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## Spectroscopy of <sup>10</sup>N with the invariant-mass method

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Proton decays of <sup>10</sup>N states has been investigated with the invariant-mass technique using data from two reactions. In the first experiment, <sup>10</sup>N states were created via multi-nucleon knockout from a fast <sup>13</sup>O beam. The second experiment involved proton pickup from a <sup>9</sup>Be target to a fast <sup>9</sup>C beam. Both data sets produce similar distributions with a peak centered at a decay energy of 2.8 MeV and a width of  $\approx 2.5$  MeV. This result is consistent with a previous study using multinucleon transfer reaction which was originally fit with an  $\ell=0$  resonance but later interpreted as an  $\ell=1$  resonance. This later interpretation is affirmed as the proton pickup reaction should favor  $\ell=1$ . This strength is located near the predicted energies of two  $\ell=1$  resonances in calculations using complex scaling and the Gamow shell model. The multi-nucleon knockout data also show excess strength below the main peak which is interpreted as contributions from one or more  $\ell=0$ resonances.

Studies of exotic nuclei near and beyond the drip lines have attracted considerable interest. These nuclei can exhibit unusual properties such as extended halos and single and multi-nucleon decays. Theoretically, the role of the continuum is important for understanding their structure and thus theories must go beyond standard shell-model calculations by including the continuum via a Berggren basis [1], complex scaling [2], or other techniques [3, 4]. Light nuclei, such as the one studied here, have historically been a testing ground for advanced theoretical techniques as they are computationally more accessible.

The nuclide <sup>10</sup>N is located beyond the proton drip line, two neutrons away from <sup>12</sup>N, the lightest particlebound Nitrogen isotope. As one moves beyond the proton drip line, the widths of the ground and low-lying excited states increase especially for light nuclei with small Coulomb and centrifugal barriers. Eventually these levels dissolve into the continuum marking the edge of identification of nuclei and thus the edge of the chart of nuclides. The wide levels expected for <sup>10</sup>N present a challenge experimentally making it difficult to separate and resolve closely-spaced states. At present, the spectroscopy of this nuclide is poorly known with only two experimental studies [5, 6].

The mirror of <sup>10</sup>N, <sup>10</sup>Li, is neutron-unbound and there has been significant debate on its structure, specifically whether there exists a  $n+{}^{9}$ Li virtual state at threshold [7–11]. Understanding of the  $n-{}^{9}$ Li interaction is also important for understanding the iconic nucleus <sup>11</sup>Li which has an extended neutron halo [12]. In the same sense, <sup>10</sup>N resonances are important in understanding the structure of the newly discovered <sup>11</sup>O system [13, 14]. The ground and excited states of this nucleus undergo 2p decay and the <sup>10</sup>N levels represent possible intermediate states in sequential-decay branches [15, 16].

The first observation of a  ${}^{10}$ N state was make by Lépine-Szily *et al.* in the multi-nucleon transfer reaction  ${}^{10}B({}^{14}N,{}^{14}B){}^{10}N$  [5]. After subtraction of some background, a broad peak was fitted with a *R*-matrix  $p+{}^{9}C$  line shape with  $\ell=0$  [17]. Based on calculated twonucleon transfer amplitudes, the Tilley *et al.* evaluation subsequently argued that the state observed was actually a  $\ell=1$  resonance, the analog of the 1<sup>+</sup> first excited state in <sup>10</sup>Li [18].

As the width of a line shape increases, different definitions of the resonant energy and width can diverge. It is thus important to use a consistent definition in comparing fitted line shapes from different works. Lépine-Szily *et al.* give a resonance energy of  $E_r^{\pi/2} = 2.6(4)$  MeV where  $E_r^{\pi/2}$  corresponds to the energy where the phaseshift contribution from this level changes by odd integer multiples of  $\pi/2$  in *R*-matrix theory [17]. Alternatively, the resonance energy from the real part of the pole  $(E_r^{pole} - i\Gamma^{pole}/2)$  in the S matrix obtained from the fitted parameters is  $E_r^{pole}$ =2.2(5) MeV. The line profile has a maximum in between these two resonance energies at  $E_r^{max} = 2.4(4)$  MeV. We will use this latter resonance energy for comparisons between different experiments as it is more intuitive and we found it is less sensitive to the assumed  $\ell$  value used in the fits. For the width, we will use the FWHM of the fitted line profile for comparison for similar reasons. However theoretical models which include the continuum such as complex scaling and the Gamow Shell model find the poles of the S-matrix and so for comparisons to these one must use the  $E_r^{pole}$  and  $\Gamma^{pole}$  values derived from the fitted *R*-matrix parameters. Table I lists the different resonance parameters obtained from this work and from previous experimental studies.

