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Negative components in the representation $\frac{d^2(\sigma E)}{dE^2}$ of heavy-ion fusion cross sections

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Negative components are observed in the representation $d^2(\sigma E)/dE^2$ in heavy-ion fusions related to recent studies of noticeable oscillations in the high-energy region of the fusion excitation functions. A modified multi-Gaussian model has been developed, which includes contributions for both positive and negative components in the $d^2(\sigma E)/dE^2$ representation. The model reproduces the heavy-ion fusion excitation functions very well, including oscillations. The property of these negative components in the $d^2(\sigma E)/dE^2$ spectrum and the reaction mechanism of fusion at energies above the Coulomb barrier are investigated. The model also reproduces well light heavy-ion fusion reactions between ^{12}C and ^{16}O at high energy; results imply that the compound nucleus channel effect may have an important **contribution** at the high energy region for all fusion systems.

Fusion reactions are the most complex ones in heavy-ion collisions. The understanding of the reaction dynamics has been improved progressively during the last sixty years. Many international conferences have been devoted to the topic [1] and have provided for important new developments in heavy-ion fusion studies. A number of review articles can be found in the literature. Some recent ones are listed as Ref. [2–7] together with the data collection in Ref. [8].

In this article the influence of compound nuclei on the fusion cross section (the compound channel effect) is discussed. It is of interest with regard to structures in fusion excitation functions; but yet not investigated so often as the entrance channel effect in the literature.

After many historical developments, Stelson [9] presented a new method, calculating the fusion excitation function with the expression $\sigma(E) = \int \sigma(E, B)D(B)dB$, and fitting experimental data. Where $\sigma(E, B)$ is a *known* (previously obtained) excitation function with a single barrier height B . Most importantly, Stelson assumed that there are more than one barrier with different heights during the collision, and introduced the barrier height distribution $D(B)$. The function $D(B)$ contains adjustable parameters which will be determined in fitting experimental data. In general, $\sigma(E, B)$ can be either an analytic function, a known experimental data-set, or results of a model calculation.

Rowley *et al.* [10] found that under some approximations the experimental barrier height distribution $D(B)$ can be extracted from a precise measurement of the fusion excitation function via the second derivative. (Barrier height is a measurement of energy, $D(B)$ is often expressed as $D(E)$):

$$D(E) = \frac{1}{\pi R^2} \frac{d^2(\sigma(E)E)}{dE^2}, \quad \text{with} \quad \int D(E)dE = 1. \quad (1)$$

Here, R is the barrier radius, After that, many studies have concentrated on obtaining the barrier height distribution, and the understanding of the corresponding re-

action dynamics by comparing experimental results with coupled channels (CC) calculations. CC calculations did reproduce some peaks of the second derivative in the low energy region, with couplings to the excitation of the low-lying states of the entrance channel and/or some transfer-reaction channels, thus validating Rowley's idea.

However, CC calculations often do not reproduce the experimental second derivative at higher energies, even though they describe the average cross sections there quite well. One might argue that the 'non-perfect' behavior at higher energies will be solved by more refined CC calculations in the future. But there are other important issues.

In Ref. [11], an empirical model of heavy-ion fusion cross sections has been suggested. By applying the *reciprocal* on another fusion representation:

$$B_{\text{test}}(E) = \frac{d^2(\sigma E)}{dE^2} = \pi R_0^2 D_{\text{test}}(E), \quad \text{with} \quad \int D_{\text{test}}(E)dE = 1, \quad (2)$$

one obtains the cross section formula:

$$\begin{aligned} \sigma(E)E &= \int_{E_0}^E \left(\int_{E_0}^{E'} \frac{d^2(\sigma E'')}{d(E'')^2} dE'' \right) dE' \\ &= \pi R_0^2 \int_{E_0}^E \left(\int_{E_0}^{E'} D_{\text{test}}(E'') dE'' \right) dE', \end{aligned} \quad (3)$$

where R_0 is a normalization constant for the function $D_{\text{test}}(E)$. **The lower integration limits in Eq. 3 are $E_0 = -Q$ or 0, depending on whether the fusion Q value is negative or positive.**

Any reasonable analytic function $D_{\text{test}}(E)$ with adjustable parameters can be used in Eq. (3). Comparing results with the experimental fusion excitation function by using least-squares fittings, the optimum parameters for the assumed $D_{\text{test}}(E)$ function can be determined.

