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# $\mathrm{O}(6)$-symmetry breaking in the $\gamma$-soft nucleus ${ }^{126} \mathrm{Xe}$ and its evolution in the light stable xenon isotopes 

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#### Abstract

Low-lying collective states in ${ }^{126} \mathrm{Xe}$ have been investigated via the ${ }^{12} \mathrm{C}\left({ }^{126} \mathrm{Xe},{ }^{126} \mathrm{Xe}\right.$ ) projectile Coulomb excitation reaction at 399 MeV . The $\gamma$ decays were detected with the Gammasphere array. Coulomb excitation cross sections relative to the $2_{1}^{+}$state were obtained. Twenty-two absolute E2 transition strengths have been deduced. An $s d$-IBM 1 fit agrees well with the new experimental data. This makes a quantitative test of $\mathrm{O}(6)$-symmetry breaking in ${ }^{126} \mathrm{Xe}$ possible. The measured absolute $B(E 2)$ values indicate a preservation of $O(5)$ symmetry, while the $O(6)$ symmetry is broken. The evolution of $\mathrm{O}(6)$-symmetry breaking and of $\mathrm{O}(5)$-symmetry conservation in the ${ }^{124,126,128} \mathrm{Xe}$ isotopic chain is discussed.


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## I. INTRODUCTION

Atomic nuclei are many-body fermionic quantum systems whose dynamics is governed by the strong nuclear interaction. The strongly-coupled many-body nature leads to collective motion of nucleons governed by the symmetries of the residual interactions between them. Those symmetries manifest themselves as regular sequences of excited levels and through certain simple selection rules for their electromagnetic decays. The question of how such well pronounced, simple behaviour arises from the complex motion of many strongly interacting fermions and of how this behavior evolves as a function of nucleon number has been and is still one of the central issues in nuclear structure research. There are two extreme approaches to this question. One is to utilize the semi-classical concept of nuclear shapes at the expense of sacrificing the link to the contribution of individual nucleons. The second approach is to perform microscopic calculations. The solution of this problem in the full configuration space remains a considerable computational challenge which, in practice, can be solved for heavy nuclei only after considerable truncations. One intermediate approach for describing the low-lying quadrupole collective modes in even-even nuclei was suggested in terms of the Interacting Boson Model (IBM) [1, 2]. The IBM-1 model considers the valence nucleons coupled into boson pairs with spins $J=0$ ( $s$-boson) and $J=2$ ( $d$-boson), only.

The IBM-1 Hamiltonian posseses inherent $\mathrm{U}(6)$ symmetry which can be used as a powerful algebraic tool for classifying the nature of low-lying collective states of even-even nuclei into three symmetry classes: $\mathrm{U}(5)$ [3], $\mathrm{SU}(3)$ [4], and $\mathrm{O}(6)$ [5]. The presence of a symmetry is related to the conservation of certain quantum num-
bers. At these dynamical symmetries, the IBM Hamiltonian is analytically solvable $[1,2]$ and the solutions are directly related to analytically solvable cases of the geometrical Bohr Hamiltonian [6]- the harmonic vibrator, the quadrupole-deformed axial rotor, and the $\gamma$-unstable rotor [7], respectively. The existence of any of the dynamical symmetries is experimentally established by observing the specific energy level patterns and decays which follow the associated selection rules. Exact realizations of the dynamical symmetries are not manifested in nature, but finding nuclei that behave closely to them is an important task because such systems serve as benchmarks for the evolution of nuclear collectivity. Moreover, such benchmark nuclei also show to what extent the bosonic approximation holds for the fermionic nuclear many-body problem.

