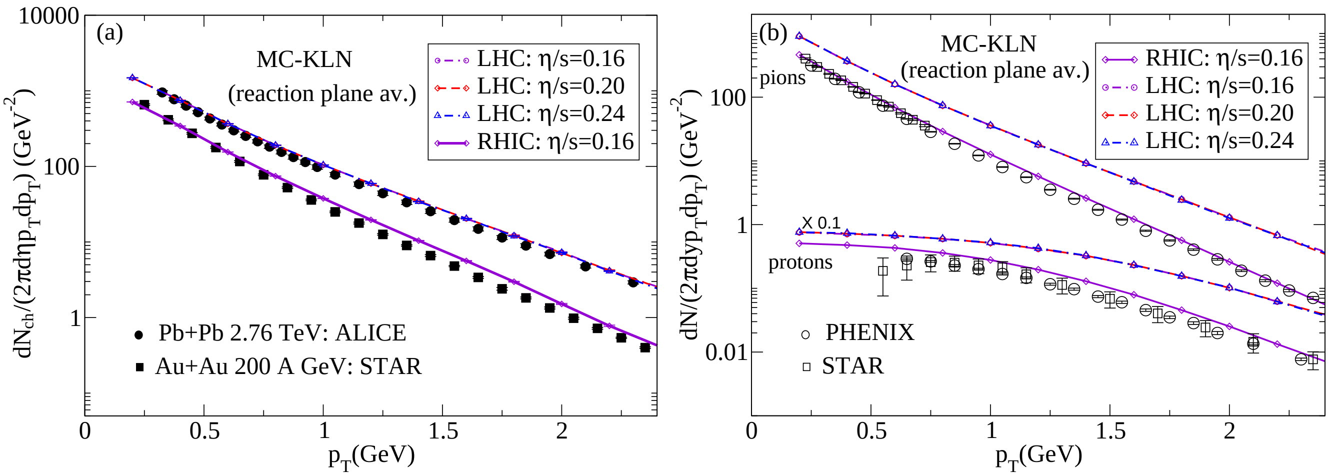

FIG. 1: (Color online) Left: p_T spectra for all charged hadrons. Experimental data are from ALICE [33] and STAR [32]. Right: p_T spectra for pions and protons. Experimental data are from STAR [34] and PHENIX [35].

glecting the bulk viscosity[1] and assuming zero net baryon density and heat conductivity. In our calculations we shall either use a constant η/s at RHIC and LHC energies (which represents the *averaged* shear viscosity effects during the whole evolution of the QGP fireball) or input different temperature dependencies for $(\eta/s)_{\rm QGP}(T)$ in the temperature range covered by both RHIC and LHC energies. The corresponding relaxation time is set as $\tau_\pi = 3\eta/(sT)$.[2]

In order to start a `VISHNU` calculation, initial conditions are required. The two most popular geometric models for initial particle production including fluctuation effects for high-energy heavy ion collisions are the Monte-Carlo Glauber Model (MC-Glauber) [17, 18] and the Monte-Carlo KLN Model (MC-KLN) [17, 19]. Following Ref. [4, 5, 18], we account for event-by-event fluctuations on average by using an initial entropy density profile that is the result of averaging over a large number of fluctuating initial entropy density distributions. Such an average can be done in two ways: either by recentering and rotating each Monte-Carlo event to align the major and minor axes of each initial density distribution before averaging (*initialization in the participant plane*) or by directly averaging without re-centering and rotating (*initialization in the reaction plane*). In this paper, we use the reaction plane method since we will compare our theoretical results with the elliptic flow data extracted by STAR and ALICE using the 4-particle cumulant method, $v_2\{4\}$, which minimizes non-flow effects [21] and measures v_2 in the reaction plane under the assumption of Gaussian fluctuations [22]. The assumption of Gaussian fluctuations was challenged in Ref. [23] for the most central and peripheral collisions; a recent analysis [24] confirmed, however, that for collision centralities up to about 40% reaction-plane averaged initial conditions produce initial eccentricities that are very close to $\varepsilon\{4\}$, the presumed driver for $v_2\{4\}$ [22]. Only for more peripheral collisions does this shortcut break down [24].