When Rowley *et al.* derived Eq. (1), the approximations were, that the classical expression of fusion cross

section for a barrier of radius R was used, and that the radius R is independent of the barrier height $B(E)$. Eq. (2) is similar to Eq. (1), but is only a definition without any approximations. All approximations are contained in the assumption of the function $B_{test}(E)$.

Excellent results have been obtained by assuming $D_{test}(E)$ to be a sum of L Gaussian (LG) distributions (Eq. (15) of Ref.[11]), where parameters V_i , W_i , and ω_i are the centroids, the standard deviations, and the weights of the i -th Gaussian functions, with the condition, $\sum \omega_i = 1$. L is the number of Gaussians used in $D_{test}(E)$. With this assumption, the cross sections are given by an analytic function with $Z_i = (E - V_i)/\sqrt{2}W_i$ (Eq. (16) of Ref.[11]).

This model reproduces heavy-ion fusion excitation functions well, including the deep sub-barrier region (fusion hindrance phenomenon); much better than the previous models (including the CC calculations).

This multi-Gaussian model is an empirical one, based on Rowley's idea of the barrier distribution. The contribution from any barrier component during the collisions is 'real', being positive in physics. For any component in the multi-Gaussian model, its contribution is proportional to $R_i^2 = R_0^2 \omega_i$, and is positive. However, a puzzle appeared in fitting results. All Gaussian components concentrate at energies lower or nearby the Coulomb barrier. Nothing appears at higher energies.

In Ref. [12], it was mentioned that at energies higher than the Coulomb barrier, there are weak, but noticeable oscillations/structures in the heavy-ion fusion excitation functions. In that study, the question was raised, what is the role of those negative components appearing in the experimental $d^2(\sigma E)/dE^2$ spectra at high energies. Fig. 1, is a close reproduction of Fig. [1] in Ref. [12] for the fusion reaction $^{40}\text{Ca} + ^{90}\text{Zr}$ [13]. In Fig. 1(a) a three-Gaussian fit (black, with parameter set g34090 in Table I) compares with the experimental $d^2(\sigma E)/dE^2$ spectrum. The theoretical calculations show two bumps containing three peaks (the lowest bump is a combination of two Gaussian peaks) located at energies lower than 100 MeV. Above 100 MeV, the calculated curve is a flat line of zero value. Obviously, this is due to the reason that the experimental data in the energy region higher than 100 MeV have negative and positive components mixed. The flat, zero-value line is the average result in the fitting procedure since the model includes only positive components. But the oscillations/structures in the fusion cross sections are generated from both negative and positive peaks in the representation $d^2(\sigma E)/dE^2$.

For heavy-ion fusion reactions at higher energies, negative components have shown on experimental second-derivative spectra very often, e.g. Ref.[2]. As mentioned above, CC calculations reproduce positive components well at low energy in the second-derivative spectra and confirm Rowley's finding. CC calculations do not give a good reproduction for the higher energy part, and have