The experimental observation of $\mathrm{O}(6)$ symmetry in nuclei is scarce. The best established case is ${ }^{196} \mathrm{Pt}[8,9]$. Also, the $\mathrm{Xe}-\mathrm{Ba}-\mathrm{Ce}$ nuclei in the $A \approx 130$ region are considered as exhibiting $\mathrm{O}(6)$-like behaviour [10]. The stable even-even xenon isotopes, ${ }^{124-132} \mathrm{Xe}$, were considered to be part of this extensive $\mathrm{O}(6)$-like region. Indeed, the low-spin structures of ${ }^{128} \mathrm{Xe}$ [11], ${ }^{126} \mathrm{Xe}$ [12] and ${ }^{124} \mathrm{Xe}$ [13] manifest $\mathrm{O}(6)$-like arrangements of energy levels [10] and E2 branching ratios reflecting the selection rules of the $\mathrm{O}(5)$ symmetry, where $\mathrm{O}(5)$ is a subgroup of $\mathrm{O}(6)$. However, this interpretation was challenged very recently in the case of ${ }^{124} \mathrm{Xe}$ [14]. It has now been shown quantitatively that the $\mathrm{O}(6)$ symmetry is severely broken, while the $\mathrm{O}(5)$ symmetry holds well. This surprising result for ${ }^{124} \mathrm{Xe}$ [14] naturally leads one to question to what extent the $\mathrm{O}(6)$ symmetry is broken/preserved in the neighboring xenon isotopes. Such an unambiguous and quantitative study of symmetry breaking/preservation requires a complete set of absolute $E 2$ transition strengths, especially the ones associated
with transitions between states from different $\mathrm{O}(6)$ quantum numbers $\sigma$ and $\mathrm{O}(5)$ quantum numbers $\tau$. These absolute $E 2$ transition strengths test the validity of the selection rules directly; in the cases of $\mathrm{O}(6)$ and $\mathrm{O}(5)$ symmetries, $E 2$ transitions are allowed between states with $\Delta \sigma=0$ and $\Delta \tau= \pm 1$. This experimental information is rarely available, but it has been shown [14, 15, 17] that projectile Coulomb excitation can serve as a powerful tool to obtain it. The purpose of the present study is to investigate the ${ }^{126} \mathrm{Xe}$ low-spin states with projectile Coulomb excitation. The information obtained on the absolute $E 2$ transition strengths allows to quantify the degree of $\mathrm{O}(6)$ symmetry breaking/preservation in this nucleus, following the method outlined in Ref. [14]. The same analysis is carried out for ${ }^{128} \mathrm{Xe}$ using the results from Ref. [15]. Altogether, information is provided on the evolution of $\mathrm{O}(6)$-symmetry breaking/preservation in the chain of xenon isotopes when the neutron number approaches the $N=82$ shell closure.

## II. EXPERIMENT

The experiment was performed at Argonne National Laboratory. The superconducting ATLAS linear accelerator provided a beam of ${ }^{126} \mathrm{Xe}$ ions with an energy of 399 MeV , which corresponds to $\sim 80 \%$ of the Coulomb barrier for reactions on a ${ }^{12} \mathrm{C}$ target [16]. The beam intensity was $\sim 1 \mathrm{pnA}$. The beam was pulsed ( 12 MHz ) and impinged on a natural ${ }^{12} \mathrm{C}$ target of $1 \mathrm{mg} / \mathrm{cm}^{2}$ thickness. The emitted $\gamma$ rays were detected by the Gammasphere array which consisted of 98 high-purity Comptonsuppressed Germanium detectors arranged in 16 rings $[18,19]$. The event trigger was defined by detection of a single $\gamma$ ray, but higher-multiplicity events were recorded as well. The average trigger readout rate was 15000 events/s (readout dead time was $\sim 30 \%$ ). This count rate compares with a "beam-off" rate of 600 events/s. The total number of recorded events was $1.3 \times 10^{9}$ for a running time of $\sim 29 \mathrm{~h}$ and approximately $1.8 \times 10^{7}$ events of $\gamma$-ray fold higher than 1 were sorted into a $\gamma \gamma$-coincidence matrix. Doppler correction (recoiling velocity $\beta=6.4(2) \%$ ) and time-random background subtraction was applied. As the beam energy was relatively low, the dominant "beam-off" count rate came from natural sources. This background was identified and subtracted by selecting events occuring between the beam bursts, scaled to eliminate the $1461-\mathrm{keV}$ decay from ${ }^{40} \mathrm{~K}$. The singles spectrum is displayed in Fig.1.

## III. DATA ANALYSIS

All the $\gamma$ transitions observed have been placed in the ${ }^{126}$ Xe level scheme and their corresponding intensities are listed on Table I. They were reported previously in [12, 20, 21], except for the $1373-\mathrm{keV}$ line originating from the decay of the $3_{2}^{-}$state at 2315 keV to the $4_{1}^{+}$level at


FIG. 1: (Color online) Background-subtracted and Dopplercorrected single $\gamma$-ray spectrum for the sum over all Ge detectors following ${ }^{126} \mathrm{Xe}$ Coulomb excitation on a ${ }^{12} \mathrm{C}$ target.

942 keV . The $\gamma$-ray intensities have been normalized to the $2_{1}^{+} \rightarrow 0_{1}^{+}$groundstate transition, which dominates the spectrum by three orders of magnitude. The population yields of each state have been deduced from $\gamma$ singles and $\gamma \gamma$-coincidence data. The contribution from the electron conversion decays was small in comparison to the systematic errors $(<1.5 \%,[22])$ and has been neglected. The contributions of known transitions that were not observed (e.g. due to their too low energy or the presence of contaminants), have been determined from previously published branching ratios [23].

