This is the accepted manuscript made available via CHORUS. The article has been published as:

In-beam γ-ray spectroscopy of $^{35}$Mg and $^{33}$Na  
Phys. Rev. C 83, 044305 — Published 6 April 2011  
DOI: 10.1103/PhysRevC.83.044305
In-beam $\gamma$-ray spectroscopy of $^{35}$Mg and $^{33}$Na


1 National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824, USA
2 Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA
3 Department of Physics, Florida State University, Tallahassee, FL 32306, USA
4 Department of Physics and Center for Nuclear Study, University of Tokyo, Hongo, Tokyo 113-0033, Japan
5 RIKEN, Hiyoshi, Wako-ku, Saitama 351-0198, Japan
6 Japan Atomic Energy Agency, Tokai, Ibaraki 319-1195, Japan

Excited states in the very neutron-rich nuclei $^{35}$Mg and $^{33}$Na were populated in the fragmentation of a $^{38}$Si projectile beam on a Be target at 83 MeV/u beam energy. We report on the first observation of $\gamma$-ray transitions in $^{35}$Mg, the odd-$N$ neighbor of $^{34}$Mg and $^{36}$Mg, which are known to be part of the “Island of Inversion” around $N = 20$. The results are discussed in the framework of large-scale shell-model calculations. For the $A = 32$ nucleus $^{33}$Na, a new $\gamma$-ray transition was observed that is suggested to complete the $\gamma$-ray cascade $7/2^+ \rightarrow 5/2^+ \rightarrow 3/2^+$ connecting three neutron 2p-2h intruder states that are predicted to form a close-to-ideal $K = 3/2$ rotational band in the strong-coupling limit.

PACS numbers: 23.20.Lv, 29.38.Dh, 21.60.Cs, 27.30.+t
Keywords: $^{33}$Na, $^{35}$Mg, Island of Inversion

I. INTRODUCTION

The structure of nuclei in the so-called “Island of Inversion” [1, 2], a region of the nuclear chart comprised of neutron-rich Ne, Na and Mg isotopes around neutron number $N = 20$, has provided much insight into the driving forces of shell evolution. In this region, driven largely by the spin-isospin parts of the nuclear force [3], neutron np-nh “intruder” configurations of $\nu(sd)^{-n}(fp)^{+n}$ character (labeled $nh\omega$) gain “correlation energy” [4] with respect to the normal-order configurations and dominate the wave functions of low-lying states including the ground states – signaling the breakdown of $N = 20$ as a magic number in these nuclei. The resulting onset of collectivity was quantified with inelastic scattering experiments [5–10] and inferred from measurements of the $2^+_1$ excitation energies [11–14] on even-even nuclei out to $^{32}$Ne and $^{30}$Mg at $Z = 10$ and $Z = 12$, respectively. These collective properties of even-even nuclei are rather well described by state-of-the-art Monte Carlo Shell-Model (MCSM) calculations (SDPF-M effective interaction) that allow for unrestricted mixing of neutron particle-hole configurations across the $N = 20$ shell gap. However, experimental information on their odd-$N$ neighbors, $^{31,33}$Mg [15–22] and $^{30}$Na [23], for example, has posed significant challenges for these calculations and led to controversial interpretations, indicating that odd-$A$ and odd-odd nuclei pose very stringent benchmarks for a shell model description of this region.

In this paper we present results from the first $\gamma$-ray spectroscopy of $^{35}$Mg$^{29}$ and report a new $\gamma$-ray transition observed in the $A/Z = 3$ nucleus $^{33}$Na ($N = 22$). Excited states in these very neutron-rich nuclei were populated in the fragmentation of an intermediate-energy $^{38}$Si rare-isotope beam on a $^{9}$Be target. The results are confronted with large-scale shell-model calculations.

