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Neutron stars with a crossover equation of state

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The question of whether quark matter exists in neutron stars is a long standing one. Generally one finds that a first order phase transition from baryons to quarks softens the equation of state so much that the star would collapse into a black hole. We consider a crossover equation of state, similar to the crossover that is found in lattice QCD studies at finite temperature and zero or small baryon chemical potentials. We find that with reasonable parameters it may be possible to support neutron stars up to about 2.2 solar masses. In that case 1 to 10% of the pressure would be contributed by quark matter in the central core of the highest mass stars.

Neutron stars are an interesting astrophysical laboratory to study strong gravitational fields and the properties of matter at the highest attainable baryon densities in the universe. The LIGO/Virgo collaboration has observed gravitational waves from the merger of neutron stars [1]. Such mergers are predicted to have produced many of the heavy elements in the universe [2]. The central baryon densities are theoretically estimated to be about ten times greater than in atomic nuclei. Such baryon densities will likely be reached or exceeded in the Beam Energy Scan II at RHIC (Relativistic Heavy Ion Collider) [3]. The advantage of heavy ion collision experiments is that they are measurable and repeatable on earth. Their disadvantage is that the volume in which these densities are achieved is transient and involve a length scale of 10 fm, whereas the length scale in the centers of neutron stars is 1 km, a difference of 17 orders of magnitude.

For a long time it was thought that neutron stars might have an upper limit of about 1.5 to 1.6 solar masses $M_\odot$ [4]. There were a few with larger masses but they had significant uncertainties. Then stars with masses $1.97 \pm 0.04 M_\odot$ [5] (subsequently updated to $1.928 \pm 0.017 M_\odot$ [6]), $2.01 \pm 0.04 M_\odot$ [7], and $2.14 \pm 0.10 M_\odot$ [8] were discovered. These require substantial interaction pressure for them to be held up from gravitational collapse. For comparison, an equation of state with relativistic but noninteracting protons, neutrons, and electrons can only support a 0.7 solar mass star.

The equation of state for dense matter relevant to neutron stars has been studied for more than fifty years. There exists a vast literature which we do not intend to review here. One approach is relativistic mean field theory [9, 10]. In this approach baryons move independently in the mean scalar and vector meson fields generated by the baryons themselves. Although not entirely satisfactory for several reasons, it has the advantage that the known properties of nuclear and neutron matter near normal nuclear density $n_0 = 0.153 \text{ fm}^{-3}$ can be fit with the parameters in the Lagrangian, and then the equation of state can be extrapolated to higher densities while respecting special relativity and thermodynamic relationships. As the density and therefore chemical potential increase it becomes thermodynamically favorable to form hyperons. But the strength of the interactions between

hyperons and nucleons and among hyperons is poorly constrained by hypernuclei, which is true of all theoretical approaches. Adding degrees of freedom softens the equation of state, meaning that at the same energy density the pressure is decreased. This makes it difficult to make massive neutron stars with most choices of coupling constants [11]. Reaching at least $2 M_\odot$ requires parameter choices that effectively eliminate hyperons [12]. A first order phase transition to quark matter would also soften the equation of state since the quark phase has higher energy density than the hadronic phase at the same pressure. Hence it is highly unlikely that a first order phase transition to quark matter could occur before the star would collapse to a black hole [11]. A second order transition also softens the equation of state, but not by much [11].

Precision lattice QCD simulations at finite temperature and zero or small chemical potential clearly show a crossover transition from hadronic matter to quark-gluon plasma [13]. The equation of state was accurately modeled in Ref. [14] using a switching function to transition smoothly from a hadronic gas to quark-gluon plasma.

$$P(\mu, T) = S(\mu, T)P_{qg}(\mu, T) + (1 - S(\mu, T))P_h(\mu, T) \quad (1)$$

Here $P_h(\mu, T)$ is the hadronic pressure and $P_{qg}(\mu, T)$ is the quark-gluon pressure as functions of the baryon chemical potential μ and temperature T . The $S(\mu, T)$ is a switching function which ranges between 0 and 1 as μ and T increase. It must be an even function of μ and infinitely differentiable so as not to introduce an artificial phase transition of any order. Motivated by this and by the discussion of neutron stars above we entertain the possibility that the transition from cold nuclear or neutron matter to quark matter is also a crossover and study the general implications for neutron stars. Reference [14] suggests the form at $T = 0$

$$S(\mu) = \exp [-(\mu_0/\mu)^4] \quad (2)$$

with the reasonable estimate that $\mu_0 \approx 1.65 \text{ GeV}$. This parameter could not be precisely determined by comparison with the lattice results but simply as a baseline estimate. This approach allows freedom in the choice of hadron and quark pressures.