The observed relative yields measure the Coulomb Excitation (CE) cross sections relative to the $2_{1}^{+}$state. The multiple-Coulex code CLX, based on the Winther-De Boer theory [24] has been used to determine the set of matrix elements that reproduce the observed relative cross sections. The previously known $B\left(E 2 ; 2_{1}^{+} \rightarrow\right.$ $\left.0_{1}^{+}\right)=0.152(5) e^{2} b^{2}$ value from Ref. [23] sets the absolute scale. The energy loss of the beam inside the target ( $\sim 40$ MeV ) was taken into account. The unknown quadrupole moments of excited states were allowed to vary between the extreme rotational limits $(\mathrm{Q}= \pm 2.78 \mathrm{eb})$, adding uncertainties of $5 \%$ on average to the transitional matrix elements. The input matrix elements in CLX were also constrained by the known branching and multipole mixing ratios. The choice of sign of the matrix elements is not always unique in a fit to multi-step Coulomb excitation processes. However, constraints come from the requirement that the relative phases must be "quantum mechanically coherent". The signs of the E2 matrix elements were chosen to be in agreement with those predicted by the IBM-1 calculation described below and are also listed in Table I for clarity $(\sigma)$. Besides the statistical uncertainties the final results in Table I also include uncertainties from all quantities varied or constraining the

TABLE I: Measured properties of the levels and $\gamma$-ray transitions in ${ }^{126} \mathrm{Xe}$. The absolute $E 2$ strengths are compared to $s d$-IBM 1 calculations.

${ }^{a}$ The multipole mixing ratios are taken from Ref. [23].
${ }^{b} B(E 2)$ values are given in W.u. (1 W.u. $\left.(E 2)=0.003752 e^{2} b^{2}\right), B(M 1)$ in $\mu_{N}^{2}$ and the $B\left(E 3 ; 0_{1}^{+} \rightarrow 3_{1}^{-}\right) \uparrow$ value is given in $e^{2} b^{3}$.
${ }^{c}$ Adopted value taken from Ref. [25].
${ }^{d}$ This transition is not observed. Contrary to Ref. [14], this transition does not play a relevant role for the population of the $4_{1}^{+}$ state.
${ }^{e}$ These transitions are doublets, the individual intensities have been separated using the known branching ratios taken from Ref. [23].
${ }^{f}$ This transition is not observed directly, but it is included in the calculations for the Coulomb cross-sections for a best match with the data.
${ }^{g}$ Upper limits since the mixing ratio was unknown, $B(E 2)$ and $B(M 1)$ upper limits are given under the assumption of either pure $E 2$ or pure M1 transitions, respectively.
${ }^{h}$ Determined through the branching ratio from Ref. [23].
${ }^{i}$ This transition was observed only in coincidence spectra.
${ }^{j}$ In Ref. [26], a $B(E 3) \uparrow=0.085(13) e^{2} b^{3}$ value is reported.
${ }^{k}$ Upper limit since only a lower limit of the total width $\left(\Gamma_{t o t}\right)$ of the state has been measured.
variations in the procedure described above, e.g. branching ratios, multipole ratios and unknown quadrupole moments.

## IV. RESULTS

The resulting $B(E 2)$ transitions strengths are given in Table I. The present analysis resulted in 22 absolute values and 7 upper limits for the $B(E 2)$ strengths. Due to their small values, the $B(M 1)$ transitions strengths have been omitted in Table I, except for the relevant upper limit obtained for the transition of the $2^{+}$state at 2455 keV to the $2_{1}^{+}$level.

## - $4_{1}^{+}$level at 942 keV

The unobserved $4_{1}^{+} \rightarrow 2_{2}^{+}$transition at 62 keV does not play a significant role for the population of the $4_{1}^{+}$state at 942 keV . Another issue with the population of the $4_{1}^{+}$is the contribution of a one-step $E 4$ excitation. Since it was impossible to quantify or estimate this $B(E 4)$ transition strength, the $B\left(E 2 ; 4_{1}^{+} \rightarrow 2_{1}^{+}\right)$value in Table I assumes no $E 4$ transition from the ground state.

- $0_{2}^{+}$level at 1314 keV and $3_{1}^{+}$level at 1317 keV
The $0_{2}^{+}$state at 1314 keV decays through the $434-\mathrm{keV} \gamma$ ray to the $2_{2}^{+}$level and through the $925-\mathrm{keV}$ transition to the $2_{1}^{+}$level. These lines are doublets with the two $\gamma$ rays coming from the $3_{1}^{+}$ level at 1317 keV . This $3_{1}^{+}$state decays also to the $2_{2}^{+}$level with $\mathrm{E}_{\gamma}=438 \mathrm{keV}$ and to the $2_{1}^{+}$level with $\mathrm{E}_{\gamma}=925 \mathrm{keV}$. The energy resolution achieved here did not permit to resolve these two doublets. Therefore, the intensities of each doublet were fitted and the previously known branching ratios [23] were used to obtain the individual respective intensities.


