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Beyond modified Urca: the nucleon width approximation for flavor-changing processes in dense matter

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Flavor-changing charged current ("Urca") processes are of central importance in the astrophysics of neutron stars. The standard approach is to approximate the Urca rate as the sum of two contributions, direct Urca and modified Urca. Attempts to make Urca calculations more accurate have been blocked by an unphysical divergence in the mUrca rate at the dUrca threshold density. We argue that to make progress the traditional separation into "direct" and "modified" contributions should be replaced by a single calculation of the Urca rate that consistently includes collisional broadening of the nucleons in the medium. In this paper we describe how to perform such a calculation in a systematically improvable way. In the simplest approximation this means including an imaginary part of the nucleon mass (nucleon width). The total Urca rate is then obtained via a straightforward generalization of the dUrca calculation, yielding results that agree with both direct and modified Urca at the densities where those approximations are valid. At low densities, we observe an enhancement of the rate by more than an order of magnitude, with important ramifications for neutron star cooling and other transport properties.

INTRODUCTION

The emission and absorption of neutrinos via Urca (charged-current neutrino-nucleon) processes plays a crucial role in the formation [1] and thermal evolution [2, 3] of neutron stars, and in neutrino transport and proton fraction equilibration in supernovas and neutron star mergers [4–9].

At densities and temperatures where neutrinos are trapped and equilibrated the dominant neutrino creation/absorption mechanism is the direct Urca (dUrca) process [10], (Fig. 1(a)),

$$\begin{aligned} \mathrm{n} + &\leftrightarrow \mathrm{p} + e^- + \bar{\nu}_e \,, \\ \mathrm{n} + \nu_e &\leftrightarrow \mathrm{p} + e^- \,, \end{aligned} \tag{1}$$

However, at lower temperatures where neutrinos are free-streaming, some nuclear equations of state have a direct Urca threshold density n_{dUrca}. At baryon density n_B below n_{dUrca} the process (1) is suppressed, and the standard approach is to add the rate of a separate "modified Urca" (mUrca) process [11]

$$\begin{aligned} \mathrm{n} + N &\leftrightarrow \mathrm{p} + N + e^- \bar{\nu}_e \,, \\ \mathrm{p} + N + e^- &\leftrightarrow \mathrm{n} + N + \nu_e \,, \end{aligned} \tag{2}$$

whose Feynman diagrams are shown in Fig. 1(b). The "spectator" nucleon N undergoes a strong interaction with the participant nucleons. The standard expression for the mUrca rate comes from Friman and Maxwell (F&M) [12]. It is used across a wide range of astrophysical contexts [13, 14], including calculations of neutron star cooling [15–17] that are used to constrain the properties of nuclear matter, like the direct Urca threshold [18, 19], or the nuclear superfluid gap [20].

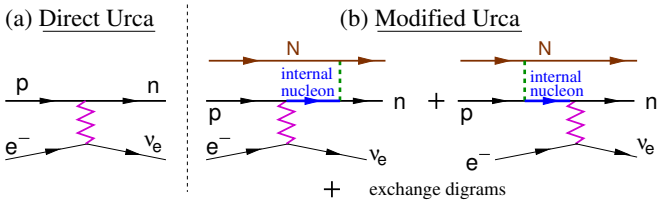


FIG. 1: Feynman diagrams for direct Urca and modified Urca contributions to the electron capture (neutrino creation) rate. The line labeled N is a spectator nucleon. The green dashed line represents a strong interaction. The modified Urca rate diverges when the internal nucleon (blue) goes on shell.

Although it is widely used, the F&M expression relies on crude approximations. As we will see, there is strong evidence that it is inaccurate by a factor of 10 or more. Frustratingly, efforts to improve on the F&M expression have been blocked by the appearance of an infrared divergence in the mUrca rate. In this Letter we show how to overcome this obstacle by dropping the idea of mUrca and dUrca as separate processes. We describe how to formulate the Urca rate as a single calculation in a systematically improvable framework. We evaluate the rate in this approach, including the leading collisional broadening contribution, which we call the nucleon width approximation (NWA), and show that it reproduces dUrca and the best estimates of mUrca without any unphysical divergence. Moreover, it is straightforward to evaluate and easy to generalize to other contexts such as arbitrary neutrino distributions, non-zero magnetic fields, high temperatures, etc. As well as eliminating the need for a separate mUrca calculation it points the way for-

ward for further improvement in the accuracy of Urca rate calculations.