II. EXPERIMENT

The $^{38}$Si secondary beam was produced by fragmentation of 140 MeV/nucleon $^{48}$Ca primary beam delivered by the Coupled Cyclotron Facility of the National Superconducting Cyclotron Laboratory onto a $752$ mg/cm$^2$ primary $^{9}$Be fragmentation target. The isotope of interest was selected in the A1900 fragment separator [25], using a $750$ mg/cm$^2$ Al wedge degrader for purification. The $^{38}$Si secondary beam interacted with a $376(4)$ mg/cm$^2$ thick $^{9}$Be reaction target positioned at the pivot point of the large-acceptance S800 spectrograph [26]. The reaction residues were identified on an event-by-event basis from the time of flight taken between two scintillators and the energy loss measured in the S800 ionization chamber. The flight times were corrected for the reaction residue’s trajectories as reconstructed from the position and angle information provided by the cathode readout drift chambers of the S800 focal-plane detector system. The particle identification spectrum for the reaction residues produced in $^{9}$Be($^{38}$Si,$^4$Z)X is shown in Fig. 1.

Since the measurement’s main focus was the study of $^{36}$Mg in the two-proton knockout from $^{38}$Si [13, 27, 28], the magnetic rigidity of the S800 spectrograph was set to center the longitudinal momentum distribution of $^{36}$Mg
in the S800 focal plane. Therefore, $^{35}$Mg and $^{33}$Na entered the focal plane off-center and were impacted by acceptance cuts (estimated to be of the order of 20-30%). The cross sections for the production of these nuclei from $^{38}$Si were measured to be $\sigma(^{36}$Mg) = 0.10(1) mb [13], $\sigma(^{35}$Mg) $\gtrsim 0.05$ mb and $\sigma(^{33}$Na) $\gtrsim 0.03$ mb.

The secondary $^9$Be target was surrounded by SeGA, an array of 32-fold segmented high-purity germanium detectors [29]. The high degree of segmentation was used to event-by-event Doppler reconstruct the $\gamma$ rays emitted by the reaction residues in flight. For this, the location of the segment that registered the largest energy deposition determines the $\gamma$-ray emission angle relative to the target position. Sixteen detectors were arranged in two rings (at 90° and 37° central angles with respect to the beam axis). The 37° ring was equipped with seven detectors while nine detectors occupied positions at 90°. The array was calibrated for energy and efficiency with $^{152}$Eu and $^{226}$Ra calibration standards.

### III. RESULTS

Figure 2 shows the Doppler-reconstructed $\gamma$-ray spectra measured in coincidence with $^{35}$Mg and $^{33}$Na, respectively. In the following sections we discuss the experimental results in comparison to Monte-Carlo Shell-Model calculations using the SDPF-M effective interaction [24] that allows for unrestricted mixing of neutron particle-hole configurations across the $N = 20$ gap and to large-scale shell-model calculations with the SDPF-U effective interaction [30] that does not include neutron intruder configurations in the model space.

### A. $^{35}$Mg

For $^{35}$Mg, a $\gamma$-ray transition at 446 keV is clearly visible and possibly indications of two other transitions at 621 and 670 keV. There is no evidence for $\gamma$-ray peaks at higher energies; in fact, the spectrum has remarkably few counts beyond 700 keV, likely indicative of a low neutron separation energy ($S_n(^{35}$Mg) = 1020(200) keV reported in [31] and $S_n(^{35}$Mg) = 730(460) keV estimated in the compilation by G. Audi et al. [32]).

With its even-even neighbors $^{34}$Mg and $^{36}$Mg shown to be part of the “Island of Inversion”, one expects neutron particle-hole intruder excitations to dominate the structure of $^{35}$Mg as well. The Monte-Carlo Shell-Model calculation with the SDPF-M effective interaction predicts eight excited states below 1.2 MeV, with a 5/2− ground state almost degenerate with the first 3/2− level at 30 keV excitation energy (see Figure 3 and Table I). The 446 keV transition observed in the experiment could correspond to the decay of any of the excited 7/2−, 3/2+, 1/2− or 7/2− states between 350 keV and 560 keV to either the ground state or the low-lying excited state predicted by theory. We note that it was impossible for the measurement presented here to detect a $\gamma$-ray transition below $\sim 80$ keV due to the energy threshold settings of the SeGA electronics for this run. Assuming that the neutron separation energy is indeed low for $^{35}$Mg, the possible transitions at 621 keV and 670 keV likely proceed to one or both of the $5/2^−$ or $3/2^−$ near-degenerate...
states. It might be possible that the two transitions originate from the same level and that their energy difference of 49(11) keV corresponds to the energy spacing of the alleged 3/2+–5/2− doublet near the ground state. The possible scenarios of a decay level scheme are summarized in Fig. 3, assuming that the tentatively proposed 621 and 670 keV transitions are not feeding each other based on the threshold setting of the ScGA array.