## - $4_{2}^{+}$level at 1488 keV

Similarly, the $546-\mathrm{keV} \gamma$ ray from the $4_{2}^{+}$level at 1488 keV to the $4_{1}^{+}$level is a doublet with the strong $553-\mathrm{keV}$ transition. The individual intensity of the $546-\mathrm{keV} \gamma$ ray has been deduced from the known branching ratio [23] and the measured intensity of the $4_{2}^{+} \rightarrow 2_{2}^{+}, 609-\mathrm{keV}$ transition. Again, no $E 4$ transition from the ground state was considered in the Coulomb excitation process.

## - $2_{3}^{+}$level at 1678 keV

The mixing ratio for the $2_{3}^{+} \rightarrow 2_{1}^{+}$transition is unknown. Due to low statistics, the present data did not enable an angular distribution measurement. In Table I, the mixed transitions coming from this state have been calculated by
assuming pure $E 2$ character, hence the resulting upper limits. However assuming that the $2_{3}^{+} \rightarrow$ $2_{1}^{+}$transition is of pure $M 1$ character, results in a $B\left(M 1 ; 2_{3}^{+} \rightarrow 2_{1}^{+}\right) \leq 4.35(87) \times 10^{-4} \mu_{N}^{2}$ value. This small strength rules out the possibility that this state could be a sizeable fragment of the one-phonon $2_{1, \mathrm{~ms}}^{+}$mixed-symmetry state [27].

## - $2_{5}^{+}$level at 2086 keV

This state does not decay to the $2_{1}^{+}$level, which rules out the possibility that it could have mixedsymmetry character. The $2_{5}^{+} \rightarrow 0_{3}^{+}$is not observed either. However, the existence of this transition cannot be ruled out, due the large energy suppression factor for its intensity $\left(\mathrm{E}_{\gamma}=326 \mathrm{keV}\right)$. This transition was included in the CLX calculations showing that the yields of the $2_{5}^{+}$and the $0_{3}^{+}$are barely influenced by the size of the $\left\langle 0_{3}^{+}\|E 2\| 2_{5}^{+}\right\rangle$ matrix element. A $B\left(E 2 ; 2_{5}^{+} \rightarrow 0_{3}^{+}\right)$value below $\sim 50 \mathrm{~W} . \mathrm{u}$ will influence the yields of the $2_{5}^{+}$and $0_{3}^{+}$states within their experimental error bars only.

- $2^{+}$level at 2455 keV

Since the mixing ratio for this $2^{+} \rightarrow 2_{1}^{+}$transition is unknown [23], a pure $E 2$ transition was assumed, leading to an upper value $B\left(E 2 ; 2^{+} \rightarrow\right.$ $\left.2_{1}^{+}\right) \leq 1.82(54)$ W.u. Assuming pure $M 1$ multipolarity results in $B\left(M 1 ; 2^{+} \rightarrow 2_{1}^{+}\right) \leq 0.020(6) \mu_{N}^{2}$. This small value rules out a mixed-symmetry character for this state.

The experimental low-lying energy level scheme of ${ }^{126} \mathrm{Xe}$ is displayed on the left side of Fig 2. The right side provides the corresponding energy level spectrum steming from the $s d$-IBM 1 fit described in the next section.

## V. DISCUSSION

In order to quantify the degree of symmetry breaking in ${ }^{126} \mathrm{Xe}$, the procedure outlined in Ref. [14] was followed. The $s d$-IBM 1 Hamiltonian was used:

$$
\begin{equation*}
H=\epsilon n_{d}+\left(\lambda+\frac{2}{5} \beta\right) L L+\kappa Q^{\chi} \cdot Q^{\chi}+4 \beta T^{(3)} T^{(3)} \tag{1}
\end{equation*}
$$

similar to the recent studies [13, 14] with the fitted parameters $\epsilon_{0}=0.712 \mathrm{MeV}, \beta_{0}=-23.0 \mathrm{keV}, \lambda_{0}=13.5$ $\mathrm{keV}, \chi_{0}=0.180, \kappa_{0}=-41.9 \mathrm{keV}$ and $e_{B}=0.12246$ $e^{2} b^{2}$. The positive sign of the parameter $\chi_{0}$ is chosen in order to reproduce consistently the Coulomb excitation yields of states populated by multi-step processes, only. For a more detailed discussion on the procedure for obtaining the Hamiltonian's parameters, the reader is referred to Ref. [13, 28]. The results from this numerical IBM calculation (done with the code PHINT [29]) are presented in the right-most column of Table I and on the right side of Fig. 2. All calculated absolute E2 transition