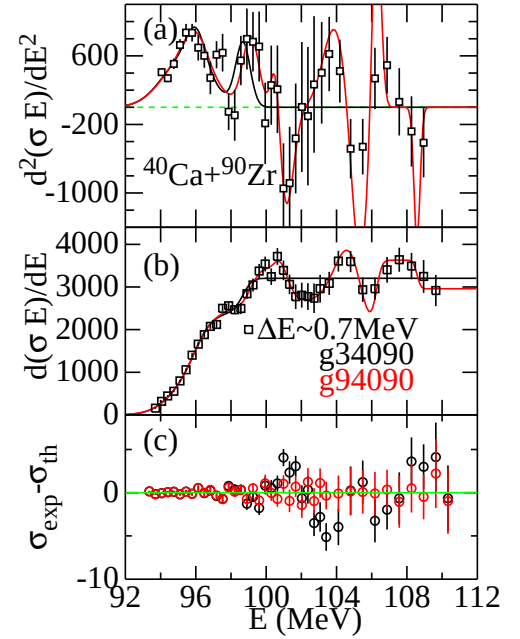


FIG. 1: Spectra for the fusion system $^{40}\text{Ca} + ^{90}\text{Zr}$ [13]: (a) $d^2(\sigma E)/dE^2$, (b) $d(\sigma E)/dE$ and (c) $\sigma_{exp} - \sigma_{th}$. Curves are calculations with Eq. (5). black: g34090; red: g94090 (see Table I). Units of the y-axis are mb/MeV, mb, mb, respectively. Green lines are horizontal one with value 0.

not explained those negative components. To our knowledge, it was only mentioned in Ref. [14] for CC calculations, that the $d^2(E_{c.m.}\sigma_{fus})/dE_{c.m.}^2$ is expected to become slightly negative, and to remain so, at energies above the barrier. But that is only a minute effect without structures.

These negative components in the $d^2(\sigma E)/dE^2$ spectrum have to be investigated in order to understand the reaction mechanism at the energy region above the Coulomb barrier. At lower energies, barrier distributions have been obtained from $d^2(\sigma E)/dE^2$. However, at higher energies, $d^2(\sigma E)/dE^2$ can be something else.

In the following we will develop a new model starting from the original multi-Gaussian model, but including also the negative Gaussian components. The assumed second derivative of the energy weighted σ is:

$$(d^2(\sigma E)/dE^2)_{assum} = \sum_{i=1}^L P_i \exp\left[-\left(\frac{E - V_i}{C_i}\right)^2\right]. \quad (4)$$

For each Gaussian component there are three parameters, P_i , V_i , and C_i , that correspond to $(\pi\omega_i R_i^2)/W_i$, V_i , and $\sqrt{2}W_i$ in the previous multi-Gaussian model, respectively. Now, parameter P_i can be either positive or negative. Similarly, by using the first part of Eq. 3, another analytic cross-section expression is obtained:

$$\sigma(E)E = \sum_{i=1}^L P_i \frac{C_i^2}{2} \left[\sqrt{\pi} Z_i \text{erfc}(-Z_i) + \exp(-Z_i^2) \right], \quad (5)$$

TABLE I: Least-squares fitting results from the modified multi-Gaussian model for fusion systems studied in this article. For example, g94090 is the name of a parameter set for the fusion reaction $^{40}\text{Ca} + ^{90}\text{Zr}$, which has 9 Gaussian components. Each component has three parameters (defined in Eq. [4]). There is a '/' between Gaussian component sets. Some of these parameter sets (like g34090) have only positive components, others have both positive and negative components.

