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Photoproduction of pion pairs at high energy and small angles

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Abstract

We explore the photoproduction mechanisms for charged and neutral pion pairs off a heavy nucleus at threshold. We calculate the production of charged pairs in the Coulomb field of the nucleus in the Born approximation using explicit expressions for the differential cross sections and their connection with the total cross section $\sigma(\gamma\gamma \rightarrow \pi^+\pi^-)$. The production of σ mesons at threshold and their subsequent decay into pions is an important mechanism and is investigated quantitatively in a framework that can be used for both charged and neutral pions. σ mesons can be produced via electromagnetic (photon exchange) or strong (ω exchange) interactions. For neutral pions, another production mechanism consists of the two-step process where charged pions are produced first and then charge-exchange into two neutral pions. Differential and total cross sections are computed for each production mechanism.

1 Introduction

The photoproduction of charged pion pairs on the proton $\gamma p \rightarrow \pi^+\pi^- p$ has been widely investigated beginning from the seminal experiments at SLAC [1]. At high energies $E_\gamma \gtrsim 3\text{ GeV}$, the pairs are primarily produced diffractively (Pomeron exchange) in the P -wave (orbital momenta $L=1$) producing $\rho(770)$ vector mesons and the $f_2(1270)$ tensor mesons (D -wave), which are easily observable in the $\pi^+\pi^-$ invariant mass spectrum. In addition, the charged pion pairs may be produced by photon or vector (ω , ρ) meson exchanges with the target nucleus (Fig. 1). Such pairs are produced in the S -wave ($L = 0$) resulting in scalar mesons: $f_0(500)$ and $f_0(980)$. The threshold region is dominated by the $f_0(500)$ meson, which we will refer to as

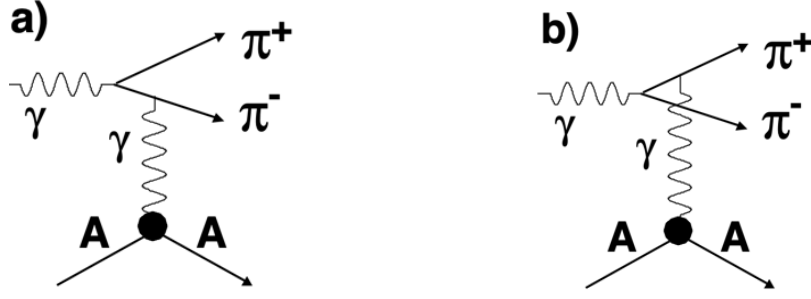


Figure 1: Diagrams for the Primakoff production of charged pion pairs.

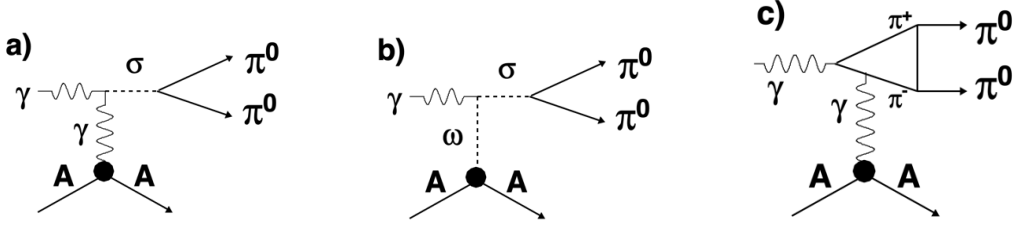


Figure 2: Diagrams for the production of neutral pion pairs through a) Primakoff σ -meson production and decay, b) strong σ -meson production and decay, and c) production via charge exchange.

the σ -meson in this paper. The production of neutral pion pairs is a result of scalar meson production decaying to $\pi^0\pi^0$ (Fig. 2a, b) or the production of charged meson pairs with subsequent charge exchange ($\pi^+\pi^- \rightarrow \pi^0\pi^0$, Fig. 2c).

Understanding double pion photoproduction at threshold is crucial for experiments whose goal is to measure the pion polarizability using the Primakoff production of two pions [2, 3]. In addition, it can also be an important background to single meson Primakoff production experiments of η, η' with the goal of determining their lifetimes with high precision [4]. Therefore, this paper presents the formalism and calculations for electromagnetic and strong production of two pions in the field of a nucleus. We start by estimating the contribution to the photoproduction cross section of charged pions in the Coulomb field and then present the relevant expressions that describe the production of neutral pion pairs.

2 Photoproduction of charged pion pairs in the Coulomb field

The reaction of interest can be expressed as

$$\gamma(k) + A(p) \rightarrow \pi^+(q_+) + \pi^-(q_-) + A(p'), \quad (1)$$

where the four-vectors of each particle are given in parentheses. The differential cross section for the photoproduction of charged pion pairs in the Coulomb field of nucleus

(Fig. 1) can be obtained using the techniques from Ref. [5] for $M \gg 2m$, where m and M are the single pion and pion pair invariant masses:

$$d\sigma = \frac{8\alpha^3 Z^2}{(2\pi)^2} \frac{d^2 q}{Q^4} \left(\vec{q}_- (\vec{q}_+^2 + m^2) + \vec{q}_+ (\vec{q}_-^2 + m^2) \right)^2 \frac{\beta_- (1 - \beta_-) d\beta_- d^2 q_-}{(\vec{q}_-^2 + m^2)^2 (\vec{q}_+^2 + m^2)^2} |F_{em}(q, q_L)|^2. \quad (2)$$

Here and below we are using the following notation:

- $k = (E, \vec{k})$ are the incident photon 4-momentum, energy and 3-momentum;
- $k' = (E', \vec{k}')$ are the pion pair 4-momentum, energy and 3-momentum;
- Ω, θ are the solid and polar angles of the produced pion pair;
- $\vec{q}_{+,-}$ the transverse part of the positive/negative pion 3-momenta relative to the incident photon direction;
- $\vec{q} = \vec{q}_- + \vec{q}_+$ the transverse part of the transfer momentum;
- $q_L = \frac{M^2}{2k}$ is the longitudinal transfer momentum;
- $F_{em}(q, q_L)$ is the nucleus electromagnetic form factor;
- β_- is the fraction of photon energy taken by the π^- ;
- Z is the charge of the nucleus;
- $Q^2 = \vec{q}^2 + q_L^2$;
- and $t = (k - k')^2$ is the Mandelstam variable.