Questions about the validity of the direct comparison of the reaction-plane averaged theoretical results with $v_2\{4\}$ data from peripheral collisions engender corresponding uncertainties for the extracted values of the QGP viscosity. Additional 5−10% uncertainties arise from the recent observation [24–26] (see also [27]) that single-shot hydrodynamic evolution of a smooth averaged initial profile (as employed here) slightly overpredicts the elliptic flow compared to an event-by-event hydrodynamic evolution of each fluctuating and highly inhomogeneous initial profile separately where the average over the event ensemble is taken only at the end. In contrast to Ref. [4, 5], we here do not aim to extract the QGP viscosity at RHIC and LHC energies with reliable uncertainty estimates, but rather to investigate the relative change of the QGP viscosity from RHIC to LHC energies. Assuming that distortions from non-Gaussian eccentricity fluctuations and the replacement of event-by-event evolution by single-shot hydrodynamics are similar for the STAR and ALICE data, their presence will not affect the conclusions regarding the variation of the QGP viscosity with collision energy drawn from a comparison of $v_2\{4\}$ data with our calculations. For the same reason and for the sake of computing efficiency, we only utilize the MC-KLN initialization in the reaction plane,[3] and

[1] Ref. [15] shows that the bulk viscosity results in less than 20% contamination for the extracted QGP shear viscosity value, due to the critical slowing down of the bulk relaxation time near T_c.

[2] The choice of τ_π in the QGP phase has small to negligible effects on the final spectra and elliptic flow [11].

[3] Studies show that the choice of MC-Glauber or MC-KLN initializations will only affect the absolute value of the extracted specific shear viscosity at RHIC and LHC energies, but that the

leave the case for a detailed extraction of QGP viscosity at LHC energies to future studies.

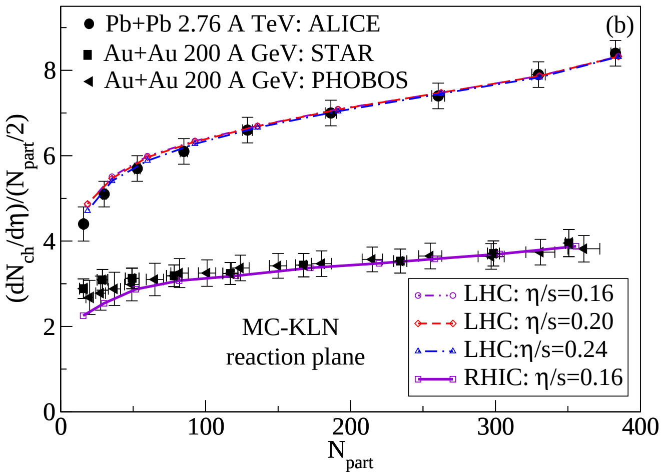

FIG. 2: (Color online) Centrality dependence of the charged hadron pseudo-rapidity density per participant pair $(dN/d\eta)/(N_{\rm part}/2)$. Experimental data are from ALICE [2], STAR [30] and PHOBOS [31]. Theoretical lines in both panels are from `VISHNU` with different constant η/s as input (see the text for the details of other inputs and parameters).

The initial time τ_0 and the normalization of the averaged initial entropy density profile need to be fixed from experimental data. Following Refs. [4, 5], we use the following parameter sets for the shear viscosity to entropy ratio η/s and hydrodynamic starting time τ_0: (0.16, 0.9 fm/c), (0.20, 1.05 fm/c) and (0.24, 1.2 fm/c). Please note that for a larger value of the QGP viscosity, we use a later starting time τ_0 to compensate for the additional radial flow generated by that larger viscosity [4, 29]. After tuning the normalization of the initial entropy density to approximately reproduce the final state charged hadron multiplicity per unit of pseudo-rapidity in 200 A GeV central Au+Au collisions ($dN/d\eta \simeq 690$ [30, 31]) and in 2.76 A TeV central Pb+Pb collisions ($dN/d\eta \simeq 1600$ [1]), we find that our calculation provides a good description of the data on p_T spectra for all charged hadrons in most central collisions for STAR [30, 32] and ALICE [33] as shown in Fig. 1a. Ref. [5] also shows that with the above parameters one can obtain a good fit to the p_T-spectra for identified hadrons (such as pions and protons) from most central collision to most peripheral collisions at RHIC energies. We find that, with the above adjustment of the starting time τ_0 when changing η/s, these p_T-spectra are rather insensitive to the QGP viscosity. In Fig. 1b, we show the p_T-spectra for pions and protons in most central collisions, and compare the RHIC results with the STAR [34] and PHENIX data [35]. Due to the current lack of ALICE data for identified hadrons, the corresponding LHC results are predictions.