FIG. 2: (Color online) (a) Low-energy, positive-parity levels of ${ }^{126} \mathrm{Xe}$; The levels observed in the present experiment are represented by solid lines. Dashed lines indicate energy levels known from the literature [23]. (b) sd-IBM 1 calculation for ${ }^{126} \mathrm{Xe}$ (see text). The eigenstates are arranged in $(\tau, \sigma)$ multiplets according to the $\mathrm{O}(6)$ dynamical symmetry. The arrows represent the $E 2$ transitions for off-yrast, quasi- $K=0$ levels of particular interest. The thickness of the arrows and the numbers associated with them correspond to the absolute $B(E 2)$ values in W.u. The dashed arrows are transitions with $B(E 2) \leq 0.1$ W.u.
strengths must be considered as predictions, except for the $2_{1}^{+} \rightarrow 0_{1}^{+}$transition. The agreement between the calculated and the experimental $B(E 2)$ values is good for all observed $E 2$ transitions. The agreement for the energy levels is also satisfactory (Fig. 2). It is worth emphasizing that the IBM parameterization obtained here is capable of describing the present comprehensive data set in a consistent way. This provides confidence that the calculated wave functions represent the nuclear quantum states in the framework of the IBM correctly. The structure including the ground state band, the quasi- $\gamma$ band, the quasi- $K=4$ band and the quasi- $K=0$ band residing on top of the $0_{2}^{+}$ state reflects the expected pattern of a $\sigma=N$ structure in the $\mathrm{O}(6)$ symmetry. Correspondingly, the structure denoted by quasi- $\beta$ band could be expected to represent a realization of a structure with the $\mathrm{O}(6)$ quantum number $\sigma=N-2$. Surprisingly, this is not the case. For a quantitative analysis of the presence of the $O(6)$ and $\mathrm{O}(5)$ symmetries, the wave functions for the first few $0^{+}$and $2^{+}$IBM states have been projected onto the $\mathrm{O}(6)$ basis $\left\{\left|J^{\pi}\right\rangle^{(\sigma, \tau)}\right\}$ and the results are presented in Fig. 3.

From Fig. 3, it is obvious that neither $\tau$ nor $\sigma$ are perfect quantum numbers. However, $\tau$ quantum numbers appear to be quite well preserved which indicates that
$\mathrm{O}(5)$ is the relevant symmetry. The components with a "correct" $\tau$ quantum number exhaust about $90 \%$ or more of the total wave functions. The best example is the ground state where more than $99 \%$ of the wave function contains the right $\tau=0$ number. The small admixtures with different $\tau$ s values are such that the deviations from the $\mathrm{O}(5)$ selection rules can easily be explained. For example, the wave function of the $0_{2}^{+}$state, the bandhead of the three phonon $K=0$ structure (Fig. 2(b)), contains a small component with $(\sigma=7, \tau=0)$ which has an amplitude of about $1.4 \%$ (see Fig. 3). This component makes an allowed $(\Delta \sigma=0, \Delta \tau=-1) E 2$ transition to the main component of the $2_{1}^{+}$state possible. This $2_{1}^{+}$level in turn has the "correct" $(\sigma=7, \tau=1)$ quantum numbers and an amplitude of about $70 \%$. At the same time, the main component of the $0_{2}^{+}$state with ( $\sigma=7, \tau=3$ ) quantum numbers can proceed through an allowed transition to the component of the $2_{1}^{+}$state with $(\sigma=7, \tau=2)$. Analogously, the components with ( $\sigma=5, \tau=0$ ) and ( $\sigma=3, \tau=0$ ) of the $0_{2}^{+}$level can make allowed transitions to the components with $(\sigma=5, \tau=1)$ and $(\sigma=3, \tau=1)$ of the $2_{1}^{+}$state (see Fig. 3). All these $\Delta \tau$ allowed contributions can add up and result in the mildly collective $0_{2}^{+} \rightarrow 2_{1}^{+}$transition observed experimentally (see Fig. 2(a)). In the same way, the small components with "incorrect" $\tau$ and $\sigma$ in the wave functions of the $0_{3}^{+}$and the $2_{1}^{+}$IBM states are


FIG. 3: (Color online) Squared amplitudes of the components with different ( $\sigma, \tau$ ) values in the $0_{1,2,3}^{+}$(a) and the $2_{1,2,4}^{+}$(b) $s d$-IBM 1 wave functions.
the main reasons for the existence of the otherwise forbidden $0_{, "}^{+}{ }_{\beta}$ " $\rightarrow 2_{1}^{+}$transition while main contributions to the collective $E 2$ transition between the $2_{4}^{+}$and the $0_{3}^{+}$ states come mostly from components with $\Delta \tau= \pm 1$.