system	parameter set, P_i , V_i and C_i (mbMeV $^{-1}$, MeV and MeV)
$^{40}\text{Ca} + ^{90}\text{Zr}$	g34090= 772.7, 95.85, 2.004/ 658.6, 98.95, .3803/ 318.6, 107.0, .2876 g94090= 574.5, 95.53, 1.938/ 337.8, 96.01, .9217/ 816.1, 99.13, .6407/ 838.6, 100.55, .3733/ -1193, 101.11, .6256/ 924.3, 103.9, .9389/ -1803, 105.3, .6656/ 2502, 106.3, .3463/ -1564, 108.57, .2420
$^{16}\text{O} + ^{154}\text{Sm}$	G416154= 120.9, 55.11, 2.016/ 237.7, 58.84, 2.979/ 354.2, 60.28, 2.581/ 15.62, 60.91, .0921 G616154= 130.3, 54.82, 1.822/ 479.2, 58.59, 2.152/ 877.8, 61.02, .5733/ 819.2, 63.63, .4146/ -1314, 65.44, .5557/ 822.9, 66.96, .7557
$^{34}\text{S} + ^{168}\text{Er}$	G334168= 172.9, 117.36, 3.927/ 267.5, 123.58, 3.974/ 137.4, 127.27, 1.682 G534168= 168.7, 117.34, 4.045/ 275.3, 123.56, 3.914/ 176.3, 128.59, 1.632/ 64.52, 132.84, 1.230/ -50.93, 143.15, 7.949
$^{40}\text{Ca} + ^{94}\text{Zr}$	G34094= 523.1, 94.95, 2.545/ 101.9, 90.45, 2.121/ 361.6, 99.76, .5957 G74094= 518.5, 95.07, 2.556/ 108.6, 90.71, 2.280/ 433.7, 100.57, .9388/ -506.2 102.45, 1.179/ 693.8, 104.09, .6682/ -1337, 108.88, .6378/ 1099, 107.74, .4862
$^{12}\text{C} + ^{16}\text{O}$	g11216= 853.0, 7.74, 1.081 g91216= 853.0, 7.74, 1.081/ 1600, 12.31, .6720/ -4492, 13.68, .5335/ 4671, 14.86, .4611/ -2184, 16.71, .8231/ 3095, 20.58, 1.284/ -6280, 21.06, .6976/ 3096, 22.41, .5727/ -2321, 25.07, 1.069
$^{12}\text{C} + ^{12}\text{C}$	g11212= 825.6, 6.03, .9391 g71212= 825.6, 6.03, .9391/ 3367, 14.08, .6043/ -3973, 14.29, .5762/ 3175, 19.77, .6010/ -4456, 20.40, .5512/ 2191, 23.84, .4832/ -2427, 25.52, .4634
$^{16}\text{O} + ^{16}\text{O}$	g11616= 846.1, 9.52, 1.032 g81616= 840.2, 9.91, 1.187/ 2985, 17.99, .5171/ -8823, 18.89, .5729/ 4807, 19.24, .6683/ -13219, 27.38, .2992/ 2782, 28.04, .1786/ -5482, 31.05, .5522/ 3130, 30.07, .3231

with $Z_i = (E - V_i)/C_i$.

In our least-squares fitting, the fitting is for the experimental cross sections, not for the derivatives of the experimental data. Similar to Ref. [11], smallest values of χ_0^2 and χ^2 are used to select the best results.

In Fig. 1, red curves are results from the least-square analysis done with the modified multi-Gaussian model. The best result was obtained with nine Gaussian components, with three P_i being negative. Evidently, reproductions with the modified multi-Gaussian model (red curves) are better than with the previous multi-Gaussian model (black curves) for all three representations. For the comparisons in (a) and (b), one has to consider the ambiguity problem for mathematical calculations of differentiation as discussed in Refs. [11, 15], and the uncertainties for the experimental data of the first derivative ($d(\sigma E)/dE$) are much less than of the second derivative ($d^2(\sigma E)/dE^2$). One can see in (c), remaining oscillations/structures appear for the case of the black curve, but they have been reproduced in red for the whole energy range. That is, the calculations are within the uncertainties of the data points. The parameter sets used in Fig. 1 are labeled g34090 and g94090 (3 and 9 Gaussian components). Each component has three parameters as mentioned above, values are listed in Table I.

In Fig. 2, results for three other fusion systems are shown: $^{16}\text{O} + ^{154}\text{Sm}$ [16], $^{34}\text{S} + ^{168}\text{Er}$ [17], and $^{40}\text{Ca} + ^{94}\text{Zr}$ [18]. Results for the previous model and for the new model are shown with different colors (black for pure positive and red for the modified one). Fits (with 6, 5 or 7 Gaussian, respectively, including negative components) give good reproductions of the fine structures. Parameter sets, g616154, g534168, and g74094 together with g416154, g334168, and g34094 are given in Table I.