In the MC-KLN initialization, we use the standard parametrization for the saturation scale $Q^2_{s,A}$ as shown in [18], which is tuned to reproduce the centrality dependence of the charged hadron multiplicity for 200 A GeV Au+Au collisions. Fig. 2 shows that such aparametrization also leads to a good description for the slope of the $(dN_{\rm ch}/d\eta)/(N_{part}/2) - N_{part}$ curve in 2.76 A TeV Pb+Pb collisions. However, we have to point out that the same parametrization of $Q^2_{s,A}$ will lead to a slight overprediction of the value of $dN_{\rm ch}/d\eta$ at LHC energies as shown in Ref. [2]. To avoid the over-generation of elliptic flow from over-predicted final multiplicities, we tune the normalization of initial entropy density as described above to fit the $dN/d\eta$ in the 0−5% centrality bin. This leads to a good fit on the overall magnitude of the $(dN_{\rm ch}/d\eta)/(N_{\rm part}/2)$ vs. $N_{\rm part}$ curve.

Having fixed all parameters, we calculate the differential elliptic flow at RHIC and LHC energies for different constant values of η/s as input. Fig. 3 shows that with $\eta/s = 0.16$, `VISHNU` nicely fits the STAR $v_2(p_T)\{4\}$ data from 0 to 2 GeV for different centrality bins. In contrast, the same $\eta/s = 0.16$ significantly over-shoots the ALICE $v_2\{4\}$ data (for $p_T > 0.5\,{\rm GeV}/c$). After increasing η/s to $0.20 - 0.24$, `VISHNU` can roughly fit the ALICE data at higher p_T, but still under-predicts the data for $p_T < 0.5\,{\rm GeV}/c$. This effect of under-prediction of the low p_T data is also found in other hydrodynamics-based calculations, including the (3+1)-d ideal hydrodynamics + hadron cascade simulations by Hirano et. al [9], the (2+1)-d viscous hydrodynamic calculations by Bozek [7]

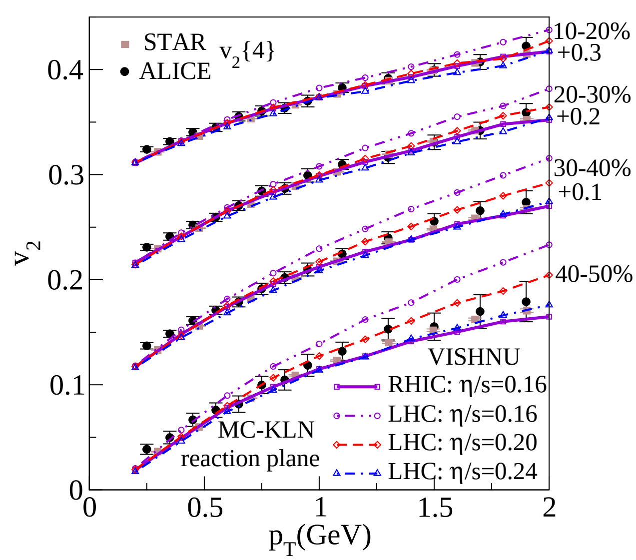

FIG. 3: (Color online) $v_2(p_T)$ at 10-20%, 20-30%, 30-40% and 40-50% centrality. Experimental data are from STAR [20] and ALICE [3] obtained from 4 particle cumulant method. Theoretical lines are from `VISHNU` calculations with different constant η/s as input. See text for details.

trend on how the QGP viscosity changes from RHIC to LHC is very similar for MC-Glauber and MC-KLN initializations [28].

and the event-by-event simulations with a (3+1)-d viscous hydrodynamics with fluctuating initial conditions done by Schenke *et al.* [8]. In Ref. [7], the deviation is interpreted to be due to non-thermalized particles stemming from jet fragmentation. While the origin of this deviation is still under debate, we conclude from Fig. 3 that one needs a larger averaged QGP specific viscosity to fit the ALICE $v_2(p_T)$ at $p_T > 0.5\,\mathrm{GeV}/c$ than the one used to fit the corresponding STAR data. This conclusion rests on the assumption that the theoretical model correctly describes the slopes of the charged hadron p_T-spectra at all the centralities shown in Fig. 3.