There have been previous studies of neutron stars with a crossover equation of state. Reference [15] expressed P

as a function of baryon density n with a crossover from hadronic matter to quark matter, the latter described by the Nambu–Jona-Lasinio (NJL) model, with a tanh switching function. Reference [16] performed a similar study by expressing the energy density ϵ as a function of n using the same form of switching function. Reference [17] expressed P as a function of n using a hadronic equation of state below some density n_1 , a perturbative QCD equation of state above some density n_2 , and used a pair of polytropes to interpolate between them in the region $n_1 < n < n_2$. References [18, 19] expressed P as a function of μ using a hadronic equation of state below some μ_1 , an NJL equation of state above some μ_2 , and a fifth order polynomial in μ to interpolate between them. The advantage of our approach is that it is motivated by a fit to lattice QCD results at finite temperature, it expresses the pressure in terms of its natural variables μ and T so that thermodynamic self-consistency is guaranteed, the switching function adds only one free parameter (at $T = 0$), no phase transition of any order can occur with the chosen switching function, there is no intermediate interpolated density range where the degrees of freedom are ambiguous, and the inclusion of finite temperature is easily possible.

Based on the previous discussion we shall, for simplicity and sake of illustration, use a relativistic mean field equation of state including neutrons interacting via scalar σ and vector ω plus ρ mesons. The potential $U(\sigma)$ includes quadratic, cubic, and quartic terms. This amounts to five independent parameters. They are fit to the properties of normal isospin symmetric nuclear matter to reproduce the saturation density n_0 , binding energy 16.3 MeV, Landau mass $0.83 m_N$, compressibility $K = 250$ MeV, and symmetry energy $S = 32.5$ MeV. See sections 11.1, 11.3 and 16.2 of Ref. [10] for the numerical values of the parameters in the Lagrangian which reproduce these properties. The perturbative QCD quark matter pressure comes from Ref. [20]. These include contributions up to and including α_s^2 . The strong coupling $\alpha_s(\mu)$ satisfies the renormalization group equation using the 3-loop β function. The renormalization scale is taken from fits to the lattice QCD results as in Ref. [14]. All quark masses are set to zero. Thus both phases are electrically neutral without the need for electrons or muons. The only free parameter is μ_0 whose scale is already known.

Figure 1 shows the pressure versus energy density for four values of μ_0 . The influence of quark matter becomes apparent between 0.5 and 1.0 GeV/fm³ depending on the value of μ_0 . On a log–log plot they all appear very similar.

A more differential probe is the slope $dP/d\epsilon$. At $T = 0$ this is the adiabatic speed of sound squared c_s^2 and is plotted in Fig. 2. This behavior can be understood as follows. For the relativistic nuclear mean field theory the asymptotic behavior at large density or chemical poten-

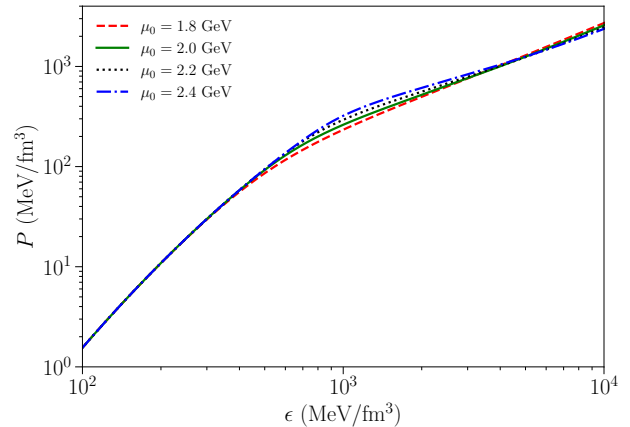


FIG. 1. Pressure versus energy density for different choices of μ_0 .