The only other experimental study of <sup>10</sup>N is by Hooker et al. using  $p+{}^{9}$ C resonance scattering with the thicktarget inverse-kinematics technique [6]. Within the *R*matrix theory, it was not possible to fit the shape and magnitude of their excitation function without introducing two  $\ell=0$  resonances. Two solutions were produced depending on the energy ordering of the expected  $2^{-}$  and  $1^{-} \ell=0$  resonances. In both solutions, the excited state was located at  $E_r^{max}=2.8(2)$  MeV consistent with the peak fitted by Lépine-Szily  $[E_R^{max}=2.4(4) \text{ MeV}]$ . The fitted widths of this excited state ( $\Gamma=2.0^{+0.7}_{-0.3}$  MeV for the  $J^{\pi}=2^{-}$  or  $\Gamma=1.2^{+0.6}_{-0.4}$  MeV for the  $J^{\pi}=1^{-}$  assignment) are also consistent with the value of 1.9(9) MeV from the fit of Lépine-Szily *el al.* 

TABLE I. Resonance parameters associated with R-matrix fits in this work and in Refs. [5, 6]. The resonance energy  $E_r$  is given from the three definitions and the decay width  $\Gamma$ from the two definitions discussed in the text. All values are in MeV. For the present work, two fits were made where the excited state is considered as either a  $\ell=0$  or  $\ell=1$  resonance. For the Lépine-Szily entry, the value of  $E_r^{\pi/2}$  is taken from [5] while the other values are derived from their fitted R-matrix parameters for  $\ell=0$ . Results for the two fits of Hooker *et al.* are listed with two possible energy-orderings of the 1<sup>-</sup> and 2<sup>-</sup> states. The values for  $E_r^{max}$  and  $\Gamma^{FWHM}$  are those listed by Hooker *al et.* while the other values were derived from their Rmatrix parameters. For the latter no uncertainty estimation was attempted.

state	$E_r^{pole}$	$\Gamma^{pole}$	$E_r^{max}$	$\Gamma^{FWHM}$	$E_r^{\pi/2}$				
This work with $\ell = 0$ excited state									
g.s.	0.96(2)	0.72(3)	1.12(3)	1.11(4)	1.34(3)				
1st	2.45(2)	2.48(3)	2.79(2)	2.51(3)	3.05(2)				
This work with $\ell = 1$ excited state									
g.s.	1.04(2)	0.97(4)	1.31(3)	1.46(7)	1.64(4)				
1st	2.37(2)	2.01(3)	2.77(2)	2.45(4)	3.16(2)				
Lépine-Szily et al. [5]									
	2.2(5)	1.8(8)	2.4(4)	1.9(9)	2.6(5)				
Hooker $et \ al. \ [6]$ solution 1									
$1^{-}$	1.3	1.2	1.9(2)	$2.5^{2.0}_{-1.5}$	1.8				
$2^{-}$	2.5	1.8	2.8(2)	$2.0^{+0.7}_{-0.5}$	2.8				
Hooker <i>et al.</i> [6] solution 2									
$2^{-}$	1.4	1.6	2.2(2)	$3.1^{+0.9}_{-0.7}$	2.2				
$1^{-}$	2.8	1.3	2.8(2)	$1.2_{-0.4}^{+0.6}$	2.9				