Using VISHNU we find that, even at LHC energies where almost all of the final momentum anisotropy is generated hydrodynamically in the QGP stage, the charged hadron $v_2(p_T)$ continues to grow somewhat during the hadronic stage. This hadronic increase of $v_2(p_T)$ is smaller at the LHC than at RHIC, in agreement with earlier findings [36] using an ideal hydrodynamic + cascade hybrid code. At RHIC energies, some of this hadronic increase is driven by the creation of additional overall momentum anisotropy which has not yet quite saturated in the QGP phase. At LHC energies, it is mostly caused by a hadronic redistribution of the momentum anisotropy already established in the QGP phase in p_T and among the various different hadronic species, due the hadronic increase in radial flow that pushes v_2 to larger p_T, especially for heavy particles [37]. This effect is sensitive to the chemical composition in the hadron gas [38–41], and a corresponding hadronic increase of $v_2(p_T)$ is not observed in purely hydrodynamic calculations with an equation of state that (incorrectly) assumes chemical equilibrium among the hadrons even below the chemical decoupling temperature $T_{\mathrm{chem}} \approx 165\,\mathrm{MeV}$ [8].

Fig. 4 shows the comparison of experimental and theoretical integrated v_2, obtained from integrating $v_2(p_T)$ with the corresponding p_T spectra as weighting functions. Following the STAR [20] and ALICE [3] analysis, we use the same p_T and pseudo-rapidity cut in our VISHNU calculations ($0.15 < p_T < 2\,\mathrm{GeV}/c$ and $|\eta| < 1$ at RHIC energy, and $0.2 < p_T < 5\,\mathrm{GeV}/c$ and $|\eta| < 0.8$ at LHC energy). One finds that VISHNU is capable of fitting the experimental data with $\eta/s = 0.16$ at RHIC and $\eta/s = 0.20$ at LHC, except for the most peripheral centrality bins. Comparing our calculation to the STAR $v_2(p_T)$ data with $\eta/s = 0.16$, the solid purple curve with square symbols is slightly above the STAR data due to the slight overprediction of the p_T-spectra around 1 GeV as shown in Fig. 1a. Similarly, the value of $\eta/s = 0.20$ from the fit to the ALICE integrated v_2 is slightly below the extracted value of $\eta/s = 0.22$ from ALICE $v_2(p_T)$, mainly because of the under-fitting of the ALICE $v_2(p_T)$ at lower p_T.

In Ref. [5], we discussed that integrated v_2 is better suited than differential v_2 for the extraction of the QGP viscosity, due to it being directly related to the fluid momentum anisotropy and insensitive to other details of hydrodynamic calculation such as chemical components of

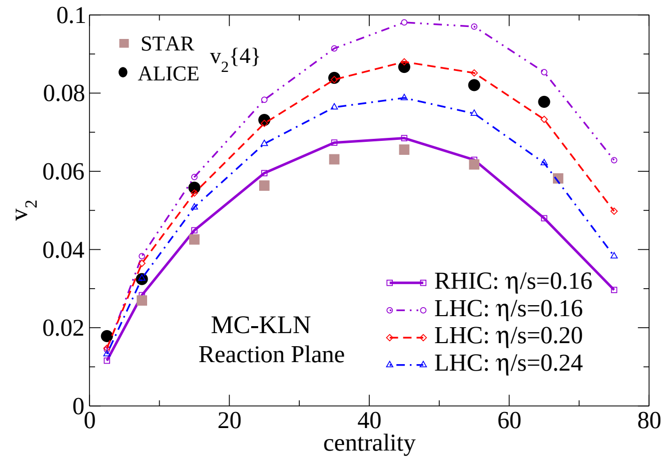

FIG. 4: (Color online) Integrated v_2 as a function of centrality. Same illustrations for theoretical and experiential lines as in Fig. 3.