tial is

$$P \rightarrow \epsilon \rightarrow \frac{1}{2} \left[\frac{g_\omega^2}{m_\omega^2} + \frac{g_\rho^2}{4m_\rho^2} \right] n^2 \quad (3)$$

where g_ω and g_ρ are the baryon–vector meson coupling constants, m_ω and m_ρ are the vector meson masses, and $n = dP/d\mu$ is the baryon density. Hence $c_s^2 \rightarrow 1$ in the absence of quark matter. Since QCD is asymptotically free $c_s^2 \rightarrow 1/3$ for quark matter, apart from logarithmic corrections. With a crossover equation of state it is therefore natural that c_s^2 would at first increase with energy density, reach a maximum less than 1, and then decrease towards an asymptotic limit of 1/3. To determine the behavior at asymptotically high density consider the quark pressure at two-loop order

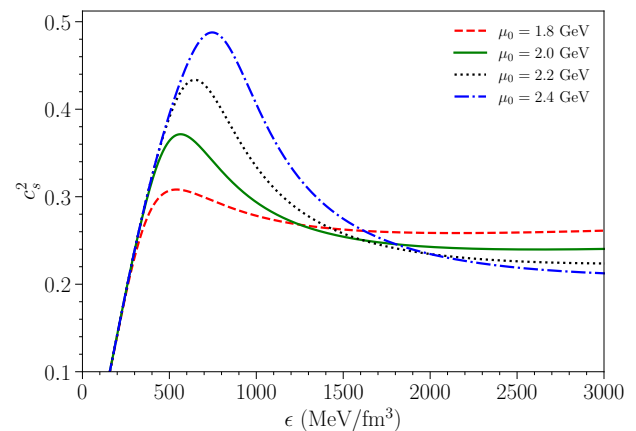


FIG. 2. Speed of sound squared versus energy density for different choices of μ_0 .

$$P_q = \frac{N_f}{4\pi^2} \left(\frac{\mu}{3}\right)^4 \left(1 - \frac{2\alpha_s}{\pi}\right) \quad (4)$$

where N_f is the number of massless quarks. Using the one-loop β function this leads to

$$c_s^2 = \frac{n}{\mu \, dn/d\mu} = \frac{1}{3} \left[1 - \frac{(33 - 2N_f)}{9\pi^2} \alpha_s^2 \right] \quad (5)$$

Hence the limit of $1/3$ is reached from below. The behavior shown in Fig. 2 is qualitatively the same as speculated in Ref. [21]. It is also similar to the behavior of quarkyonic matter [22] although the theoretical approaches appear to be very different, with the latter resulting in a second order phase transition.

It is interesting to examine the mathematical limit $\mu \gg \mu_0$. In this limit $P_h \rightarrow \frac{1}{2} \left(\frac{g_\omega^2}{m_\omega^2} + \frac{g_\rho^2}{4m_\rho^2} \right)^{-1} \mu^2$ and so

$$(1 - S)P_h \rightarrow \frac{1}{2} \left(\frac{g_\omega^2}{m_\omega^2} + \frac{g_\rho^2}{4m_\rho^2} \right)^{-1} \frac{\mu_0^4}{\mu^2} \rightarrow 0$$

$$SP_q \rightarrow \frac{N_f}{4\pi^2} \left(\frac{\mu}{3}\right)^4 \left(1 - \frac{2\alpha_s}{\pi}\right) - \frac{N_f \mu_0^4}{(18\pi)^2} \quad (6)$$

plus a correction of order α_s in the last term above. Thus there is a constant shift in the pressure at very high density; μ_0^4 essentially acts like a bag constant.

Figure 3 shows the mass as a function of central energy density for the four choices of μ_0 . The maximum mass increases with μ_0 and with central energy density ϵ_c , which is to be expected. For any of these choices of μ_0 , the value of ϵ_c for the maximum mass star is beyond the peak in c_s^2 , as can be seen by comparison of Figs. 2 and 3. The observation of 2.01 ± 0.04 and 2.1 ± 0.10 solar mass stars means that μ_0 cannot be less than about 1.8 GeV, at least with our choice for P_h .