In the Primakoff region, where the transverse nuclear recoil can be safely neglected, $\vec{q}^2$ is small and one can approximate $\vec{q}_-^2 \approx \vec{q}_+^2$, the expression simplifies to

$$d\sigma = \frac{8\alpha^3 Z^2}{(2\pi)^2} \frac{\vec{q}^2 d^2 q}{Q^4} \frac{\beta_- (1 - \beta_-) d\beta_- d^2 q_-}{(\vec{q}_-^2 + m^2)^2} |F_{em}|^2. \quad (3)$$

Recalling that the invariant mass of the pair $M^2 = \frac{\vec{q}_-^2 + m^2}{\beta_- (1 - \beta_-)}$ and replacing $d^2 q$ with πdt we obtain:

$$\frac{d\sigma}{dt} = 2\alpha^3 Z^2 \frac{\vec{q}^2}{Q^4} \frac{dM^2}{M^4} |F_{em}|^2. \quad (4)$$

This expression was derived under the assumption of large pion pair mass M , where the asymptotic value for $\sigma(\gamma\gamma \rightarrow \pi^+\pi^-) = 2\pi\alpha^2/M^2$. A more accurate calculation is obtained by using the full expression for the cross section as a function of pair invariant mass [6]:

$$\sigma(\gamma\gamma) = \frac{2\pi\alpha^2}{M^2} \left[\sqrt{1 - \frac{4m^2}{M^2}} \left(1 + \frac{4m^2}{M^2} \right) - \frac{4m^2}{M^2} \left(2 - \frac{4m^2}{M^2} \right) \ln \left(\frac{M}{2m} + \sqrt{\frac{M^2}{4m^2} - 1} \right) \right]. \quad (5)$$

We note that the cross section $\sigma(\gamma\gamma \rightarrow \pi^+\pi^-)$ is affected by higher order corrections due to the polarizability (see for example Ref. [2]). This calculation is compared to data in Fig. 3 where the calculation is scaled by a factor of 0.6.¹ Equation 4 can be

¹This naive scaling factor accounts for the first-order correction due to the limited kinematic coverage of the experimental data.

rewritten as:

$$\frac{d\sigma}{dt} = \frac{2\alpha Z^2}{\pi} \frac{\vec{q}^2}{Q^4} \frac{dM}{M} \sigma(\gamma\gamma) |F_{em}|^2. \quad (6)$$

Equations 2–4 and 6) were obtained for $M \gg 2m$, but the differential cross section of the pair production can be expressed via the $\sigma(\gamma\gamma \rightarrow \pi\pi)$ as in Eq. 6 for any M values [6, 7]. Eq. 6 coincides with the equation (5.49) in Ref. [6]. Finally, substituting dt with $\frac{|\vec{k}||\vec{k}'|}{\pi} d\Omega$, we obtain:

$$\frac{d\sigma}{d\Omega dM} = \frac{2\alpha Z^2}{\pi^2} \frac{|\vec{k}|^2 |\vec{k}'|^2 \sin^2\theta}{Q^4} \frac{1}{M} \sigma(\gamma\gamma) |F_{em}|^2. \quad (7)$$

Thus, the photoproduction of charged pion pairs in the Primakoff region has the same angular dependence as in the case of single π^0 meson photoproduction [8], while dependence on invariant mass of the pair M is determined by the cross section of the process $\gamma\gamma \rightarrow \pi^+\pi^-$.

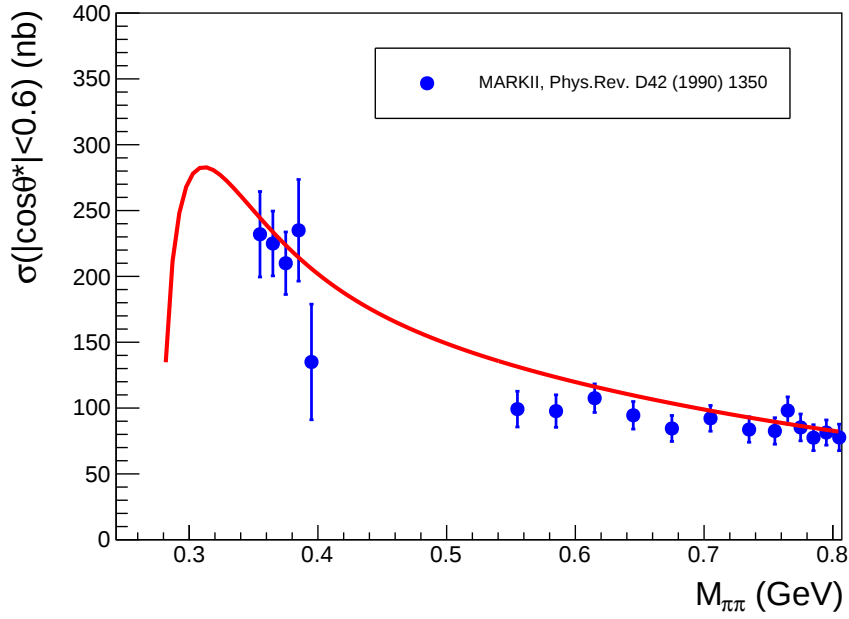


Figure 3: The total cross section of the process $\sigma(\gamma\gamma \rightarrow \pi^+\pi^-)$ (equation 5), scaled by factor of 0.6. The points with errors are the experimental data for the solid angle range $|\cos(\theta^*)| < 0.6$ from the Ref. [9] (Table III).

