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# The piston mechanism in a time-dependent two-level model

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We analyze a recent study of the one-proton knockout reaction on $^{19}$N, in the framework of a time-dependent two-level model. Focusing on the specific case of $^{18}$C, we quantify the validity of the so called "piston" mechanism in terms of the time scales involved in the process, namely the reaction time, the period of oscillation between the mixed states, and the lifetime of the excited unbound state.

*Introduction.* Pairing correlations play a crucial role in defining the properties of atomic nuclei. The evolution of these correlations in exotic nuclei is a subject which has received much attention in recent years, as new accelerator facilities are providing unique radioactive beams for study. Of particular interest is the role of neutron-neutron ($nn$) pairing in neutron-rich isotopes.

In an elegant experiment carried out at the GSI facility using the FRagment Separator (FRS) and the R$^3$B-LAND setup [1], the emission of neutron pairs from the neutron-rich $N = 12$ isotones $^{18}$C and $^{20}$O was studied following the high-energy nucleon knockout reactions $^{19}$N(-1$p$)$^{18}$C and $^{21}$O(-1$n$)$^{20}$O, to populate unbound states of the two isotones up to 15 MeV above their two-neutron emission thresholds. Their analysis of triple fragment$-n-n$ correlations display significantly different features between $^{18}$C and $^{20}$O. The authors argue that in the case of $^{18}$C, the knockout of a deeply bound proton from $^{19}$N suddenly promotes neutron pairs of the final system into the continuum without affecting the pairing properties of the ground states, and associate the process to a "piston" mechanism.

In this work we study more quantitatively the "piston" mechanism, and discuss its validity in terms of the time dependence of a two-level model [2].

*Formalism.* While we focus our discussion on the case of $^{18}$C, the approach can be trivially extended to other cases. In a $(-1p)$ reaction the population of the excited states proceed through the knockout of a deeply bound $p_{3/2}$ proton in $^{19}$N. Let us start by considering the basic ingredients that may play a role in the low-lying $2^+$ states [1] in the Carbon isotopes [3–5]. Following the picture depicted in Fig. 1, we assume that the elementary excitations, which are relevant for the description of the structure of $^{16}$C, are two-neutron particle and two-proton hole states built upon the double magic core $^{16}$O [3]. We call $|2^+_p\rangle$ the two-proton hole state in $^{14}$C, at 7 MeV excitation energy, and $|2^+_n\rangle$, the two-neutron particle state in $^{18}$O, at 1.98 MeV excitation energy.

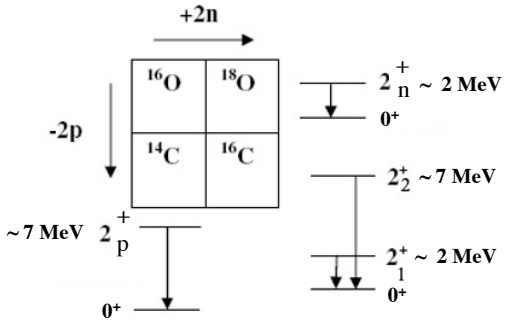



FIG. 1. Elementary $2^+$ excitations relevant for the structure of $^{16}$C. Adapted from Ref. [3].

The mixing between them in $^{16}$C can then be expressed as:

$$|2^+_1\rangle = \alpha|2^+_n\rangle + \beta|2^+_p\rangle$$
$$|2^+_2\rangle = -\beta|2^+_n\rangle + \alpha|2^+_p\rangle \quad (1)$$

In the weak coupling limit the low lying quadrupole excitation in $^{16}$C will be dominated by the two neutron component in Eq. 1 (that is $\alpha \gg \beta$ for the $|2^+_1\rangle$ state).

Concerning the structure of the heavier Carbons, the dependence of the effective single particle energies (ESPE) of the N=2 major shell with mass number [6], shown in Fig. 2, exhibits a large gap ($Z = 16$) between the $2s_{1/2}$

[1] Similar arguments can be applied to the $1^+$ state that could be populated in the proton-knockout reaction.

and $2d_{5/2}$ states and the $2d_{3/2}$, in relation to a typical pairing gap $\Delta$ indicated by the yellow shaded area. In Refs. [4, 5], a seniority inspired scheme was proposed that captures the main physical ingredients and correlates well with the available experimental data.