The reaction process leading to 35Mg from 38Si will not predominantly proceed as the direct removal of two protons and a neutron from 38Si but is likely dominated by the two-proton knockout into the continuum of 39Mg and subsequent neutron emission. Therefore one cannot expect the final-state and nuclear structure selectivity of a strictly direct reaction and would rather expect all bound low-lying states to be populated without preference for certain configurations. This consideration together with the predicted high level density in the MCSM and the exceptionally clean γ-ray spectrum, showing evidence for at most three γ-ray transitions, suggests that the neutron separation energy of 35Mg is indeed low – likely at the lower end of $S_n = 1020(200)$ keV – and that the γ-ray transitions we observe are not in coincidence and depopulate excited states at $\approx 500$ keV and $\approx 700$ keV to the ground state or the alleged near-degenerate excited state below 100 keV excitation energy (this corresponds to the scenario in Fig. 3 with $x \leq 80$ keV and $y = 0$ keV or $z \leq 80$ keV).

The neutron-separation energy predicted in the MCSM calculation is too low with $S_n \approx 300$ keV. This is likely related to the pairing matrix elements in the interaction as evidenced by an overestimation of the odd-even staggering in the calculation of $S_n$ in the magnesium isotopes (see Fig. 4).

Table I gives the $n\hbar\omega$ decomposition of the wave functions of all negative and positive-parity states at energies $E_x \leq 1.29$ MeV calculated with the MCSM. We note that the first state, with $J^\pi = 3/2^-$, that is dominated (56%) by $0\hbar\omega$ (non-intruder) configurations occurs at 790 keV excitation energy. All other states calculated in the MCSM in this energy window are almost pure intruder states with $1\hbar\omega$ (positive-parity states) or $2\hbar\omega$ (negative-parity states) neutron intruder configurations.

For comparison, the conventional shell-model calculations with the SDPF-U effective interaction predict only four excited states with $J^\pi = 5/2^-$, $7/2^-$, $3/2^-$ and $1/2^-$ on top of a $3/2^-$ ground state.

**B. 33Na**

The γ-ray spectrum taken in coincidence with 33Na shows two clear full-energy peaks that correspond to γ-ray transitions at 429(5) keV and 688(6) keV in the most neutron-rich Na isotope ($A = 33$) studied with γ-ray spectroscopy to date. A pioneering experiment at RIBF in RIKEN populated one excited state at 467(13) keV in 33Na with C-induced inelastic scattering and one-neutron removal [33]. We assume that this is the same γ-ray transition observed by us, noting that the authors of [33] obtained their value from combining two very different energies of 476(12) keV and 447(13) keV originating from their two different measurements, with the lower energy closer to the value reported energy by us. In agreement
TABLE I: Composition of the $^{35}$Mg wave functions with respect to $nh\omega$ probabilities calculated within the MCSM. Only excited states with $E_x \leq 1.29$ MeV are listed.

<table>
<thead>
<tr>
<th>$J^+$</th>
<th>$E$(MeV)</th>
<th>$1h\omega$ (%)</th>
<th>$3h\omega$ (%)</th>
<th>$5h\omega$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3/2^+$</td>
<td>0.36</td>
<td>96.5</td>
<td>3.5</td>
<td>0.0</td>
</tr>
<tr>
<td>$5/2^+$</td>
<td>0.81</td>
<td>97.1</td>
<td>2.9</td>
<td>0.0</td>
</tr>
<tr>
<td>$7/2^+$</td>
<td>1.29</td>
<td>97.4</td>
<td>2.5</td>
<td>0.0</td>
</tr>
</tbody>
</table>