The $\sigma$ quantum numbers do, however, appear to have little or no validity (again, see Fig. 3). Even the ground state contains only $63.1 \%$ of $\sigma=N=7$ value. For the states which were thought to belong to the $\sigma=N-2=5$ representation, the $0_{3}^{+}$and the $2_{4}^{+}$levels, the $\sigma$ quantum number is completely diluted. In fact, the components with $\sigma=5$ account only for $20.2 \%$ and $42.3 \%$ of the total wave functions of these IBM states (Fig. 3).

In an attempt to quantify the amount of symmetry breaking, the fluctuations $\Delta q$ in the quantum number of states are considered, where $\Delta q=\sqrt{\left\langle q^{2}\right\rangle-\langle q\rangle^{2}}$, with $q$ representing the quantum number related to the symmetry under consideration. For further details concerning the fluctuation analysis, the reader is referred to Ref. [14]. For an exact symmetry, $\Delta q=0$ for all states. All the other cases represent a situation with broken symmetry. A perturbed and dissolved symmetry is defined by the classification value $\Delta q_{\text {class. }} \equiv \delta q_{\min } /(2 \sqrt{2 \ln 2})$, where $\delta q_{\text {min }}$ is the minimum step the quantum number $q$ can change (e.g. $\delta \sigma_{\min }=2$ ). The symmetry is then considered to be perturbed, but dominant, if the fluctuations in the quantum number are $\Delta q \leq \Delta q_{\text {class. }}$. For $\Delta q>\Delta q_{\text {class. }}$. the symmetry related to this quantum number is viewed as dissolved. The classification values obtained for the $\mathrm{O}(6)$ symmetry and the $\mathrm{O}(5)$ symmetry are $\Delta \sigma_{\text {class. }}=0.849$ and $\Delta \tau_{\text {class. }}=0.425$, respectively.

In Ref. [14], it was proposed that the degree of symmetry breaking/preservation is quantified by the fluctuation in the respective quantum number ( $\sigma$ in the case of $\mathrm{O}(6)$ and $\tau$ in the case of $\mathrm{O}(5))$ for states whose decay are sensitive to a particular selection rule. For testing the $\mathrm{O}(6)$ symmetry such a state is the bandhead of the quasi- $\beta$ band, while the decay of the bandhead of the quasi- $\gamma$ band is sensitive to the $\mathrm{O}(5)$ selection rules. Indeed, the $2{ }^{\prime \prime}{ }^{+} \gamma^{\prime \prime} \rightarrow 0_{1}^{+}$transition is $\sigma$-allowed and $\tau$ -
forbidden, while the $0_{,}^{+}{ }_{\beta}{ }^{\prime}, \rightarrow 2_{1}^{+}$is a $\sigma$-forbidden and $\tau$ allowed transition. Therefore, the quantum number fluctuations in the $0_{"}{ }^{+}{ }_{\beta}$, and the $2_{"}{ }^{+}{ }_{\gamma}$, states are considered to be measures for the symmetry breaking/preservation of $O(6)$ and $O(5)$ symmetry for the entire nucleus, respectively. To trace the evolution of these observables through the parameter space of the IBM-1 from the exact $\mathrm{O}(6)$ symmetry to the point defined by the model parameters of Eq. (1), the following parametrisation was used: $\beta(a)=\beta_{O(6)}+a\left(\beta_{W}-\beta_{O(6)}\right), \epsilon(a)=\epsilon_{0} a$, and $\chi(a)=\chi_{0} a$ [14]. When $a=1, H$ is the Hamiltonian found to best describe ${ }^{126}$ Xe. For $a=0$, the IBM Hamiltonian corresponds to the exact $\mathrm{O}(6)$ symmetry. The parameter $\beta_{O(6)}$ was also arbitrarily fixed with $\beta_{O(6)}=-0.455 \mathrm{keV}$ in order to obtain the head of the $\sigma=N-2$ structure as the third $0^{+}$state in the exact $\mathrm{O}(6)$ symmetry $(a=0)$. The evolution of the two observables sensitive to the $\mathrm{O}(6)$ and $\mathrm{O}(5)$ symmetries $B\left(E 2 ; 0_{",}{ }^{+}{ }_{\beta}, \rightarrow 2_{1}^{+}\right) / B\left(E 2 ; 2_{1}^{+} \rightarrow 0_{1}^{+}\right)$ and $B\left(E 2 ; 2^{\prime}{ }_{\gamma}{ }^{\prime \prime} \rightarrow 0_{1}^{+}\right) / B\left(E 2 ; 2_{1}^{+} \rightarrow 0_{1}^{+}\right)$with $\Delta \sigma$ and $\Delta \tau$ is plotted in Fig. 4 (a) and (b), respectively.