This paper presents new data on the low-lying resonances in <sup>10</sup>N from two invariant-mass experiments performed with the HiRA apparatus [19] at the National Superconducting Cyclotron Laboratory. Both experiments used the same <sup>9</sup>Be target and almost identical setups with 14 HiRA Si-CsI(Tl)  $\Delta E$ -E telescopes covering polar angles from  $2.1^{\circ}$  to  $12.4^{\circ}$ . First, we will consider data obtained with an E/A=65.4-MeV <sup>13</sup>O beam where <sup>10</sup>N states were produced by the knockout of a proton and two neutrons. Details of the experiment can found in Refs. [13, 14, 20–22] where results for <sup>11,12</sup>N, <sup>11,12</sup>O, and <sup>13</sup>F resonances are presented. Based on studies of the mirror nucleus  ${}^{13}B$  [23–27], the ground state of  ${}^{13}O$  is expected to have significant contributions from proton  $[1s_{1/2}]_{J=0}^2$  and  $[0p_{1/2}]_{J=0}^2$  configurations. With the removal of one of the two valence protons, we expect to create both  $\ell=0$  and  $\ell=1$  proton resonances. This was confirmed in the  $p+{}^{10}C$  invariant-mass spectrum associated with the knockout of one proton and one neutron from this experiment [20]. A  $J^{\pi} = 1/2^{-}, \ell = 1$  resonance dominates this spectrum, but significant yield from the  $1/2^+ \ell = 0$  resonance was also inferred. In knocking out a proton and two neutrons we again expect to observe  $\ell = 0$ and  $\ell=1$  resonances this time in the  $p+{}^{9}C$  channel.

Data from a second experiment, using an E/A=68-MeV <sup>9</sup>C beam, was also used in the present study. Res-

onances in <sup>10</sup>N were produced via proton pickup from the <sup>9</sup>Be target. Details of this experiment can be found in Refs. [28, 29] where resonances from <sup>8,9</sup>B and <sup>9</sup>C are presented. From semi-classical models of the pickup reaction, transfer of protons to  $\ell=0$  and  $\ell=1$  orbitals in <sup>10</sup>N is poorly matched in terms of linear and angular momentum transfer leading to low cross sections for fast beams [30]. The mismatch is worse for  $\ell=0$  resonances, leading to a relative enhancement of  $\ell=1$  resonances. Both data sets are therefore expected to populate  $\ell=1$  resonances which will help address the issue raised by Tilley *et al.* as to whether the peak observed by Lépine-Szily *et al.* is  $\ell=0$  or  $\ell=1$ . Both types of resonances are predicted at low energy in theoretical models [2, 16, 31, 32].

In the two experiments, the invariant masses  $M_{inv}$  of detected  $p+{}^{9}$ C coincidences were determined and the decay kinetic energy of  ${}^{10}$ N states was obtained as by subtracting the masses of the decay products, i.e.,  $E_T = M_{inv} - M_p - M_{^9}C$ . The invariant-mass resolution is improved by selecting events where the decay proton is emitted transversely from the parent  ${}^{10}$ N fragment [33]. As such, all  $E_T$  spectra shown are restricted to events with  $|\cos \theta_p| < 0.5$  where  $\theta_p$  is the proton emission angle relative to the beam in the  ${}^{10}$ N center-of-mass frame. The raw  $E_T$  spectrum obtained in the first experiment is shown as the black histogram in Fig. 1(a). We have identified three sources of contamination which are also shown in this figure.

The light output of the CsI(Tl) crystals shows a lowenergy tail due nuclear reactions in the crystals. This tail causes some heavier carbon isotopes to be misidentified as <sup>9</sup>C. Thus some  $p+^{10}$ C and  $p+^{11}$ C events, produced largely from proton decays of <sup>11,12</sup>N states formed via one and two nucleon knockout reactions, will contribute to the observed  $p+^{9}$ C invariant-mass spectrum. To estimate their contributions, we have analyzed detected  $p+^{10}$ C and  $p+^{11}$ C events as if the detected carbon fragment was <sup>9</sup>C. These distributions were scaled by the probabilities of misidentification (~0.5%) as determined from <sup>10</sup>C and <sup>11</sup>C calibrations beams to give the red and blue histograms in Fig. 1(a). The largest contribution from this effect comes from the proton decay of the  $J^{\pi}=2_1^+$  first excited state of <sup>12</sup>N which produces the peak at  $E_T \sim 330$  keV.

Two-neutron knockout reactions produce <sup>11</sup>O states which decay to the  $2p+{}^{9}C$  channel [14]. If only one of the two protons is detected, these events will populate the experimental  $p+{}^{9}C$  spectrum. Monte Carlo simulations [33], constrained by the detected  $2p+{}^{9}C$  data, allowed us to estimate the  $2p+{}^{9}C$  and  $p+{}^{9}C$  detection efficiencies ( $\epsilon_{1p}$  and  $\epsilon_{2p}$ , respectively) for <sup>11</sup>O decay. The contribution from <sup>11</sup>O decay, shown as the green histogram in Fig. 1(a), was obtained from the detected  $2p+{}^{9}C$  events where one of the protons was randomly removed. The invariant-mass distribution from these  $p+{}^{9}C$  events was scaled by  $\epsilon_{1p}/\epsilon_{2p}$  to account for increased probability of detecting just one of the two protons.