To obtain an analytic expression, F&M made many simplifying assumptions such as the Fermi surface approximation (assuming all participating particles are on their Fermi surfaces), neglecting the neutrino momentum, and approximating the propagator of the internal nucleon (blue line in Fig. 1(b)) as $1/E_e$.

To go beyond these approximations one needs an improvable scheme. This would let us, for example, compute flavor relaxation rates in neutron star mergers where temperatures are comparable to the proton Fermi energy so the Fermi surface approximation is not valid, or compute the absorption mean free path of neutrinos with non-negligible momentum in matter below the dUrca threshold density, or generalize the rate calculation to scenarios such as a high magnetic field.

Improvements on F&M's calculation have typically focused on a better treatment of the strong interaction with the spectator nucleon (e.g. [14, 21]; for a review see Ref. [22]). However, using a more accurate representation of the internal nucleon propagator leads to an unphysical divergence in the mUrca rate [23] as the density approaches $n_{\rm dUrca}$ from below. Currently, the most complete mUrca calculation is that of Suleiman et. al. [24] who included lowest-order vertex corrections and numerically evaluated the full 10-dimensional phase space integral for the neutrino opacity, but had to introduce a phenomenological infrared cutoff in the charged current correlator [25, 26] to control this divergence.

In this paper, we propose a systematically improvable and practical alternative to the standard approach of calculating dUrca and mUrca as separate rates. This is the skeleton expansion leading to the *nucleon width approximation* (NWA), in which the nucleon masses are given a (density and temperature dependent) imaginary part. We focus on degenerate and homogeneous nucleonic matter. We neglect muons for simplicity. We use natural units where $\hbar = c = k_B = 1$.

URCA RATES AND THE CHARGED CURRENT CORRELATOR

It has been known for some time that Urca rates and neutral current processes can be formulated in terms of the imaginary part of the neutrino self-energy [25–36]. This is illustrated in Fig. 2; according to the Cutkowsky rules the imaginary part can be obtained by cutting the diagram, putting the cut lines on shell, and integrating over their momenta. The hadronic charged current correlator (W-boson self-energy) Π plays a central role: as shown in Fig. 2 it can be written in a skeleton expansion with a full charged current vertex $V^{\rm full}$ (red triangle) and one full neutron propagator $G_{\rm n}^{\rm full}$ and one full proton

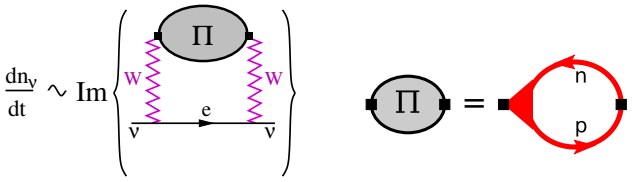


FIG. 2: Left: Feynman diagram for imaginary part of neutrino self-energy; the filled ellipse is the hadronic contribution to the in-medium charged current correlator Π. Right: Skeleton expansion for Π in terms of full vertex (red triangle) and full nucleon propagators (red lines).

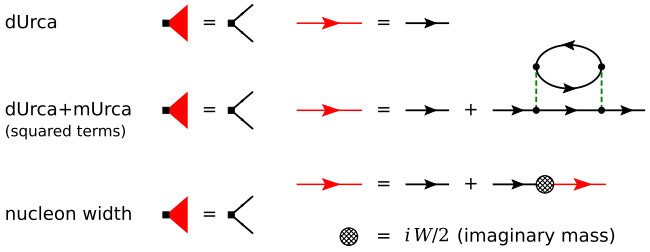


FIG. 3: Approximations used in evaluating the in-medium hadronic charged current correlator (Fig. 2). Dashed lines represent strong interactions. See text for details.

propagator $G_{\rm p}^{\rm full}$ (thick red lines),

$$\Pi_{\lambda\sigma}(q) = \int \frac{d^4k}{(2\pi)^4} {\rm Tr}\Big[V_\lambda^{\rm full} G_{\rm p}^{\rm full}(k-q) V_\sigma^W G_{\rm n}^{\rm full}(k)\Big] \ . \quad (3)$$

At nonzero temperature the integral over k_0 becomes a sum over Matsubara frequencies. In this framework we can understand the standard approaches to calculating Urca rates as different approximations to these components of the skeleton expansion (Fig. 3).