A comparison between the evolution of the $B\left(E 2 ; 0_{,}^{+}{ }_{\beta}{ }^{\prime \prime} \rightarrow 2_{1}^{+}\right) / B\left(E 2 ; 2_{1}^{+} \rightarrow 0_{1}^{+}\right)$ratio and the experimental value measured for ${ }^{126} \mathrm{Xe}$ (Fig. 4 (a)) provides the value $\Delta \sigma_{\text {exp }}\left(0_{3}^{+} ;{ }^{126} \mathrm{Xe}\right)=2.03_{-0.16}^{+0.08}$, which lies well beyond the classification value of 0.849 and is comparable to the fluctuations observed in the $\sigma$ quantum number for the other dynamical symmetries, $\mathrm{U}(5)$ and $\mathrm{SU}(3)$. The latter are also presented in Fig. 4 (a). The large value of $\Delta \sigma_{\text {exp }}$ obtained for the $0_{,{ }^{+}}{ }_{\beta}$, state of ${ }^{126} \mathrm{Xe}$ implies that the $\mathrm{O}(6)$ symmetry is severely broken, in fact to an extent comparable to the degree of breaking of this symmetry in other dynamical limits. For the $0_{3}^{+}$state of ${ }^{126} \mathrm{Xe}$ which was a candidate for the lowest $\sigma=N-2$ state, the $\mathrm{O}(6)$ symmetry is actually completely dissolved. In contrast, Fig. 4 (b) provides $\Delta \tau_{\text {exp }}\left(2_{2}^{+} ;{ }^{126} \mathrm{Xe}\right)=0.221(14)$, a value lying well below the classification value of 0.425 which indicates that the


FIG. 4: (Color online) Evolution of the $B\left(E 2 ; 2_{,{ }^{+}}{ }^{\prime \prime} \rightarrow\right.$ $\left.2_{1}^{+}\right) / B\left(2_{1}^{+} \rightarrow 0_{1}^{+}\right)\left(\right.$a) and the $B\left(E 2 ; 2^{+}{ }_{\gamma} \gamma^{\prime \prime} \rightarrow 0_{1}^{+}\right) / B\left(2_{1}^{+} \rightarrow 0_{1}^{+}\right)$ (b) ratios as functions of $\Delta \sigma$ and $\Delta \tau$ (solid curves) on the linear trajectory from the exact $\mathrm{O}(6)$ symmetry to the point defined by the IBM-1 parameters described in this work (stars). The vertical dashed lines represent the classification values of $\Delta \sigma$ and $\Delta \tau$ beyond which the respective symmetry is dissolved. The circles represent the values of the respective ratios and the fluctuations at the $U(5)$ and $S U(3)$ dynamical symmetries of the IBA. The horizontal lines represent the experimental values in ${ }^{126} \mathrm{Xe}$.
$\mathrm{O}(5)$ symmetry is preserved.
The calculations of the $\sigma$ and $\tau$ fluctuations can also be extended to ${ }^{128} \mathrm{Xe}$ by using the results of Ref. [15]. The fitted parameters of the Hamiltonian of Eq. (1) for ${ }^{124,126,128} \mathrm{Xe}$ are summarized in Table II. In the case of ${ }^{128} \mathrm{Xe}$, these parameters do not reproduce well the positions and the decays of the two first excited $0^{+}$states. Therefore, the fluctuations have been calculated by using the IBM parameters reproducing the best the experimental $B\left(E 2 ; 0_{1,}^{+} \beta^{\prime \prime} \rightarrow 2_{1}^{+}\right) / B\left(E 2 ; 2_{1}^{+} \rightarrow 0_{1}^{+}\right)$ratio (i.e for $\mathrm{a}=0.7$ ). In this case $\Delta \sigma_{\exp }\left(0_{3}^{+} ;{ }^{128} \mathrm{Xe}\right)=1.55_{-0.08}^{+0.12}$ and $\Delta \tau_{\text {exp }}\left(2_{2}^{+} ;{ }^{128} \mathrm{Xe}\right)=0.19_{-0.01}^{+0.03}$. If we would have calculated the fluctuations at the point corresponding to the parameters for ${ }^{128} \mathrm{Xe}$ in Table II, we would have
found slightly larger values of $\Delta \sigma_{\exp }\left(0_{3}^{+} ;{ }^{128} \mathrm{Xe}\right)=1.81$ and $\Delta \tau_{\text {exp }}\left(2_{2}^{+} ;{ }^{128} \mathrm{Xe}\right)=0.20$. We want to stress that in both cases the fluctuations in $\sigma$ and $\tau$ quantum numbers in ${ }^{128} \mathrm{Xe}$ are smaller than the ones for ${ }^{126} \mathrm{Xe}$.