the hadronic phase, the form of non-equilibrium distribution function δf, bulk viscosity and so on. However, due to the current deviation between calculations and the ALICE $v_2(p_T)$ data at lower p_T, which translates into corresponding errors for the integrated v_2, any extraction of the QGP viscosity from the integrated v_2 measurements alone at LHC energies cannot be considered robust.

Although both Figs. 3 and 4 indicate that the *averaged* QGP specific viscosity (constant η/s) slightly increases with collision energies, it has to be pointed out that using one constant value of η/s to fit RHIC data and a different constant value of η/s to fit LHC data is not logically consistent. In other words, one can not describe the QGP fluid created at at RHIC energies with $\eta/s = 0.16$ ($T_c < T < 2T_c$) and then use $\eta/s = 0.22$ ($T_c < T < 3T_c$) for the one created at LHC energies. It is a temperature dependent $\eta/s(T)$ that reflects the intrinsic properties of the QGP fluid, and this temperature dependence should be unique and describe the data both at RHIC and LHC energies.

However, Fig. 5 shows that one can at least find two different functional forms of $(\eta/s)_{\mathrm{QGP}}(T)$, with which VISHNU can simultaneously fit the STAR and ALICE $v_2(p_T)$ at 30-40% centrality bins.[4] $(\eta/s)(T)^{(a)}$ monotonically increases with T in the QGP phase while $(\eta/s)(T)^{(b)}$ first increases with T and then decrease with T at even higher temperature.[5] Please note that the minimum val-

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[4] $(\eta/s)(T)^{(a)}$ and $(\eta/s)(T)^{(b)}$ can also nicely fit the p_T-spectra for identified hadrons at 30-40% centrality which, due to lack of data, we obtained theoretically from VISHNU, using constant η/s as input. With p_T spectra and $v_2(p_T)$ fitted, one can also roughly fit the integrated v_2, since the latter is calculated from the former two.

[5] For the purpose of demonstration, we have chosen simple (even unrealistic) forms for the temperature dependence of $(\eta/s)(T)^{(a)}$ and $(\eta/s)(T)^{(b)}$ with 2-4 free parameters that can easily be fitted

ues of $(\eta/s)(T)$ in case (a) and (b) are below 0.16, and the the maximum values are well above 0.24.[6] Although $(\eta/s)(T)^{(b)}$ shows a smaller and subsequently negative slope at higher temperature, it has a higher minimum value of η/s than $(\eta/s)(T)^{(a)}$.

The fireball evolution may differ between $(\eta/s)(T)^{(a)}$ and $(\eta/s)(T)^{(b)}$. However, both the final v_2 and particle spectra are sensitive only to the time-integral of the QGP evolution, which in both cases apparently is very similar to the evolutions at fixed values of $\eta/s = 0.16$ at RHIC energy and $\eta/s = 0.22$ at LHC energy. Based on our current analysis, utilizing only v_2 and particle spectra, one cannot unambiguously determine the functional form of $(\eta/s)(T)$ and whether the QGP fluid is more viscous or more perfect at LHC energy.

Our parametrization $(\eta/s)(T)^{(a)}$ is similar to one of the forms used in [42] whose authors studied the sensitivity of elliptic flow measurements at RHIC and LHC to a temperature-dependent increase of $(\eta/s)_{\mathrm{QGP}}$ above the quark-hadron phase transition. The authors of [42] concluded that elliptic flow measurements at RHIC energies are insensitive to such an increase, and provided convincing evidence that this conclusion does not depend on their treatment of the late hadronic stage which they evolved hydrodynamically, with sudden Cooper-Frye freeze-out, rather than microscopically as we do here. In contrast to their work, we do find such a sensitivity; this is why in $(\eta/s)(T)^{(a)}$ we had to lower the minimal η/s value at low temperatures below the constant value of 0.16 that we used in Fig. 3, in order to preserve the theoretical description of the $v_2(p_T)$ measured by STAR. We have traced this difference to a sensitivity to initial conditions: whereas Niemi *et al.* [42] initialize the shear viscous pressure tensor $\pi^{\mu\nu}$ at either zero or a non-zero initial value that is independent of η/s, we use Navier-Stokes initial conditions $\pi^{\mu\nu} = 2\left(\frac{\eta}{s}(T)\right) s\sigma^{\mu\nu}$ ($\sigma^{\mu\nu}$ is the velocity shear tensor [11]) where $\pi^{\mu\nu}$ increases with T (i.e. towards the fireball center) if η/s does so. This increases the initial transverse pressure gradients and thereby affects the final radial and elliptic flow.