From the above analyses, we first conclude that negative components in the $d^2(\sigma E)/dE^2$ representation are not 'arbitrary' and are important. Second, one may get reaction mechanism information through the results from the new multi-Gaussian model. This will be described in the following.

The barrier distribution description responds to the entrance channel effects. At high energies, it can only simulate the average behaviors but can not describe the detailed oscillations/structures. **From present studies, in addition to entrance channel effects, contribution from other reaction mechanism should be considered.** It has suggested in Refs. [12, 19] that for heavier fusion systems, at energies higher than the Coulomb barrier, compound channel effects may be important **either**.

It is known that fusion reactions between ^{12}C and ^{16}O

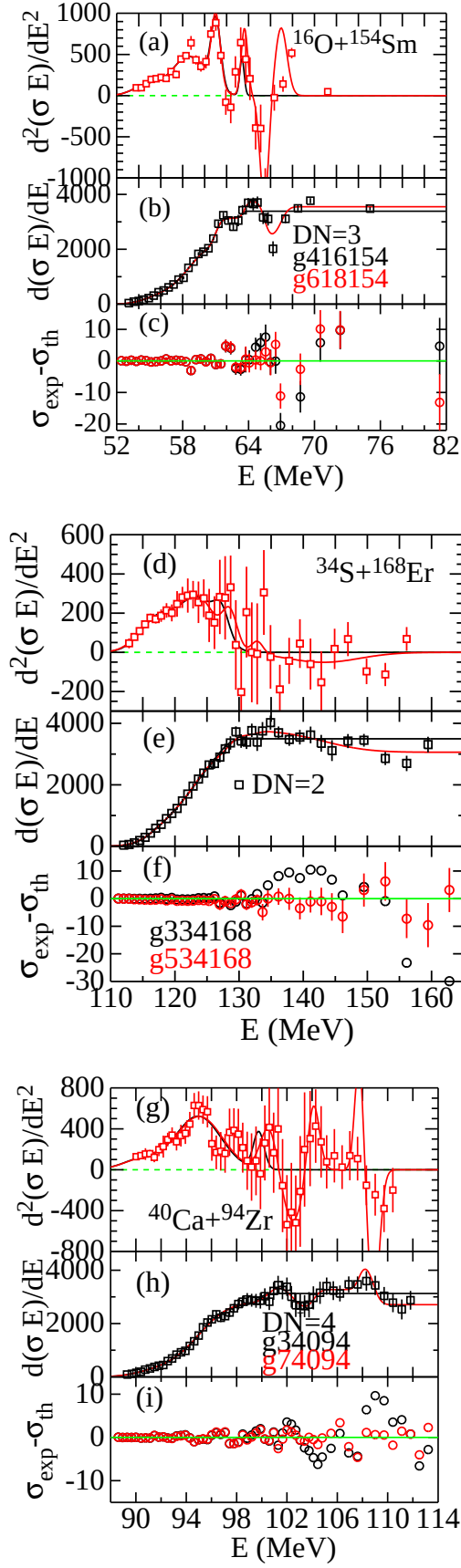


FIG. 2: Similar as Fig. 1 for systems $^{16}\text{O} + ^{154}\text{Sm}$ [16], upper, $^{34}\text{S} + ^{168}\text{Er}$, [17] middle, and $^{40}\text{Ca} + ^{94}\text{Zr}$ [18], lower. Black: g416154, g334186, or g34094. Red: g616154, g534168, g74094.