3 Neutral pion pair photoproduction in the Coulomb field

Neutral pion pairs can be produced via a two step process: the photoproduction of scalar resonance f , followed by its decay to two pions (Fig. 2a). The cross section of this process

$$\gamma(k) + A(p) \rightarrow f + A \rightarrow \pi^0 + \pi^0 + A(p'), \quad (8)$$

can be written in the factorized form as

$$\frac{d\sigma}{dt dM^2} = \sum_f \frac{1}{\pi} \frac{d\sigma_f(t)}{dt} \frac{m_f \Gamma(f \rightarrow \pi^0 \pi^0)}{(M^2 - m_f^2)^2 + m_f^2 \Gamma_{tot}^2}. \quad (9)$$

Here $\frac{d\sigma_f(t)}{dt}$ is the differential cross section of the scalar meson f off the nucleus $\gamma + A \rightarrow f + A$; $\Gamma(f \rightarrow \pi^0 \pi^0)$ and Γ_{tot} are its partial and total decay widths, respectively; M is two pion invariant mass, and m_f is f -meson mass. We adopt cross section normalization as $\int \frac{1}{\pi} \frac{d\sigma}{dt dM^2} dM^2 = B_r \frac{d\sigma}{dt}$, where B_r is the branching fraction of the f meson decay to $\pi^0 \pi^0$. The use of the Breit-Wigner formula in Eq. 9 is a poor approximation for the $f_0(500)$ meson because of its large width ($\Gamma_{tot} \sim 0.500$ GeV). However, this representation is illustrative and can be easily substituted with a more realistic description without affecting the basic results of the present work. We note that the $f_0(500)$, which is also referred to as the σ meson, dominates the cross section at small invariant masses [10].

The differential cross section of σ meson photoproduction in the nuclei Coulomb field is similar to π^0 photoproduction and reads:

$$\frac{d\sigma_f}{dt} = \Gamma(f \rightarrow \gamma\gamma) \frac{8\pi\alpha Z^2 \beta}{m_f^3} \frac{\vec{q}^2}{Q^4} |F_{em}|^2, \quad (10)$$

where β is the σ meson velocity [5, 11]. Thus the photoproduction cross section for the σ meson has the same angular dependence as the well known [8] photoproduction cross section of the π^0 with replacement of relevant masses and decay widths. Substituting this expression in Eq. 9 we get the differential cross section of two π^0 photoproduction in the Coulomb field of nucleus:

$$\frac{d\sigma}{dt dM^2} = \frac{8\alpha Z^2 \beta}{m_\sigma^2} \frac{\vec{q}^2}{Q^4} \frac{\Gamma(\sigma \rightarrow \gamma\gamma) \Gamma(\sigma \rightarrow \pi^0 \pi^0)}{(M^2 - m_\sigma^2)^2 + m_\sigma^2 \Gamma_{\sigma tot}^2} |F_{em}|^2. \quad (11)$$

This expression allows one to calculate the neutral pion pair photoproduction in the Coulomb field of the nucleus for any target, invariant mass of the pair, and transfer momentum values. The meson radiative width can be written as

$$\Gamma(f \rightarrow \gamma\gamma) = \left(\frac{m_f}{4\pi}\right)^3 \left(\frac{\alpha}{F_f}\right)^2 N_c^2 <e_f^2>^2, \quad (12)$$

where N_c the number of colors, $\langle e_f^2 \rangle$ the average of relevant quark charges and $F_\sigma = 92.4 \text{ MeV}$ is the charge meson decay constant. For the π^0 meson $N_c \langle e_\pi^2 \rangle = 3((2/3)^2 - (1/3)^2) = 1$, whereas for the σ meson $N_c \langle e_\sigma^2 \rangle = 3((2/3)^2 + (1/3)^2) = \frac{5}{3}$. The radiative decay width $\Gamma(\sigma \rightarrow \gamma\gamma)$ calculated by this expression yields $\Gamma(\sigma \rightarrow \gamma\gamma) = 1.1 \text{ keV}$. However, for our calculations we have adopted the more reliable estimate of $\Gamma(\sigma \rightarrow \gamma\gamma) = 2.05 \text{ keV}$ [12]. For the σ decay to $\pi^0\pi^0$ partial width, we use the value of $\Gamma(\sigma \rightarrow \pi^0\pi^0) = 0.210 \text{ GeV}$ [13].

4 Neutral pion pairs via the charge-exchange reaction

As it was mentioned above, the $\pi^0\pi^0$ pair can be produced as a result of charge pion pair photoproduction followed by charge exchange reaction $\pi^+\pi^- \rightarrow \pi^0\pi^0$. The expression in Eq. 6 for the differential cross section of charge pion pair photoproduction is also valid for the neutral pion pairs photoproduction if one replaces the $\sigma(\gamma\gamma)$ with the total cross section $\sigma(\gamma\gamma \rightarrow \pi^0\pi^0)$ of the two step process $\gamma\gamma \rightarrow \pi^+\pi^- \rightarrow \pi^0\pi^0$. This cross section has been calculated in the framework of the Chiral theory[14] and gives:

$$\begin{aligned}\sigma(\gamma\gamma \rightarrow \pi^0\pi^0) &= \frac{\alpha^2}{8\pi^2} \left(1 - \frac{4m^2}{M^2}\right) \left(1 + \frac{m^2}{M^2} f(M^2)\right) \sigma(\pi^+\pi^- \rightarrow \pi^0\pi^0) \\ \sigma(\pi^+\pi^- \rightarrow \pi^0\pi^0) &= \frac{1}{32\pi M^2 F_\pi^4} (M^2 - m^2)^2, \\ \text{where : } f(M^2) &= 2 [\ln^2(z_+/z_-) - \pi^2] + \frac{m^2}{M^2} [\ln^2(z_+/z_-) + \pi^2]^2, \\ z_\pm &= \frac{1 \pm (1 - 4m^2/M^2)^{1/2}}{2}.\end{aligned}\tag{13}$$

Substituting this expression in Eq. 6, one obtains the differential photoproduction cross section of two neutral pions generated by the charge-exchange reaction. We note that this contribution is about 2 orders of magnitude smaller than the Coulomb production in the threshold region (see Fig. 4).