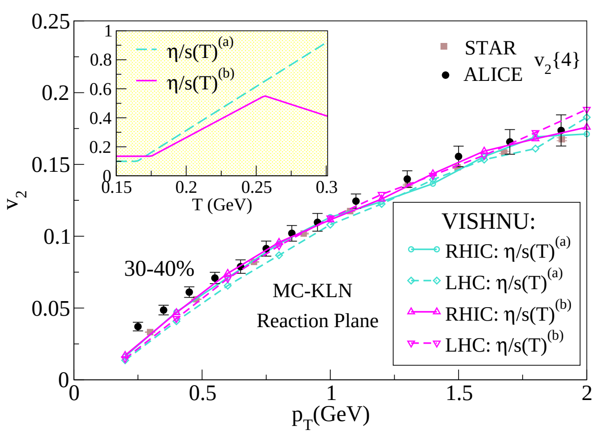

FIG. 5: (Color online) Two example of temperature dependent $(\eta/s)_{\mathrm{QGP}}(T)$, with which `VISHNU` can simultaneously fit the STAR and ALICE $v_2(p_T)$ at 30-40% centrality.

In summary, we have studied the differential and integrated v_2 at different centrality bins for 200 A GeV Au+Au collisions and 2.76 A TeV Pb+Pb collisions, using the hybrid model `VISHNU` which describes the expansion of a QGP using viscous hydrodynamics and the successive evolution of hadronic matter with a microscopic transport model. We find that, in order to describe the STAR and ALICE $v_2\{4\}$ data with reaction-plane averaged MC-KLN initial conditions, one needs an *averaged* QGP viscosity $\eta/s \approx 0.16$ at RHIC energies and $\eta/s \approx 0.20-0.24$ at LHC energies. Although this result is in qualitative agreement with expectations from weakly coupled QGP calculations [43] and from recent lattice simulations [44], both of which shows that the specific QGP shear viscosity increases with temperature, a more detailed analysis with `VISHNU` utilizing a temperature dependent $(\eta/s)_{\mathrm{QGP}}(T)$ as input shows that one cannot uniquely constrain the form of $(\eta/s)(T)$ by fitting the spectra and v_2 alone. Based on our phenomenological approach, it remains an open question whether the QGP fluid is more viscous or more perfect in the temperature regime reached by LHC energies.

Acknowledgments: We gratefully acknowledge fruitful discussions with T. Hirano, P. Huovinen, H. Masui, H. Niemi, A. Poskanzer, C. Shen, R. Snellings, and X.-N. Wang. We especially thank T. Hirano for providing the new initial conditions at LHC energies, R. Snellings for the experimental data shown in Figs. 2 and 3, and H. Buesching for the ALICE data show in Fig. 1a. This work was supported by the U.S. Department of Energy under Grants No. DE-AC02-05CH11231, DE-FG02-05ER41367, DE-SC0004286, and (within the framework of the Jet Collaboration) DE-SC0004104. We gratefully acknowledge extensive computing resources provided to us by the Ohio Supercomputer Center.

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from spectra and v_2 at RHIC and LHC energies.

[6] Since below $T_{\mathrm{chem}} = 165$ MeV the fluid is described microscopically, the behaviour of $(\eta/s)(T)^{(a,b)}$ shown in Fig. 5 in the region $T < 165$ MeV is irrelevant for our calculation. We made no attempt to model in detail the (presently unknown) form of $(\eta/s)(T)$ in the phase transition region.

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