(1) Direct Urca (Fig. 3, first row) corresponds to evaluating (3) in the approximation where the vertex is derived from the bare charged current,

$$V_\mu^W = \frac{g_w \cos\theta_C}{2\sqrt{2}} \gamma_\mu (1 - g_A \gamma_5) \ , \quad (4)$$

(which can be generalized to include additional terms such as weak magnetism [37, 38]) and $G^{\rm full}$ is replaced by the mean-field nucleon propagator, with isospin index a specifying neutron or proton,

$$G_a^{\rm mf}(k) = \frac{1}{(k_0 - U_a)\gamma^0 + k_i \gamma^i + M_a^*} \ . \quad (5)$$

At finite density one includes a chemical potential in the γ^0 term and an appropriate $i\varepsilon$ prescription. The mean-field propagators include single-particle in-medium corrections via an effective mass M_a^* and energy shift U_a.

(2) Direct + modified Urca (Fig. 3, second row), the current standard approach, e.g. F&M's approximation, uses

the bare charged current vertex and adds a second term to the nucleon propagator where a model of the strong interaction is used to dress it with a single particle-hole companion (summed over n-$\bar{n}$ and p-$\bar{p}$). By substituting this in to the skeleton expansion (Fig. 2) one can see that this is equivalent to summing over the squares of the amplitudes shown in Fig. 1(b). The interference terms between these diagrams are not included (in Fig. 2 they would correspond to vertex corrections in Π) but these interference terms are already known to be a small correction [23]. The problem with the dUrca+mUrca formulation is that it only includes one collision between the protagonist nucleon and the surrounding medium, leading to the unphysical divergence discussed above. To include the in-medium effects consistently one should sum over an arbitrary number of collisions for every participating nucleon.

(3) Nucleon Width Approximation (NWA) (Fig. 3, bottom row) is a more consistent approach in which we evaluate (3) using the bare vertex (4) and dressed nucleon propagators that include an imaginary contribution to the mass for all participating nucleons. The dressed nucleon propagator can be expressed in terms of the self-energy (hatched circle) via a Schwinger-Dyson equation (Fig. 3, third row). This corresponds to summing over an arbitrary number of collisions with the medium. The self-energy is determined by a model of the strong interaction. In general it would contain a sum of different Dirac matrix structures [39], each being a function of the energy and momentum flowing through it [27, 32, 33, 40]. In the nucleon width approximation we keep only the imaginary Lorentz scalar component $iW/2$, with no energy or momentum dependence (see Results section for further discussion). The NWA nucleon propagator then has the same form as the mean-field one, with an imaginary part $iW_a/2$ added to the mass

$$G_a^{\rm NWA}(k) = \frac{1}{(k_0 - U_a)\gamma^0 + k_i\gamma^i + M_a^* + iW_a/2} \ . \quad (6)$$

Note that this approximation, with a nucleon width but no vertex correction, is not appropriate for neutral current processes such as elastic scattering of neutrinos [27, 33]. The vector component of the neutral current is the exactly conserved baryon current J_λ^B so the vector-vector component of the neutral current correlator, $\Pi_{BB}^{\lambda\sigma}(q)$, obeys a Ward identity $q_\lambda \Pi_{BB}^{\lambda\sigma} = 0$, but giving the nucleon a width without introducing compensating corrections to the vertex will violate this condition (see Ref. [41], Ch. 7.5 and 21.3). In contrast, the vector component of the charged current is the non-conserved isospin current [42]. Thus the Ward identity for the vector-vector part of the charged current correlator is already violated in vacuum by $M_{\rm n} - M_{\rm p} \sim 1\,{\rm MeV}$, and in beta-equilibrated nuclear matter by $M_{\rm n}^* - M_{\rm p}^*$ or $U_{\rm n} - U_{\rm p}$ which can be as large as tens of MeV [25, 43]. The nucleon widths that we will use are of order $T^2/(5\,{\rm MeV})$ and so for $T \lesssim 10\,{\rm MeV}$ are no larger than the intrinsic violation of isospin symmetry.

To obtain the total Urca rate in the nucleon width approximation $\Gamma^{\rm NWA}$ we substitute nucleon propagators with widths into the dUrca expression, replacing $G_a^{\rm full} \to G_a^{\rm NWA}$ (Eq. (6)).