5 Vector mesons exchanges in $\pi^0\pi^0$ photoproduction

The photoproduction of neutral pion pairs in S-wave can also proceed through vector-meson exchange in addition to photon exchange. However, we only consider ω -meson exchange to the coherent photoproduction on nuclei because the contribution from isospin one exchange (ρ meson) is small and can be safely neglected. To estimate the contribution of ω exchange, we could proceed as in the case of π^0 meson photoproduction [15], by substituting $\Gamma(\sigma \rightarrow \gamma\gamma)$ with $\Gamma(\sigma \rightarrow \omega\gamma)$ and replacing the

139 photon propagator with the ω propagator in Eq.11. An alternative approach, which
 140 we implement here, is to use the simple parametrization from Ref. [16], which was used
 141 previously to calculate the the strong amplitude for ω exchange to the production
 142 of π^0 photoproduction on the nucleon with $f_N(\gamma N \rightarrow \pi^0 N) = L(\frac{E}{k_0})^{1.2} \sin\theta$, where
 143 $L = 10\sqrt{\mu b}$, $k_0=1$ GeV [17]. For the strong amplitude of σ meson photoproduction,
 144 one can follow the same procedure in Ref. [16] by scaling the amplitude with the co-
 145 efficient of 5/3 from Eq. 12. With this modification, the strong part of the differential
 146 cross section of σ meson photoproduction off a nucleus with atomic mass A in the
 147 Primakoff region reads as follows:

$$\frac{d\sigma_{str}}{dt} = A^2 \frac{25\pi}{9E^2} f_N^2 |F_{str}(q, q_L)|^2 \quad (14)$$

148 The full differential cross section for neutral pion pair photoproduction can be written
 149 similarly to the case of π^0 photoproduction in the Primakoff region as:

$$\frac{d\sigma}{dtdM^2} = \frac{1}{\pi} \left(\frac{d\sigma_{em}}{dt} + \frac{d\sigma_{str}}{dt} + \sqrt{\frac{d\sigma_{em}}{dt} \frac{d\sigma_{str}}{dt}} \cos\varphi \right) \frac{m_\sigma \Gamma(\sigma \rightarrow \pi^0 \pi^0)}{(M^2 - m_\sigma^2)^2 + m_\sigma^2 \Gamma_{\sigma tot}^2} \quad (15)$$

150 where $\frac{d\sigma_{em}}{dt}$ and $\frac{d\sigma_{str}}{dt}$ are given by the expressions in Eqs.10 and 14 and φ is the
 151 relative phase between the Coulomb and strong amplitudes. The difference of the
 152 strong form factor $F_{str}(q, q_L)$ from the π^0 case [16] is minor. One has to replace the
 153 pion mass in the longitudinal momenta transfer q_L with the σ meson mass and also
 154 replace the total cross section with nucleon $\sigma(\pi N) \rightarrow \sigma(\sigma N)$.

155 We briefly digress to elaborate on some of our assumptions and approximations,
 156 which are at the level of about 10% for the strong cross section. First, we do not
 157 take into account the shadowing of the initial photon, noting that the effect is very
 158 small for the Primakoff cross section. For the strong cross section it is still a small
 159 correction; for example in the case of single pion photoproduction [16] its effect is less
 160 than 10%. The final-state interactions of the produced meson are treated assuming
 161 that the σ meson travels through the nucleus with a cross section of $\sigma = 2\sigma(\pi N)$.
 162 However, a proper calculation will depend on the decay location, since the interaction
 163 of a single σ meson with the nucleon is different than that of two pions. Changes
 164 arise from different total cross sections and also due to differences in kinematics.
 165 Within the quark model, $\sigma(\sigma N) = \frac{5}{3}\sigma(\pi N)$, which is close to $2\sigma(\pi N)$. Given the
 166 limits of our knowledge of most cross sections, incorporating these details will not
 167 lead to a significantly more reliable result. However, we can estimate the size of
 168 these approximations by re-calculating with the assumption that the σ meson decays
 169 outside the nucleus and $\sigma(\sigma N) = \frac{5}{3}\sigma(\pi N)$, see Table 1. The difference coming from
 170 these two input values results in changes to the strong cross section of about 15%,
 171 depending on the dipion mass. The Primakoff cross section changes by less than 3%.
 172 The full effect of absorption on the Coulomb and strong form factors can be seen in
 173 Fig. 8 and Fig. 10, respectively.

Table 1: Percent changes to the cross section of $\sigma(\gamma\text{Pb} \rightarrow \pi\pi\text{Pb})$ for two assumptions: 1) The σ meson decays at the interaction point and the cross section $\sigma(\sigma N) = 2\sigma(\pi N)$ and 2) the σ -meson decays outside the nucleus and the cross section $\sigma(\sigma N) = (5/3)\sigma(\pi N)$. The calculations were completed for two different values of the di-pion mass.

Cross Section Affected	$M_{\pi\pi}=270$ MeV	$M_{\pi\pi}=770$ MeV
Coulomb	1.3%	3%
Strong	17%	14%
Interference	10%	7%

174 Within this context, the electromagnetic and strong form factors of the nucleus
175 read as follows [16]:

$$\begin{aligned}
F_{em}(q, q_L) = & 2\pi \frac{q^2 + q_L^2}{q} \int_0^\infty J_1(qb) b^2 db \int_{-\infty}^{+\infty} \frac{e^{iq_L z}}{(b^2 + z^2)^{3/2}} dz \\
& \times \exp\left(-\frac{\sigma' A}{2} \int_z^\infty \rho(b, z') dz'\right) \int_0^{\sqrt{b^2 + z^2}} x^2 \rho(x) dx \quad (16)
\end{aligned}$$

176

$$F_{str}(q, q_L) = -\frac{2\pi}{q} \int_0^\infty J_1(qb) b db \int_{-\infty}^{+\infty} \frac{\partial \rho(b, z)}{\partial b} e^{iq_L z} dz \exp\left(-\frac{\sigma' A}{2} \int_z^\infty \rho(b, z') dz'\right) \quad (17)$$

177 Here $\rho(b, z)$ is the nucleus density; $J_1(x)$ is the first order Bessel function; $\sigma' =$
178 $\sigma_{tot}(\sigma N)(1 - i \frac{Ref(0)}{Imf(0)})$; $f(0)$ is the forward amplitude of $\sigma + N \rightarrow \sigma + N$; the in-
179 variant transfer momenta: $t = -q^2 - q_L^2 = -4 |\vec{\mathbf{k}}| |\vec{\mathbf{k}}'| \sin^2(\frac{\theta}{2}) - (\frac{m_\sigma^2}{2E})^2$. Substituting
180 expressions from Eqs. 10 and 14 for the electromagnetic and strong cross sections into
181 Eq. 15 one can calculate the momentum transfer and invariant mass dependence of
182 the differential cross section for double π^0 production off a nucleus.