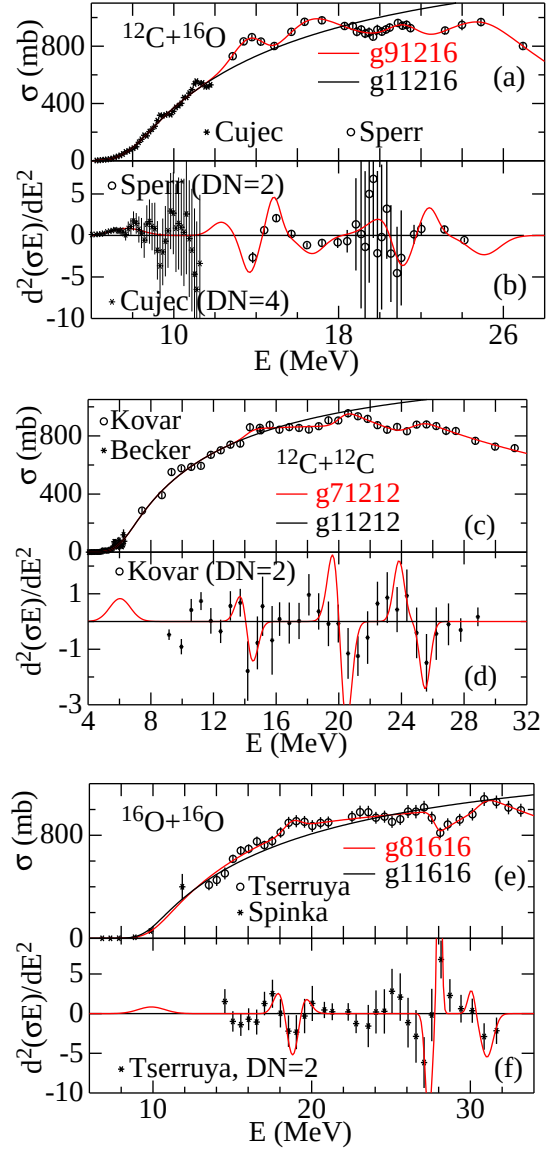


FIG. 3: Spectra for systems $^{12}\text{C} + ^{16}\text{O}$ (upper) [20] and $^{12}\text{C} + ^{12}\text{C}$ (middle) [22]. $^{16}\text{O} + ^{16}\text{O}$ (lower) [24]. Black curves are obtained with parameter sets g11216 and g11212, and g11616, and supply average reproductions for data at low energies. Well reproductions are obtained for high energy data. Results are shown with red curves, g91216 g71212, and g81616, respectively. Unit for $d^2(\sigma E)/dE^2$ is b/MeV.

nuclei are complex, and are important in astrophysical studies at low energy. Complex resonance peaks, together with the steeply rising character of the cross section present difficulties in providing explanations (e.g., see Ref. [5, 7]). For these reactions where entrance channel effects and compound channel effects, both display strongly. At higher energies, evident resonance structures were observed as early as in the 1970's [20–24].

We may ask, does the new multi-Gaussian prescription reproduce these complex resonance-like excitation func-

tions at higher energies as well? We don't expect the model will reproduce them well at low energy, since very complex structures appear. But an average description at low energies can be obtained from the single-Gaussian model, helping with that we may reproduce the structures at higher energies with the present model.

Analyses for fusion of $^{12}\text{C} + ^{16}\text{O}$ [20], $^{12}\text{C} + ^{12}\text{C}$ [22] $^{16}\text{O} + ^{16}\text{O}$ [24] are shown in Fig. 3. A single-Gaussian parameter set (g11216, g11212 or g11616) is included as the first component of the multi-Gaussian parameter set, respectively. The other components in the parameter set, are located in the the high-energy data region of interest. Fitting results are red curves with parameter sets g91216, g71212 and g81616, respectively (Table I). There are a lot of experimental data at low energies for these three reactions. For simplicity, only a few of cross sections is shown in each case. For second derivatives, a few of them is shown for reaction $^{12}\text{C} + ^{16}\text{O}$ only. Obviously these are average descriptions, respectively.

In all cases, the new multi-Gaussian model reproduces those resonance-like excitation functions and other representations at high energies very well, just like what has been seen for the heavier systems in Figs. 1 and 2. There is a difference, being that the amplitudes of variation on the cross sections are much weaker in the heavier systems. One has to use the representation $\sigma_{exp} - \sigma_{th}$ to demonstrate the phenomenon.