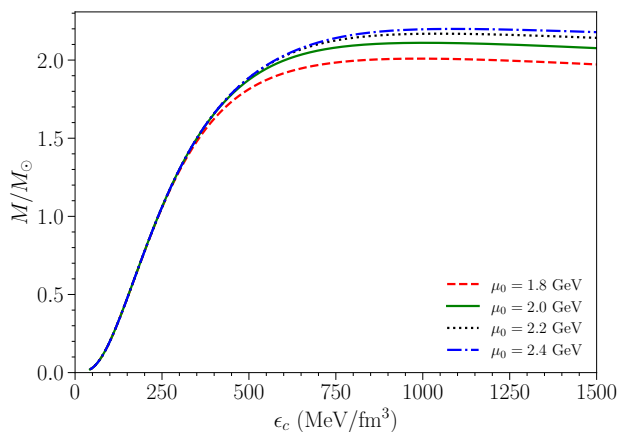


FIG. 3. Neutron star mass in units of solar mass versus central energy density for different choices of μ_0 . The maximum masses are 2.01, 2.11, 2.17 and 2.20 $M_\odot$.

Figure 4 displays the mass-radius relationship. The overall shape of the curves is typical. As μ_0 increases so does the maximum mass since larger values favor the hadronic phase. Also, as the maximum mass increases, the radius of the star at that mass decreases. This is also typical.

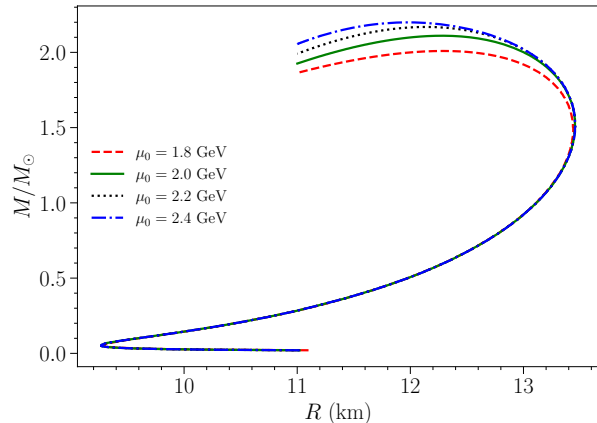


FIG. 4. Neutron star mass in units of solar mass versus surface radius for different choices of μ_0 .

How much quark matter is there in a neutron star? While that seems a natural question, in this case it is ill-posed because the switching function S is a function of μ . If n_h denotes the baryon density in the hadron phase and n_q denotes the baryon density in the quark phase, then the total baryon density at some value of μ is

$$n = \frac{dP}{d\mu} = S n_q + (1 - S) n_h + (P_q - P_h) \frac{dS}{d\mu} \quad (7)$$

which is not just a combination of the the baryon densities in the two phases, but contains an extra term due to the switching function itself. A better question to ask is what fraction of the pressure is contributed by quarks. The answer is just $S(\mu)$ which, through Eq. (7), can also be expressed as $S(n)$. Figure 5 shows S as a function of radius r for the maximum mass star with a given choice of μ_0 . Between 1 and 10% of the pressure could be provided by quark matter at the core of the star.

In conclusion, we have studied the masses, radii, and central energy densities of neutron stars using an equation that has a smooth crossover from hadronic matter to quark matter. This approach is quite flexible in that different hadronic and quark matter equations of state can be used. This obviously includes the nuclear matter in the crust or outer radius of the star as well as the possibility of including protons, electrons, muons, and hyperons. It can be directly extended to study matter at finite temperature as is relevant for supernovae, hot proto-neutron stars, and neutron star mergers. What observational effects would support the crossover equation of state to quark matter is an interesting question.

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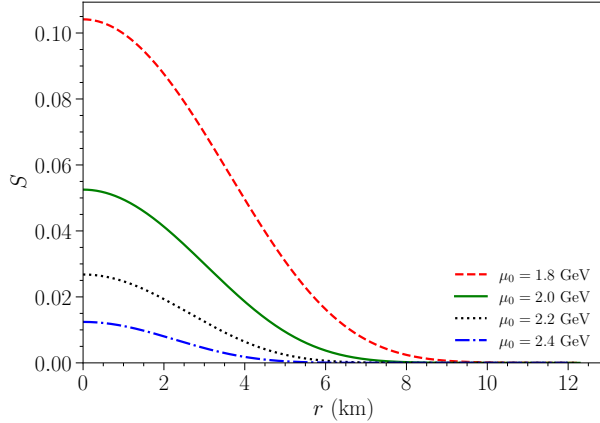


FIG. 5. Switching function versus radius for the maximum neutron star mass for different choices of μ_0 . It represents the fraction of the total pressure contributed by quark matter.

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