183 Figures 4, 5, and 6 show differential cross sections as a function of the pion pair
184 production angle and invariant mass obtained with the presented calculations for
185 lead-208 nucleus and a 5.5 GeV photon beam energy. For numerical calculations,
186 we take the interference angle between the Coulomb and strong mechanisms to be
187 $\phi = 1$ rad, which is close to the value measured for single π^0 production [18].

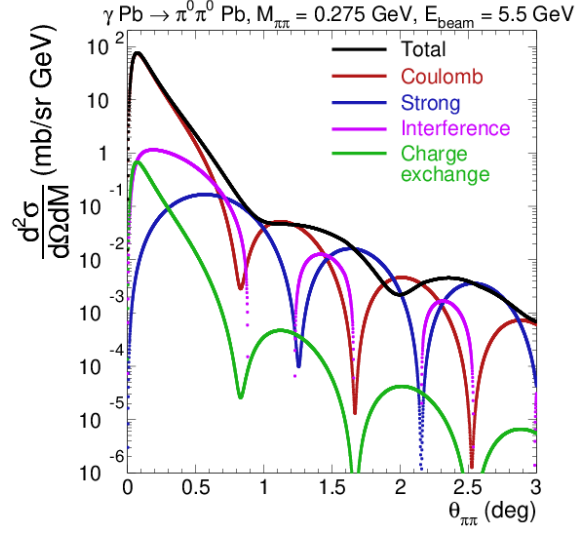


Figure 4: Double differential cross section $\frac{d^2\sigma}{d\Omega dM}$ of two neutral pion photoproduction and its components as a function of two pion production angle for lead-208 nucleus, 0.275 GeV pion pair mass, and 5.5 GeV photon beam energy. Red – Coulomb cross section for the amplitude shown in Fig. 2a; blue – strong cross section (Fig. 2b); magenta – cross section from the interference of Coulomb and strong amplitudes for 1 radian interference angle; green – cross section for the charge exchange amplitude shown in Fig. 2c; black – the total cross section, which is the sum of all contributions.

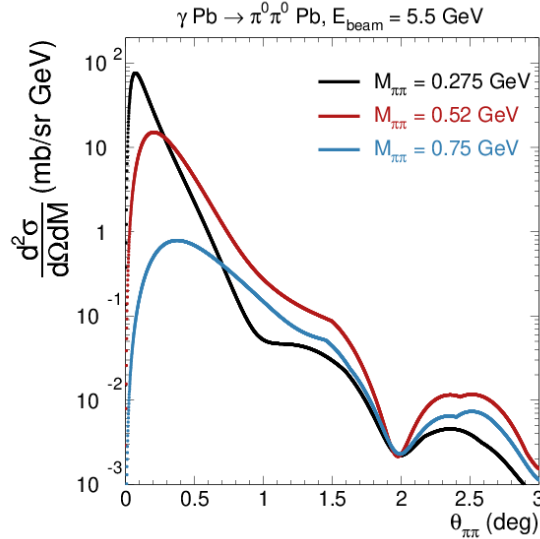


Figure 5: Total double differential cross section $\frac{d^2\sigma}{d\Omega dM}$ of neutral pair pion photoproduction as a function of the pion pair production angle for lead-208 nucleus, 5.5 GeV photon beam energy, and interference angle between Coulomb and strong mechanisms 1 rad. Black curve – 0.275 GeV pion pair mass, red – 0.52 GeV, blue – 0.75 GeV.

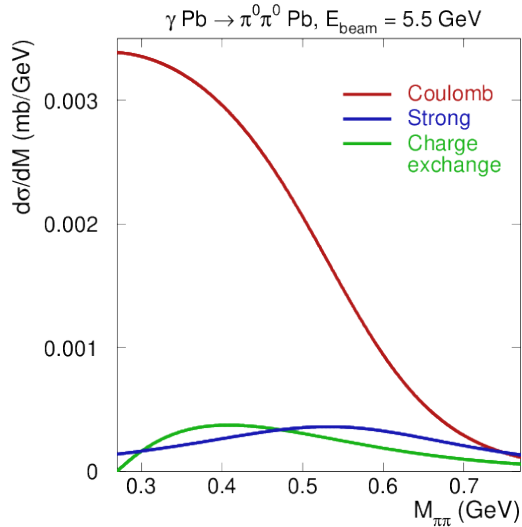


Figure 6: Components of the differential cross section $\frac{d\sigma}{dM}$ of neutral pion pair photo-production as a function of the two-pion invariant mass for lead-208 nucleus, 5.5 GeV photon beam energy (integrated over production angle). Red – Coulomb cross section for the amplitude shown in Fig. 2a; blue – strong cross section (Fig. 2b); green – cross section for the charge exchange amplitude shown in Fig. 2c.

6 Summary

In this paper we presented explicit calculations for the most important production mechanism for pion pairs at forward angles and at threshold. The production of $\pi^+\pi^-$ pairs was calculated in the Born approximation. The production of $\pi^0\pi^0$ pairs at threshold is dominated by the production and decay of the σ meson, which has been calculated for electromagnetic (photon exchange) and strong (ω exchange) mechanisms. An additional production mechanism for neutral pion pairs proceeds via the two-step process where two charged pions are produced and then rescatter into two neutral pions, which is found to be approximately 2 orders of magnitude smaller than the Coulomb contribution in the threshold region. Differential and total cross sections are computed for each production mechanism using realistic form factors in all cases.

Recently there have been several experimental programs aimed at measuring mesons at very small production angles in photoproduction in Hall D at Jefferson Lab. An ongoing experiment is measuring the Primakoff production of η mesons with the goal of determining its lifetime to high precision [4]. Experiments to measure the Primakoff production of charged [2] and neutral [3] pion pairs are scheduled to run at Jefferson Lab in the near future. Therefore, quantitative estimates of the production of pion pairs at threshold and small angles will be important for the experimental program. In the first case these pairs represent background and in the second case they are the signal.