When the structures in the light systems were first observed it was assumed to be a resonance phenomenon [20–22]. Later it was found ([23, 24]), that those structures can be reproduced from the penetration of centrifugal barriers by incident waves of different angular momentum, l . For the symmetric systems (e. g. $^{12}\text{C} + ^{12}\text{C}$ or $^{16}\text{O} + ^{16}\text{O}$) only even l waves contribute to the fusion reaction, then separations in energy between successive angular moment are relatively large. Later, Esbensen did CC calculations with this idea [25], rather good reproductions for fusion of $^{12}\text{C} + ^{12}\text{C}$ and $^{16}\text{O} + ^{16}\text{O}$ were obtained. The idea was extended to study fusion reactions as heavy as $^{28}\text{Si} + ^{28}\text{Si}$ by Esbensen [26] and Montagnoli *et al.*, [27, 28].

For fusion system heavier than, e.g. $^{28}\text{Si} + ^{28}\text{Si}$, many reaction channels are expected to open at high angular momentum and energies, and the effect of couplings to these channels will smear out the peak structure [26]. It is noted that, a) For heavier fusion systems, CC calculations never predict evident negative components in the $d^2(\sigma E)/dE^2$ spectrum, but in the experiments, they exist; b) The above mentioned CC calculations seem poorer for the system $^{12}\text{C} + ^{16}\text{O}$ than for symmetric ones, $^{12}\text{C} + ^{12}\text{C}$ and $^{16}\text{O} + ^{16}\text{O}$; c) Similarities are observed in the analyses with the modified multi-Gaussian model between the heavier fusion systems and the three light fusion systems. This may imply that one should further study the reaction mechanism for these light fusion reactions at energies well above the Coulomb barrier.

At this point, we can describe the physics meaning of the new, modified multi-Gaussian model as the following: At the energy region where the fitting result shows that all Gaussian components are positive, the model mainly describes entrance channel effects. At the higher energy region where the fitting results show that Gaussian components are mixtures of positive and negative ones, the model manifests that the reaction mechanism may need also those other than entrance channel effects; indicating that the compound channel effect could be important.

It appears important to perform more precise, smaller energy step experiments in heavier fusion systems. In Ref. [19], the overlap phenomenon for the average behavior of excitation functions to the same compound nucleus (those spectra displayed in logarithmic-scale plots) is rather promising, while the overlap between the spectra of $d(\sigma E)/dE$ (fine structure) is seen with large uncertainty, as indicated in the paper. Thus, it will be interesting to see, whether the overlap phenomenon appears only well at the level of average behavior of the excitation functions, or does appear also well at the $d(\sigma E)/dE$ level. If the later is true, then that is a strong evidence for the compound nucleus effect.

In summary, a modified multi-Gaussian model has been developed, in order to study the unexplored phenomenon of negative components in the representation $d^2(\sigma E)/dE^2$ of fusion cross sections. The model can reproduce well all three representations: cross sections, $d(\sigma E)/dE$, and $d^2(\sigma E)/dE^2$, including their fine structures for heavier fusion systems. It leads to a conclusion that, at the lower-energy region where the model fitting result shows that all Gaussian components are positive, the entrance channel effect is the main reaction mechanism. At higher-energies, where the fitting result shows that Gaussian components are a mixture of positive and negative, the model suggests that reaction mechanisms other than the entrance channel effect, e.g. the compound channel effect may be of importance.

Finally, a perhaps indirectly related remark: While the present study focused on light- and medium-mass fusion systems, the overall trends observed and reproduced with the empirical model calculations might also raise interest in detailed high-quality measurements of fusion excitation functions in heaviest-mass systems. In particular also in regard to the production and study of nuclei far from stability and very heavy compound nuclei. Where prompt fission is the dominant decay channel, leading to many orders-of-magnitude reduction of the survival of the compound-nucleus. At such level the compound nucleus channel effects discussed here, might become relevant. (See also recent model calculations, for example [29, 30].)

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