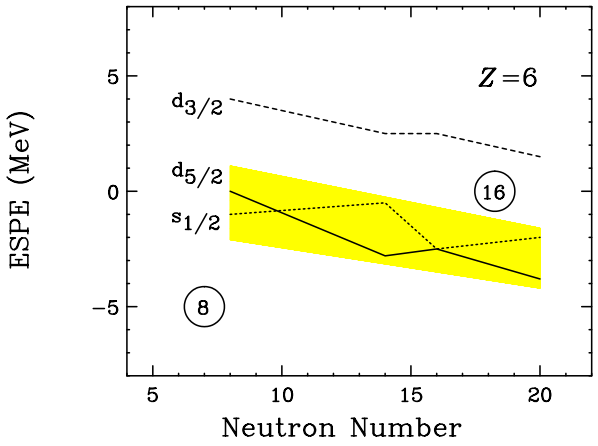



FIG. 2. ESPEs for the Carbon isotopes obtained from the WBT interaction [6]. The yellow shaded area indicates the size of the pairing gap ($\Delta$). Adapted from Ref. [4].

In this simple phenomenological model the $0^+$ and $2^+_1$ states of ${}^{18}_{6}\mathrm{C}_{8+4}$, with 4 neutrons in the $(d_{5/2}+s_{1/2})$ combined shell are:

$$|0^+;{}^{18}\mathrm{C}\rangle = |\nu(sd)^4;J=0\rangle\otimes|\pi(p_{3/2})^4;J=0\rangle;$$

$$\begin{aligned}|2^+_1;{}^{18}\mathrm{C}\rangle = \alpha|\nu(sd)^4;J=2\rangle\otimes|\pi(p_{3/2})^4;J=0\rangle+\\ \beta|\nu(sd)^n;J=0\rangle\otimes|\pi(p_{3/2})^3(p_{1/2})^1;J=2\rangle\end{aligned}$$

$$\begin{aligned}|2^+_2;{}^{18}\mathrm{C}\rangle = -\beta|\nu(sd)^4;J=2\rangle\otimes|\pi(p_{3/2})^4;J=0\rangle+\\ \alpha|\nu(sd)^4;J=0\rangle\otimes|\pi(p_{3/2})^3(p_{1/2})^1;J=2\rangle\end{aligned}\quad(2)$$

Note that in a $(-1p)$ reaction the $0^+$ state in the ${}^{18}$C is populated via the knockout of the $p_{1/2}$ proton from the ground state of the nitrogen target.

*Time dependence.* Given the fast nature of the reaction as well as the fast decay of the resonance state into $2n$, we need to consider three time scales, namely the reaction time, $t_r$, the lifetime of the excited unbound state, $\tau$, and the period of oscillation between the mixed states, $T$.

In order to estimate the time evolution of the states prepared by the reaction, which are not eigenstates of the Hamiltonian in ${}^{18}$C, we shall resort to the time dependent expression of these states [2]. We consider the Hamiltonian $H = H_0 + V$, where the states $|2^+_p\rangle$ and $|2^+_n\rangle$ are eigenstates of $H_0$ with eigenvalues $E_p$ and $E_n$ and $V$ is the interaction matrix element between them. The solution of the eigenvalue problem yields the stationary states $|2^+_1\rangle$ and $|2^+_2\rangle$ as given in Eqs. (2) with energies $E_-$ and $E_+$, corresponding to $2^+$ states in ${}^{18}$C as discussed in Refs. [4, 5]. We associate $|2^+_1\rangle$ with the first excited $2^+_1$ state and $|2^+_2\rangle$ (mixed symmetry state) with the resonance observed in Ref. [1].

Suppose that at $t=0$ we prepare the system in the state $|2^+_p\rangle$, which is not an eigenstate of $H$, then the time evolution is given by the wavefunction:

$$|\psi(t)\rangle = \beta|2^+_1\rangle e^{-iE_-t/\hbar} + \alpha|2^+_2\rangle e^{-iE_+t/\hbar} \quad (3)$$

from which the time dependence of the probabilities of finding $|\psi(t)\rangle$ in states $|2^+_n\rangle$ and $|2^+_p\rangle$, $P_{2^+_n}(t)$ and $P_{2^+_p}(t)$, can be obtained:

$$P_{2^+_n}(t) = 1 - P_{2^+_p}(t) = 4|\alpha|^2|\beta|^2\sin^2(\omega t/\hbar) \quad (4)$$

with $\omega = (E_+ - E_-)/2$.