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References

- [1] J. Ballam et al. Bubble Chamber Study of Photoproduction by 2.8-GeV and 4.7-GeV Polarized Photons. 1. Cross-Section Determinations and Production of ρ^0 and Δ^{++} in the Reaction $\gamma p \rightarrow p\pi^+\pi^-$. *Phys. Rev. D*, 5:545, 1972.
- [2] JLAB Experiment E12-13-008. Measuring the Charged Pion Polarizability in the $\gamma\gamma \rightarrow \pi^+\pi^-$ Reaction, 2013. <https://misportal.jlab.org/mis/physics/experiments/viewProposal.cfm?paperId=789>.
- [3] JLAB Experiment E12-13-008A. Measuring the Neutral Pion Polarizability, 2020. https://halldweb.jlab.org/DocDB/0043/004373/004/Pi0_Polarizability_Proposal-30.pdf.
- [4] JLAB Experiment E12-10-011. A Precision Measurement of the eta Radiative decay Width via the Primakoff Effect, 2010. <https://misportal.jlab.org/mis/physics/experiments/viewProposal.cfm?paperId=631>.
- [5] S.R. Gevorkyan, E.A. Kuraev, and M.K. Volkov. Photon and vector meson exchanges in the production of light meson pairs and elementary atoms. *J. of Phys. G: Nucl. Part. Phys.*, 40(1):015001, nov 2012.
- [6] V.M. Budnev, I.F. Ginzburg, G.V. Meledin, and V.G. Serbo. The two-photon particle production mechanism. Physical problems. Applications. Equivalent photon approximation. *Phys. Rep.*, 15(4):181–282, 1975.
- [7] A. Halprin, C.M. Andersen, and H. Primakoff. Photonic Decay Rates and Nuclear-Coulomb-Field Coherent Production Processes. *Phys. Rev.*, 152:1295–1303, 1966.
- [8] R. Miskimen. Neutral pion decay. *Ann. Rev. Nucl. Part. Sci.*, 61:1–21, 2011.
- [9] J. Boyer et al. Two photon production of pion pairs. *Phys. Rev. D*, 42:1350, 1990.
- [10] H. Marsiske et al. A Measurement of $\pi^0\pi^0$ Production in Two Photon Collisions. *Phys. Rev. D*, 41:3324, 1990.
- [11] M.M. Kaskulov and U. Mosel. Primakoff production of π^0 , η and η' in the Coulomb field of a nucleus. *Phys. Rev. C*, 84:065206, 2011.
- [12] L-Y. Dai and M. R. Pennington. Two photon couplings of the lightest isoscalars from BELLE data. *Phys. Lett. B*, 736:11–15, 2014.
- [13] S.S. Agaev, K. Azizi, and H. Sundu. The strong decays of the light scalar mesons $f_0(500)$ and $f_0(980)$. *Phys. Lett. B*, 784:266–270, 2018.

- 245 [14] J.F. Donoghue, B.R. Holstein, and Y. C. Lin. The Reaction $\gamma \gamma \rightarrow \pi^0 \pi^0$ and Chiral Loops. *Phys. Rev. D*, 37:2423, 1988.
246
- 247 [15] I. Larin et al. A New Measurement of the π^0 Radiative Decay Width. *Phys. Rev. Lett.*, 106:162303, 2011.
248
- 249 [16] S. Gevorkyan et al. Photoproduction of Pseudoscalar Mesons off Nuclei at Forward Angles. *Phys. Rev. C*, 80:055201, 2009.
250
- 251 [17] A. Browman et al. The Decay Width of the Neutral pi Meson. *Phys. Rev. Lett.*, 33:1400, 1974.
252
- 253 [18] I. Larin et al. Precision measurement of the neutral pion lifetime. *Science*, 368(6490):506–509, 2020.
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A Numerical calculations and visualization

A.1 Primakoff cross section

The Primakoff cross section proceeds via photon exchange for the process $\gamma(k) + A(p) \rightarrow f + A \rightarrow \pi^0 + \pi^0 + A(p')$. Let us calculate the two dimensional distribution of $\vec{q}^2, M^2$ using the expression

$$\frac{d\sigma}{d\Omega dM} = \frac{2M|\vec{k}||\vec{k}'|}{\pi} \frac{d\sigma}{dt dM^2}, \quad (18)$$

where $d\sigma/dt dM^2$ is given by Eq. 11. We have used the following numerical values in the calculations:

$$\beta = 1; m_\sigma = 0.5 \text{ GeV}; Q^2 = \vec{q}^2 + q_L^2; \vec{q}^2 = 4|\vec{k}|^2 |\vec{k}'|^2 \sin^2 \theta / 2 \approx (E\theta)^2; q_L = \frac{m_\sigma^2}{2k}$$

$$0.275 \text{ GeV} \leq M \leq 0.675 \text{ GeV}; 0 \leq q^2 \leq 0.01 \text{ GeV}$$

$$\Gamma_{\sigma tot} = 0.5 \text{ GeV}; \Gamma(\sigma \rightarrow \gamma\gamma) = 2.05 \cdot 10^{-6} \text{ GeV}; \Gamma(\sigma \rightarrow \pi^0\pi^0) = 0.21 \text{ GeV}$$

The Coulomb double differential cross section $d^2\sigma/d\Omega dM$ is plotted in Fig. 7. and the electromagnetic form factor is given by Eq. 16. The upper limit of integration goes to infinity since the Coulomb field covers all space. For numerical calculations we note that outside of the nucleus the absorption exponent is a constant. Thus we can take the exponent term outside the integral and this allows us to use the following analytical formulas to integrate the remaining function

$$\int_0^\infty \frac{\cos(ax)}{(b^2 + x^2)^{3/2}} dx = \frac{a}{b} K_1(ab); \int_0^\infty \frac{\sin(ax)}{(b^2 + x^2)^{3/2}} dx = \frac{\pi}{2} \frac{a}{|b|} \{L_{-1}(ab) - I_1(|ab|)\},$$

where L_{-1} is the modified Struve function, and K_1 and I_1 – the corresponding Bessel functions. In this approach, we will need to subtract the difference between this constant term and the actual value of the absorption exponent within the nucleus, which has a limited size and thus convenient for the purpose of numerical calculations. Using this method it is also easy to estimate the necessary region of integration for the variable b to get precise form factor values. For large pion pair masses, we need to integrate b up to about 50 – 80 fm for convergence. For the pion pair masses near threshold, we need to integrate to about 700 – 1000 fm to achieve a similar level of convergence.²

The plots in Fig. 8 show the real and imaginary parts of the Coulomb form factor as a function of the production angle for $M_{\pi\pi} = 0.275 \text{ GeV}$ and 0.765 GeV with and without including absorption. As seen from the plots, the presence of the absorption not only decreases the value of the real part of the form factor values but also

²Electron screening may play a role at the sub-percent level at such large distances from the nucleus.