$J^-$  | $E$(MeV) | $0h\omega$ (%) | $2h\omega$ (%) | $4h\omega$ (%) |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/2^-$</td>
<td>0.40</td>
<td>1.7</td>
<td>97.8</td>
<td>0.4</td>
</tr>
<tr>
<td>$3/2^-$</td>
<td>0.03</td>
<td>6.9</td>
<td>92.5</td>
<td>0.6</td>
</tr>
<tr>
<td>$5/2^-$</td>
<td>0.00</td>
<td>4.6</td>
<td>94.9</td>
<td>0.5</td>
</tr>
<tr>
<td>$7/2^-$</td>
<td>0.35</td>
<td>12.4</td>
<td>87.0</td>
<td>0.6</td>
</tr>
<tr>
<td>$9/2^-$</td>
<td>1.22</td>
<td>7.8</td>
<td>91.8</td>
<td>0.4</td>
</tr>
</tbody>
</table>

From the systematic studies of the onset of intruder configurations and quadrupole collectivity in the chain of Na isotopes approaching $N = 20$ [34], the question arises how the quadrupole collectivity and deformation evolves for more neutron-rich isotopes beyond $N = 20$. For this, the quadrupole moments and $B(E2)$ transition strengths for the $3/2^+$, $5/2^+$, and $7/2^+$ states in $^{33}$Na were calculated with the MCSM using $e_p = 1.3e$ and $e_n = 0.5e$ effective charges (Table II). From the experimental transition energies and the calculated $B(E2; 7/2^+ \rightarrow 3/2^+) = 67.5$ e$^2$fm$^4$, $B(E2; 2/2^+ \rightarrow 5/2^+) = 99.75$ e$^2$fm$^4$ values together with the predicted rather strong $B(M1; 7/2^+ \rightarrow 5/2^+) = 0.59\mu^2_N$ reduced M1 transition probability, the $\gamma$-ray branch for the decay to the ground state is estimated to be only 4.2% of the $\gamma$-decay branch to the $5/2^+$ state, impossible to be observed in the experiment at our level of statistics. Assuming that these states form a rotational band with $K = 3/2$ in the strong-coupling limit, the intrinsic quadrupole moments $Q_0$ were deduced. As shown in Table II, all quadrupole moments and $B(E2)$ values are well described with a common $Q_0 \sim 70$ fm$^2$. This indicates that in the MCSM calculation this structure is close to the ideal case of a well-deformed $K = 3/2$ rotational band in the strong coupling limit. This is further supported by the excitation energies; the ratio of the energy differences $E(7/2^+) - E(3/2^+)$ and $E(5/2^+) - E(3/2^+)$ in an ideal $K = 3/2$ rotational band is expected to be 2.4, while the measured energies in $^{33}$Na give a very similar ratio of 2.6. For comparison, the same energy ratio in $^{31}$Na [33] gives 3.1, indicating that the low-lying states in Na at $N = 22$ are closer to an ideal $K = 3/2$ rotational band than at $N = 20$.

To put the predicted deformation into perspective, for the $N = 22$ isotope $^{34}$Mg, the MCSM calculates $B(E2; 0^+ \rightarrow 2^+_1) = 552$ e$^2$fm$^4$ (in agreement with experiment [7, 9]) and $Q(2^+_1) = -21.4$ fm$^2$, yielding an intrinsic quadrupole moment of $Q_0 = 75$ fm$^2$, very similar to $^{33}$Na. At $N = 20$, $^{32}$Mg, the calculated $B(E2; 0^+ \rightarrow 2^+_1) = 447$ e$^2$fm$^4$ (in agreement with experiment [5–7, 9]) and $Q(2^+_1) = -17.1$ fm$^2$ lead to $|Q_0| = 67$ fm$^2$ and 60 fm$^2$, respectively, with the small discrepancy potentially indicative of triaxiality or gamma-softness.

The level scheme predicted by the conventional shell model calculations based on the SDPF-U effective interaction, which does not allow for neutron intruder configurations, has the levels in the same order but predicts the $5/2^+$ first excited state to be within 123 keV of the $3/2^+$ ground state (see Fig. 5). A strong $\gamma$-ray transition above 90-100 keV would have been observed in the present measurement.