The contamination-subtracted distribution is plotted as the data-points in Figs. 1(b) and 1(c). To compare to



FIG. 1. (a) Raw experimental  $p+{}^{9}$ C invariant-mass distribution (black) and the estimated contamination from  $p+{}^{10}$ C and  $p+{}^{11}$ C events where the carbon fragment was misidentified as  ${}^{9}$ C. The contamination from  ${}^{11}O \rightarrow 2p+{}^{9}$ C events where only one of the two protons is detected is also shown. (b,c) Contamination-subtracted spectrum (data points) with two-level fits (solid-red curve). The fits were obtained using the  $\ell=0$  line shapes constrained by Hooker *et al.* in their analysis of resonance elastic scattering excitation function. Two fits are shown corresponding to the two fits by Hooker *et al.* with different energy-ordering of the  $1^{-}$  and  $2^{-} \ell = 0$  resonances. The fitted individual level contributions are indicated by the dotted curves and the blue-dashed curves show contributions from an addition linear background which was allowed in the fits. The latter contribution is very small.

previous studies, we have tried fitting these spectra with two  $\ell=0$  resonances using line shapes fixed by the analysis of Hooker *et al.* The two fits correspond to the two solutions of Hooker *et al.* where the ground state was either a 1<sup>-</sup> or a 2<sup>-</sup> state. The experimental resolution was incorporated via Monte Carlo simulations [33] and has a FWHM of 290 keV (100 keV) at  $E_T = 2.5$  MeV (1 MeV) which is much smaller than the intrinsic widths of the fitted states. Both fits give a reasonable description of our data, but could be improved if the ground-state strength was shifted down in  $E_T$  and the excited state was wider. To explore this further, we have performed two-peak fits where the line shapes of both peaks are allowed to vary and both  $\ell=0$  and  $\ell=1$  *R*-matrix parametrizations [17] were considered. However for the ground state, fits with  $\ell=1$  were not acceptable as wide line shapes are necessary requiring  $\ell=0$  and the maximum width at the Wigner limit [34]. For the excited state, reproduction of the data with both  $\ell$  values was achieved.

The fitted resonance parameters are listed in Table I where the listed uncertainties are just the statistical errors. The *R*-matrix fits were performed with a channel radius of  $a_c=5$  fm and we have varied this by  $\pm 0.5$  fm to gauge its effect. The variation in the resonances parameters was generally of the same magnitude or smaller that the statistical errors, with the two resonances energies  $E_r^{max}$  and  $E_r^{\pi/2}$  having the least dependence of  $a_c$ . The only exception was for the excited state region with  $\ell=1$  where the fit with  $a_c=4.5$  fm was somewhat worse and the extracted widths for this state were reduced by 130 keV.

Figure 2(a) show the fit with a  $\ell=1$  resonance for the excited state. The fitted resonance energy of this state  $E_r^{max} \approx 2.8$  MeV is consistent with both the result from Hooker *et al.* [2.8(2) MeV] and that from Lépine-Szily *et al.* [2.4(4) MeV]. The fitted width of this state,  $\Gamma \approx 2.6$  MeV, is consistent with that from Lépine-Szily *et al.* [1.9(9) MeV] and for the result of Hooker *et al.* [2.0 $^{+0.7}_{-0.5}$  MeV] when this excited state is assigned as  $J^{\pi}=2^{-}$ . The width from the  $J^{\pi} = 1^{-}$  assignment of Hooker *et al.* (1.2 $^{+0.6}_{-0.4}$  MeV) is about  $2\sigma$  below our value. We only need to consider this resonance as either  $\ell=0$  or 1 as its large width is incompatible with larger values. For example, the FWHM of an  $\ell=2$  resonance at the Wigner limit [34] is only 0.7 MeV, well below our fitted width.