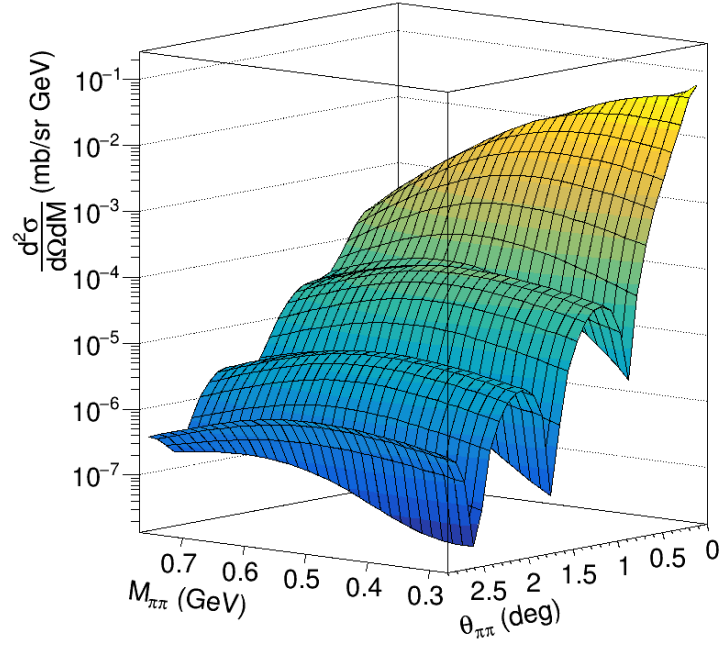


Figure 7: Double differential cross section $\frac{d^2\sigma}{d\Omega dM}$ of the Coulomb neutral pion pair photoproduction as a function of the pair production angle and invariant mass for lead-208 nucleus, and 5.5 GeV photon beam energy.

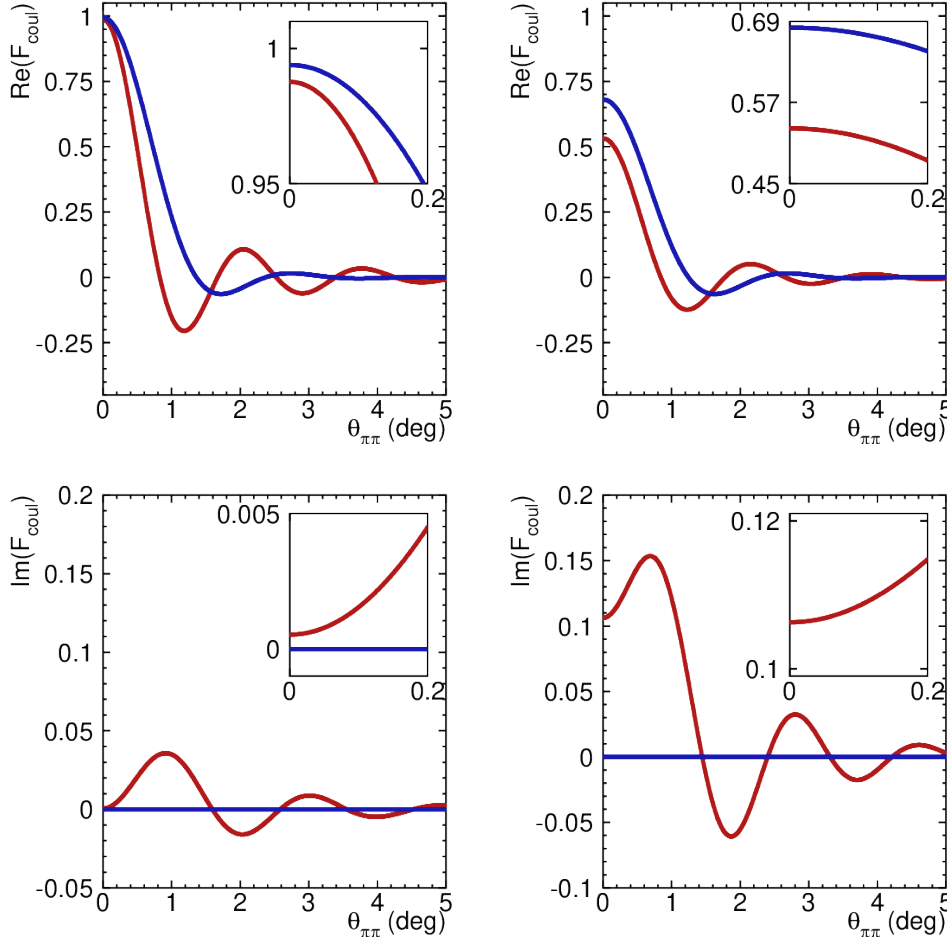


Figure 8: Real (top plots) and imaginary (bottom plots) part of the Coulomb form factor as a function of production angle for the invariant mass values: 0.275 GeV (left plots) and 0.765 GeV (right plots). Red curves – full calculation, blue curves – no absorption. The function behavior near the zero zoomed in the insertions.

enhances the oscillatory behavior. We note that the interaction cross section in the absorption term is actually a function of the pion energy and depends on the energy split between the pions in the pair. Including this dependence properly could change our calculations by more than a percent.