The propagator for a fermion with nonzero width can be written as a mass-spectral decomposition [44] in terms of propagators with zero width,

$$G_a^{\rm NWA}(k, M_a^*, W_a) = \int_{-\infty}^{\infty} dm\, G_a^{\rm mf}(k, m)\, R_a(m) \ , \quad (7)$$

with the Breit-Wigner spectral function

$$R_a(m) \equiv \frac{1}{\pi} \frac{W_a/2}{(m - M_a^*)^2 + W_a^2/4} \ . \quad (8)$$

In the limit $W_a \to 0$, $R_a(m) \to \delta(m - M_a^*)$, so $G_a^{\rm NWA} \to G_a^{\rm mf}$.

Substituting (7) into (3),

$$\Pi_{\lambda\sigma}^{\rm NWA}(q) = \int_{-\infty}^{\infty} dm_{\rm n} dm_{\rm p}\, \Pi_{\lambda\sigma}^{\rm mf}(q, m_{\rm n}, m_{\rm p}) R_{\rm n}(m_{\rm n}) R_{\rm p}(m_{\rm p}) \ . \quad (9)$$

Since the Urca rate is just an integral over q of the correlator $\Pi(q)$ multiplied by a function of q that comes from the leptonic part of the neutrino self-energy (Fig. 2), and the dUrca rate is obtained by using the mean-field propagators in the correlator, this leads to a particularly simple form for the NWA Urca rate: one just "smears" the dUrca rate over a range of nucleon masses

$$\Gamma^{\rm NWA} = \int_{-\infty}^{\infty} dm_{\rm n} dm_{\rm p} \Gamma^{\rm dUrca}(m_{\rm n}, m_{\rm p})\, R_{\rm n}(m_{\rm n}) R_{\rm p}(m_{\rm p}) \ . \quad (10)$$

This expression is applicable to any Urca process, i.e. any process obtained by cutting the neutrino self-energy diagram (Fig. 2). Once the width has been obtained from a model of the strong interaction, or by a phenomenological fit, the NWA Urca rate can be straightforwardly calculated from the dUrca rate for general nucleon masses.

RESULTS

Fig. 4 shows the density dependence of the neutron decay rate in matter described by the IUF equation of state [45] at $T = 1\,{\rm MeV}$ in cold chemical equilibrium $\mu_n = \mu_p + \mu_e$ [8, 46, 47]. The NWA calculation (red line) uses nucleon widths $W_a = T^2/(5\,{\rm MeV})$ obtained from a Brueckner theory calculation for pure neutron matter using the Paris NN potential. For details see Ref. [31], Eq. (69), which also shows that for nucleons near the Fermi surface (those with surplus energy $\omega \sim T$, which are expected to make the dominant contribution) the

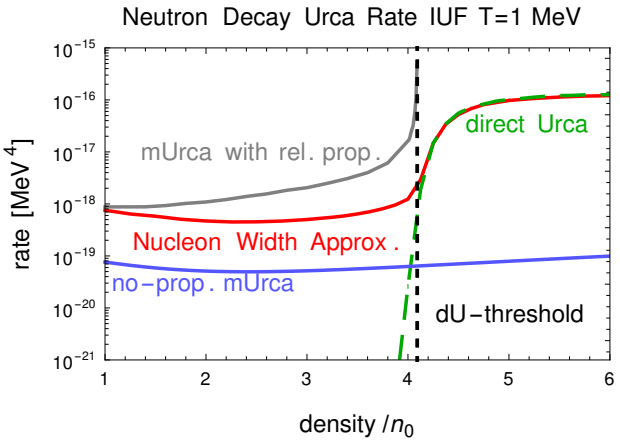


FIG. 4: Neutron decay rate as a function of density for IUF matter at $T = 1\,\mathrm{MeV}$, comparing NWA with standard approximations. The blue "no-propagator mUrca" (no-prop.) line is the traditional F&M mUrca rate where the propagator is approximated by a constant [12]. The gray "mUrca with relativistic propagator (rel. prop.)" curve is a relativistic version of the improved modified Urca calculation of Ref. [23].