*Results.* The proton knockout reaction prepares the system in the state $|2^+_p\rangle$ which undergoes oscillations with probabilities following Eq. 4. We show in Fig. 3 the time evolution of these probabilities with $t$ given in units of the characteristic oscillation period, $T = \pi\hbar/\omega = 2\pi\hbar/(E_+ - E_-)$. The amplitude of the oscillations, $4\alpha^2\beta^2$, reaches its maximum of 1 when $\alpha = \beta = 1/\sqrt{2}$.

Based on the analysis of Refs. [4, 5], we have $\beta \approx 0.27$ and $\omega = (E_+ - E_-)/2 \approx 3.4$ MeV, giving the oscillation period $T \approx 6\ 10^{-22}$ s. For reference, we also indicate in Fig. 3 the reaction time, which at a bombarding energy of 400 MeV/A is of the order of $2\ 10^{-24}$ s, and the lifetime of the resonance state, $\tau$, which is of the order of $6\ 10^{-22}$ s [1]. Given these estimates it appears that, although the reaction is very fast, the lifetime is long enough to average the probability of finding the state vector, $|\psi(t)\rangle$, in $|2^+_p\rangle$ to:

$$\langle P_{2^+_p}(t)\rangle = 1 - 2|\alpha|^2|\beta|^2 \approx 0.85 \quad (5)$$

We can then conclude that it is only for the cases where $\tau \sim t_r \ll T$, or in the unlikely scenario of negligibly mixing, $\beta \sim 0$, that the concept of the piston mechanism might be applicable and the decay properties of the populated resonance truly reflect the neutron correlations in the ground state of the final nucleus, in this case ${}^{18}$C. This could be confirmed experimentally by a measurement of the spectroscopic factors directly probing the amplitudes $\alpha$ and $\beta$. These arguments are, of course, applicable to other similar systems.

Finally, if we consider the fact that $|2^+_2\rangle$ is a decaying state with energy $E_+ - i\Gamma_+/2$, the results get modified [7] as shown with the dotted line in Fig. 3, where we have used $\Gamma_+ = \hbar/\tau$. We also show the results for a lifetime $\frac{1}{2}\tau$ (dashed line) to illustrate the effects of a faster decay.

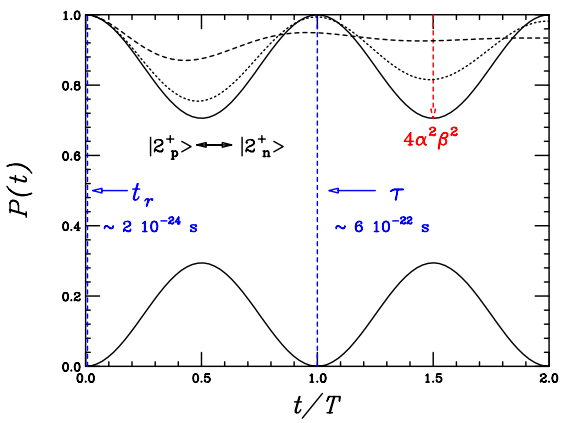



FIG. 3. Time oscillations between the states $|2_p^+\rangle$ and $|2_n^+\rangle$, with time in units of $T$. The vertical (blue) dashed lines indicate the estimates of the reaction time, $t_r$, and the lifetime of the resonance state, $\tau$. The amplitude of the oscillations is also shown in red. The dotted and dashed lines include the effects of the decay for lifetimes $\tau$ and $\frac{1}{2}\tau$ respectively.

In the limit $\tau \to 0$, the probability approaches the value $1 - |\alpha|^2|\beta|^2$ .

*Conclusions.* We have studied the interplay between two-proton hole and neutron particle states, excited via a high-energy proton knock-out reaction on $^{19}$N leading to unbound states in $^{18}$C [1]. The "piston" mechanism concept offers a unique motivation to use $(p, 2p)$ reactions to systematically study the evolution of $nn$ correlations in exotic neutron-rich nuclei. It seems clear to us that such a program will feature prominently in current and future rare-isotopes facilities worldwide. With this in mind, we used a two-level mixing model and its time dependence, to assess the validity of the "piston" mechanism and suggested some indicators for its applicability. In particular, differences in the time scales of the reaction, the lifetime and the oscillation period of the decaying state should be considered in relating the measured $n - n$ correlations to the pairing properties of the ground state in the final nucleus. While an extension of the two-level system to include more mixing states will be more realistic, our scenario captures the main physical ingredients of the problem.

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