A.2 Electromagnetic and Strong cross sections

The electromagnetic and strong cross sections integrated over invariant mass:

$$\frac{d\sigma}{d\Omega} = \frac{|\vec{k}||\vec{k}'|}{\pi} \frac{d\sigma}{dt}; \quad \frac{d\sigma}{dt} = \frac{d\sigma_{em}}{dt} + \frac{d\sigma_{str}}{dt} + \sqrt{\frac{d\sigma_{em}}{dt} \frac{d\sigma_{str}}{dt}} \cos \varphi, \quad (19)$$

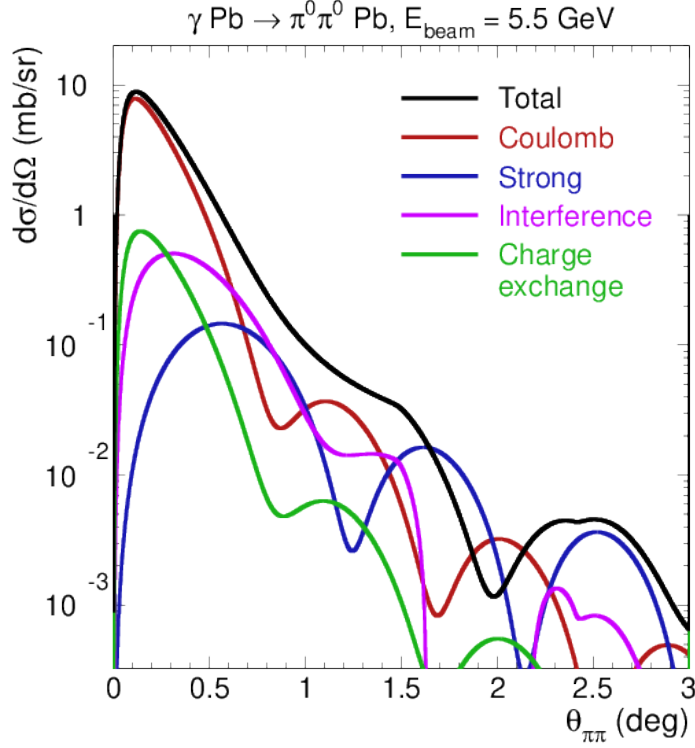


Figure 9: Differential photoproduction cross section $\frac{d\sigma}{d\Omega}$ of neutral pion pairs for each component as a function of two pion production angle for a lead-208 nucleus, 5.5 GeV photon beam energy, and integrated over the pion pair mass range between 0.27 and 0.77 GeV. Red–Coulomb part, blue–strong, magenta–their interference for 1 radian interference angle (negative values are omitted on the log scale), green – charge exchange, black – total sum.

where the expressions for the two components can be found in Eqs. 10 and 14. They are computed for $E = 5.5 \text{ GeV}$, and the strong form factor is given by Eq. 17. For the pion pair invariant mass integration range of 0.27–0.77 GeV, the resulting cross section $\frac{d\sigma}{d\Omega} = \int_{0.27 \text{ GeV}}^{0.77 \text{ GeV}} \frac{d^2\sigma}{dM d\Omega}$ shown in Fig. 9 for each term separately as well as for the total. The strong form factor as a function of production angle value is plotted in Fig. 10.

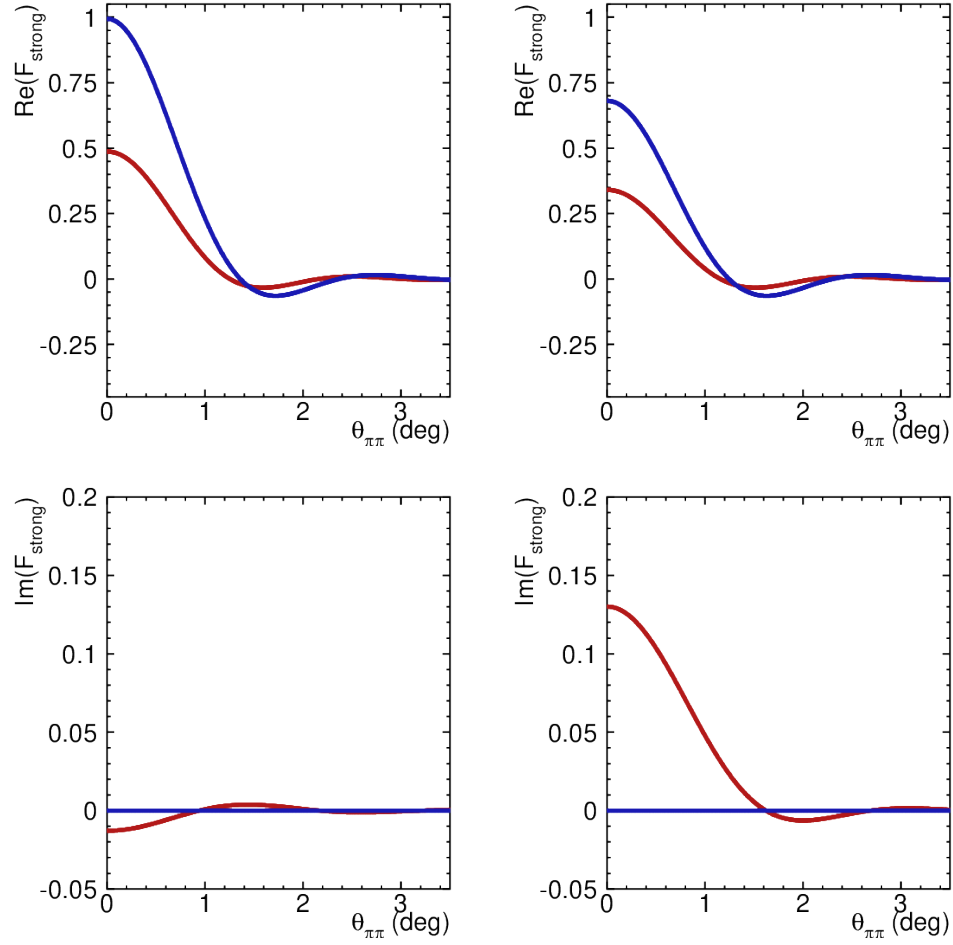


Figure 10: Real (top plots) and imaginary (bottom plots) part of the strong form factor as a function of production angle for the invariant mass values: 0.275 GeV (left plots) and 0.765 GeV (right plots). Red curves – full calculation, blue curves – no absorption.