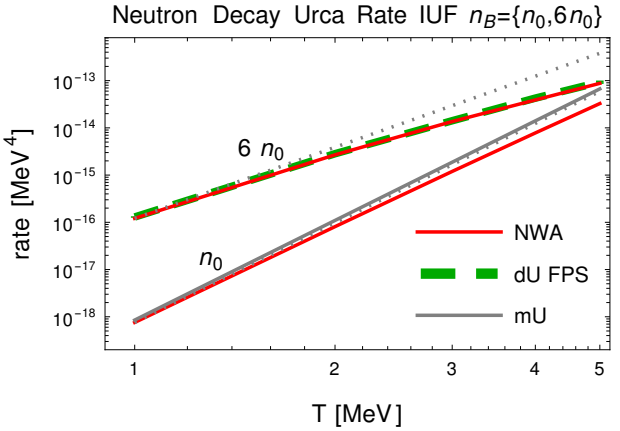


FIG. 5: Neutron decay rate as a function of temperature for IUF matter at densities $n_B = n_0 \equiv 0.16\,\mathrm{fm}^{-3}$ (well below the dUrca threshold) and $n_B = 6\,n_0$ (above threshold), showing that NWA gives the expected temperature dependence. NWA is shown in red. At the higher density we compare it to the full phase space direct Urca calculation (green dashed line). At n_B=n_0 we compare it to the improved modified Urca calculation (gray line). The dotted lines indicate the T^5 and T^7 power laws of direct and modified Urca predicted by the Fermi surface approximation.

energy dependence is a small correction to the energy-independent T^2 term. Ref. [31] also found that the width depends only weakly on density.

One could estimate the widths using other models of the strong interaction, or set the width phenomenologically by matching Γ^{NWA} to an mUrca-with-propagator calculation at a density well below the dUrca threshold. We find that the rate far below the threshold depends

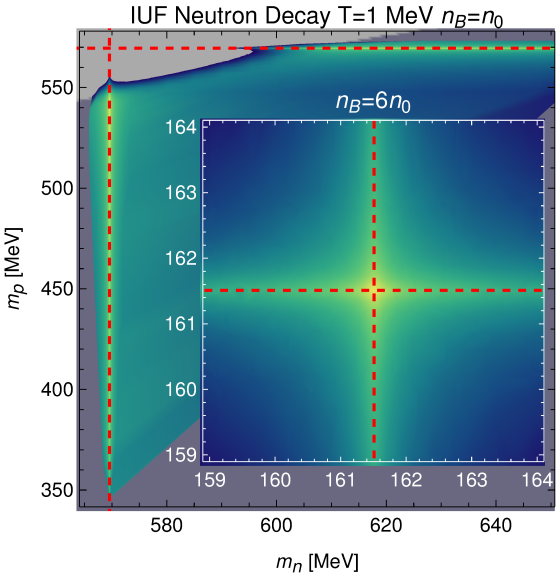


FIG. 6: The NWA Urca rate integrand (10) for the IUF EoS. Brighter (yellow) color indicates a larger integrand. Main plot is for a density below the dUrca threshold, insert is above the threshold. Dashed red lines show the effective masses M_a^*. In the outer dark gray-colored area we do not calculate the integrand because its contribution to the integrand is quadratically suppressed by the Breit-Wigner functions. In the lighter gray-shaded area in the top left corner, the integrand is exponentially suppressed by kinematic constraints.

linearly on the widths, so any update in the width calculation will propagate linearly to the rates.

In Fig. 4 the dUrca rate is calculated by evaluating the full phase space integral with relativistic matrix element as in Ref. [8]. We see that the NWA result agrees with dUrca above the dUrca threshold. Far below the threshold, it is fairly close to our version (gray line) of Ref. [23]'s improved mUrca calculation in which we include the internal nucleon propagator with relativistic kinematics and propagators for the nucleons, use the Fermi surface approximation, and model the strong interaction via one-pion exchange as in Ref. [14]. As expected, the improved mUrca rate diverges at the dUrca threshold, while NWA smoothly matches to dUrca.

Fig. 5 shows the temperature dependence of the neutron decay rate for IUF matter at two densities. Firstly $n_B = n_0$ (saturation density), far below the dUrca threshold. Secondly $n_B = 6n_0$, above the dUrca threshold. The NWA calculation uses the same width estimates as in Fig. 4. We see that above threshold NWA (solid red line) agrees very well with the dUrca calculation (dashed green line).

Far below threshold, NWA (solid red line) agrees well with the improved mUrca calculation (solid gray line). In both cases, the dotted lines show the predicted power-law behavior (T^5 for dUrca, T^7 for mUrca) at low temperatures where the Fermi surface approximation can be used

to simplify the integrals [12, 14]. We see that NWA captures the expected temperature dependence above and below the threshold.