FIG. 5: (Color online) Level schemes of $^{33}$Na as calculated in the shell model (left) compared to the measured $\gamma$-ray transitions. Assuming that the observed 429 keV and 688 keV transitions are in coincidence, the experimental level scheme agrees very well with the MCSM using the SDPF-M effective interaction.
TABLE II: Quadrupole moments, $E2$ transition strengths and extracted intrinsic quadrupole moments $Q_0$ for the $3/2^+$, $5/2^+$, and $7/2^+$ lowest-lying states in $^{33}$Na calculated with the MCSM (SDPF-M with proton and neutron effective charges of $e_p = 1.3e$ and $e_n = 0.5e$). $Q_0$ was deduced assuming that this is a $K = 3/2$ rotational band within the strong-coupling limit. With this assumption, the quadrupole moments and transition strengths in this band are very well described with a common intrinsic quadrupole moment of $Q_0 \sim 70$ fm$^2$, indicating that this structure in $^{33}$Na is indeed a rather well-deformed $K = 3/2$ rotational band within the MCSM calculation.

<table>
<thead>
<tr>
<th>$J$</th>
<th>$Q$ (fm$^2$)</th>
<th>$Q_0$ (fm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3/2^+$</td>
<td>+14.3</td>
<td>72</td>
</tr>
<tr>
<td>$5/2^+$</td>
<td>-4.3</td>
<td>60</td>
</tr>
<tr>
<td>$7/2^+$</td>
<td>-14.6</td>
<td>73</td>
</tr>
</tbody>
</table>

$J_i \rightarrow J_f^*$ $B(E2)$ | $|Q_0|$ (e$^2$fm$^4$) (fm$^2$)
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$3/2^+ \rightarrow 5/2^+$</td>
<td>263</td>
</tr>
<tr>
<td>$3/2^+ \rightarrow 7/2^+$</td>
<td>135</td>
</tr>
<tr>
<td>$5/2^+ \rightarrow 7/2^+$</td>
<td>133</td>
</tr>
</tbody>
</table>

IV. SUMMARY

In summary, in-beam $\gamma$-ray spectroscopy of the very neutron-rich nuclei $^{35}$Mg and $^{33}$Na was performed following the fragmentation of a $^{38}$Si projectile beam on a $^9$Be target at intermediate beam energy. In $^{35}$Mg, the odd-$N$ neighbor of the “Island of Inversion” nuclei $^{34}$Mg and $^{36}$Mg, $\gamma$-ray transitions were measured for the first time. In comparison to Monte-Carlo Shell-Model calculations with the SDPF-M effective interaction, the transitions are interpreted as connecting excited states around $\sim 450$ keV and $\sim 700$ keV to the $5/2^-$ ground state and/or the first excited $3/2^-$ state that is predicted to be within $30$ keV of the ground state. For the $N = 2Z$ nucleus $^{33}$Na, the most neutron-rich Na isotope studied to date with $\gamma$-ray spectroscopy, a new $\gamma$-ray transition at $688(6)$ keV was measured in addition to the known transition at $429(5)$ keV. The two transitions, if in coincidence, are in very good agreement with MCSM calculations and proposed to establish the $7/2^+ \rightarrow 5/2^+ \rightarrow 3/2^+_g$ cascade of the ground-state band in $^{33}$Na – predicted in the MCSM to be an almost ideal $K = 3/2$ rotational band structure (of intruder states) in the strong-coupling limit. Coulomb excitation or excited-state lifetime measurements are needed to confirm the degree of deformation and rotational character and remain a challenge for future experiments. All low-lying states in these two nuclei are predicted to be intruder states, putting $^{35}$Mg and $^{33}$Na inside the “Island of Inversion”.

Acknowledgments

This work was supported by the National Science Foundation under Grants No. PHY-0606007 and PHY-0758099 and in part by the JSPS Core-to-Core Program EFES and by a Grant-in-Aid for Young Scientists (No. 21740204) from the MEXT of Japan and by a Grant-in-Aid for Specially Promoted Research (No. 13002001) from the MEXT.