TABLE II: Parameters used in the $s d$-IBM 1 Hamiltonian for ${ }^{128,126} \mathrm{Xe}$ and comparison with the parameters found in [13] for ${ }^{124} \mathrm{Xe}$.

| $\epsilon_{0} / \kappa_{0}$ | $\chi_{0}$ | $\beta_{0} / \kappa_{0}$ | $\lambda_{0} / \kappa_{0}$ | $\kappa_{0}[\mathrm{keV}]$ | $\mathrm{e}_{B}[\mathrm{eb}]$ |
| ---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{128} \mathrm{Xe}$ | -16.8 | +0.173 | 0.76 | -0.245 | -53.0 |
| 0.115 |  |  |  |  |  |
| ${ }^{126} \mathrm{Xe}$ | -17.0 | +0.180 | 0.55 | -0.322 | -41.9 |
| ${ }^{124} \mathrm{Xe}$ | -20.9 | -0.257 | 0.563 | -0.284 | -34.91 |



FIG. 5: (Color online) Evolution of the $\tau$ (a) and $\sigma$ (b) fluctuations characterizing the degree of $\mathrm{O}(5)$-symmetry preservation and $\mathrm{O}(6)$-symmetry breaking, in ${ }^{124,126,128} \mathrm{Xe}$. The horizontal dashed lines represent the classification values above which the symmetry is broken and under which the symmetry is preserved.

The $\tau$ and $\sigma$ fluctuations for the three isotopes ${ }^{124,126,128}$ Xe are plotted in Fig. 5, (a) and (b) respectively. In these three Xe isotopes, it is observed that the $\mathrm{O}(6)$-symmetry is broken (Fig. 5 (b)); with the breaking more pronounced in ${ }^{124} \mathrm{Xe}$ and gradually decreasing with increasing number of nucleons. On the contrary, the $\tau$ fluctuations lie always below the classification value (Fig. 5 (a)), which indicates that the $\mathrm{O}(5)$ symmetry is only slightly perturbed in ${ }^{124,126,128} \mathrm{Xe}$. It is worth stressing that such an analysis is made possible only by the comprehensive set of absolute values of $E 2$ transition rates available from projectile Coulomb excitation measurements on a light target combined with an $s d$-IBM 1 fit. The equivalent $B(E 2)$ decay rates from the $0_{3}^{+}$level are at present unknown in ${ }^{130} \mathrm{Xe}$ and ${ }^{132} \mathrm{Xe}$. Thus, the degree of $\mathrm{O}(6)$ symmetry breaking in these two isotopes cannot be quantified in the same way. However, these nuclei already lie beyond the region of the $\mathrm{O}(6)$ symme-
try. Indeed, in Ref. [15, 30] ${ }^{130}$ Xe was proposed as the best candidate among stable even-even Xe isotopes for the $\mathrm{E}(5)$ symmetry and ${ }^{132} \mathrm{Xe}$, with only two pairs of valence neutrons away from the closed shell $N=82$, is already close to the $U(5)$ symmetry; a feature applying even more to ${ }^{134} \mathrm{Xe}$.
To conclude, it has been shown that none of the stable even-even Xe isotopes should be considered as possessing the $\mathrm{O}(6)$ symmetry of the IBM. The exclusion of the stable xenon isotopes from the set of nuclei with dominant $\mathrm{O}(6)$ symmetry raises questions about the extent to which any nucleus from the $A=130$ mass region can achieve $O(6)$ symmetry.

## VI. SUMMARY

${ }^{126}$ Xe has been studied using projectile Coulomb excitation. In total, 22 absolute $E 2$ transition strengths between low-spin states have been determined. The experimentally observed level energies, branching ratios and the absolute transition strengths are reproduced well by a general $s d$-IBM 1 calculation outside any dynamical symmetry. Symmetry breaking (or preservation) was investigated by relating the fluctuations in the quantum numbers directly to the experimental observables, as suggested in Ref. [14]. In ${ }^{126} \mathrm{Xe}$, the $\mathrm{O}(6)$ symmetry was found to be completely dissolved while the $\mathrm{O}(5)$ symmetry is only slightly perturbed. By using the same approach in ${ }^{128} \mathrm{Xe}$, a similar result was obtained. The evolution of the $\mathrm{O}(6)$ and $\mathrm{O}(5)$ symmetry breaking/preservation as a function of neutron number for the light stable xenon isotopes indicates a tendency of approaching the $\mathrm{O}(6)$ limit, but without ever reaching it in the xenon isotopes. As a result a similar, detailed investigation of the $\mathrm{Ce}, \mathrm{Ba}$ and Pt isotopes which have been previously associated with the $O(6)$ symmetry appears to be warranted.

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