From Figs. 4 and 5 we see that at low densities the NWA rate is enhanced by at least an order of magnitude compared to the widely used no-propagator mUrca calculation (blue line in Fig. 4). This enhancement was discovered in the improved mUrca calculation of Ref. [23], where it was accompanied by a divergence at the dUrca threshold. In NWA we see the enhancement in a well-behaved calculation that can be applied across a wide range of densities and temperatures. This may have important ramifications for any scenario where charged current neutrino interactions play an important role, such as neutron star cooling [16, 48], transport in neutron stars and neutron star mergers [6, 14, 49–51] and for the thermal states of compact stars in low-mass X-ray binaries [15, 23, 52].

To understand how NWA implements the physics that mUrca attempts to capture, we show in Fig. 6 the integrand from (10) plotted in the (m_n, m_p) plane. Below the dUrca threshold (Fig. 6, main plot) the dUrca rate $\Gamma^{\rm dUrca}(M^*_{\rm n}, M^*_{\rm p})$ is exponentially suppressed in the $T \to 0$ limit because $(M^*_{\rm n}, M^*_{\rm p})$ (intersection of dashed red lines) lies outside the kinematically allowed region. Specifically, the proton and electron Fermi momenta are too small to produce a neutron on its Fermi surface, violating the dUrca criterion $k_{F\rm p} + k_{Fe} \geq k_{F\rm n}$. We shade this kinematically suppressed area in light gray. But in the NWA integral (10) there are contributions from lower m_p (or higher m_n) (bright yellow regions) where, since the chemical potential is held constant in the spectral mass integral, the proton Fermi momentum is larger, or neutron Fermi momentum is smaller, and hence obeys the dUrca criterion. These contributions are moderately suppressed because they lie in the tails of the Breit-Wigner distribution (8), yielding the slower (than dUrca) rate that is seen in the improved mUrca calculation. Far away from both physical masses the rate becomes too small to calculate, even in the kinematically allowed region, due to the strong Breit-Wigner suppression. We shade this area in dark gray to contrast it from the kinematically forbidden, exponentially suppressed region in light gray.

At a density above the dUrca threshold (inset in Fig. 6) the in-medium masses (M^*_n, M^*_p) enter the kinematically allowed region. The Breit-Wigner function is then effectively just a slightly smeared delta function, and the mass integral yields almost the same result as setting $m_a = M^*_a$ in the rate. Thus instead of the unphysical divergence seen in mUrca calculations we obtain a smooth crossover to the standard dUrca rate.

CONCLUSIONS

In this Letter we have proposed the nucleon width approximation (NWA) as a replacement for the traditional dUrca+mUrca approximation. NWA is a preferable approach because it captures the relevant physics, namely collisional broadening of the nucleons, in a consistent fashion, and implements the first step in a systematically improvable scheme. The rate calculated using NWA, with widths taken from a Brueckner theory calculation, shows the expected behavior above and below the dUrca threshold and varies smoothly across it. The rate below threshold is enhanced by an order of magnitude compared to standard mUrca calculations.

NWA can be applied in any context where dUrca rates can be calculated, opening the door to a consistent study of Urca rates at finite temperature, which has not been possible up to this point, and other scenarios outside the scope of standard mUrca calculations such as matter with non-equilibrium neutrino distributions, strong magnetic fields, decays in some models of dark matter [53–57], or weak decays in hyperonic matter [58–62] or quark matter [63–65].

We evaluated the dUrca rate by performing the full 4D numerical phase space integral with the vacuum matrix element [8] so the NWA rate is a 6D numerical integral which is tractable because the integrand is well behaved. At low temperatures ($T \lesssim 1\,\text{MeV}$) one could alternatively use an analytic approximation for the dUrca rate based on the Fermi surface approximation with either a constant [14] or an angle-averaged matrix element. These approaches agree with the full integral to within a factor of about $2-7$ at low T. Then the NWA rate becomes a 2D numerical integral of a well-behaved peaked integrand as shown in Fig. 6.

As well as investigating applications of NWA, for example, to neutrino opacities in mergers or neutron star cooling, a natural next step would be to pursue the systematic improvement scheme outlined in this letter: (1) explore other estimates of the nucleon width, e.g. from chiral effective theory [66]; (2) include other contributions (with different Dirac index structures) to the nucleon self-energy; (3) allow for momentum and energy dependence of the self-energy; (4) include vertex corrections, for example an RPA resummation along the lines of Ref. [67].

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