To compare to single-particle estimates, we have found the poles of the S-matrix for a potential consisting of Coulomb, nuclear, and spin-orbit terms with standard parameters ( $r_C=1.4$  fm,  $r_0=r_{s.o.}=1.2$  fm,  $a=a_{s.o.}=0.6$ fm,  $V_{s.o.} = 6$  MeV). The depth of the nuclear potential was adjusted to match the experimental  $E_r^{pole}$  value. For an  $\ell=1$  ( $p_{1/2}$ ) excited-state resonance, the deduced singleparticle width was  $\Gamma^{pole}_{s.p} =$  1.14 MeV and this can be increased to match the experimental value of 2.01(3) MeV by increasing the diffusenesses to  $a=a_{s.o.}=0.9$  fm. Hence if this excited state is an  $\ell=1$  resonance, then it has very strong single-particle character or alternatively the large width may be a result of two overlapping  $\ell = 1$  resonances (see later). No single-particle  $\ell=0$  resonance could be found at the resonance energy of the excited state. Indeed the single-particle  $\ell=0$  resonance energy reaches a maximum of  $E_r^{pole} = 1.74$  MeV as the depth of the nuclear potential is decreased at which point the single-particle width is  $\approx 4$  MeV. This observation indicates that the  $\ell = 0$  solution for this state corresponds to something more complicated than a single-particle resonance.

While the data can be fit with two  $\ell=0$  resonances with parameters similar to those obtained by Hooker *et al.*, this is not entirely satisfactory as  $\ell=1$  resonance(s) were expected to be produced. To further explore this, let us look at the second data set from the <sup>9</sup>C beam. The  $E_T$  distribution associated with this is displayed in

TABLE II. Comparison of resonance parameters in MeV for low-lying states in <sup>10</sup>N from three theoretical models.

$J_{\pi}$	Aoyama $E_r^{pole}$	et al. [2] $\Gamma^{pole}$	$\begin{array}{c} \text{Garrido} \\ E_r^{pole} \end{array}$	et al. [31] $\Gamma^{pole}$	Mao $e_r$ $E_r^{pole}$	t al. [32] $\Gamma^{pole}$
$1^{-}$					1.92	0.9
$2^{-}$	1.51	3.47	1.74	3.94	2.39	0.3
$1^+$	2.84	1.89	2.62	1.68	2.9	0.3
$2^+$	3.36	2.82	2.89	2.21	2.68	0.36

et al. [2, 35] and Garrido and Jensen [31] give similar predictions. The third study by Mao *et al.* is with the Gamow Shell Model. All theoretical models predict a  $1^+$ , 2<sup>+</sup> doublet of  $\ell$ =1 resonances at  $E_r^{pole} \approx 2.8$  MeV. The centroids of these states are separated by  $\approx 0.5$  MeV in the work of Aoyama *et al.* and by  $\approx 0.2$  MeV in the other two studies. This doublet is located close to our fitted excited state at  $E_r^{pole} = 2.37(2)$  MeV for  $\ell = 1$  and the two states would not be resolved in our experimental data, so the fitted excited state could very likely have contributions from both of these levels. The predicted widths from the two complex-scaling calculations in Table II are also similar to the fitted width of  $\Gamma^{pole}=2.01(3)$  MeV, but the widths predicted with the Gamow shell model,  $\approx 0.3$  MeV, are much smaller and difficult to reconcile with the data.

A 1<sup>-</sup>, 2<sup>-</sup> doublet of  $\ell=0$  resonances is predicted below the  $\ell=1$  doublet in the Gamow Shell Model calculations. However, in the complex scaling calculations of Garrido et al., there is only a 2<sup>-</sup> resonance as their  $J^{\pi}=1^{-}p+{}^{9}C$ potential is purely repulsive. The other complex-scaling study of Aoyama *et al.* does not explicitly address the existence of this  $1^-$  resonance, but we suspect the situation is similar. To the extent that both  $\ell=0$  resonances exits, our fitted ground-state resonance may also have contributions from each of them. The fitted ground-state energy of  $E_r^{pole} \approx 1.0$  MeV is below all the predictions for the  $2^-$  states in Table II and also below that in the  $1^-$  ground state for the Gamow shell model. This state is also lower than the ground state fitted by Hooker et al. for both  $1^-$ ,  $2^-$  spin assignments. We note that this state is not well populated or resolved in either of our experiments which would lead to further uncertainties if there are other unknown sources of background present.

An assignment of the excited state as  $\ell=1$  leads to some tension with the conclusions of Hooker *et al.* who also require an  $\ell=0$  resonance in this region of  $E_T$  to describe their data. Perhaps there is both  $\ell=0$  and  $\ell=1$  strength in this energy region which cannot be resolved due to their large widths. Hooker *et al.* did find that contributions from a  $\ell=1$  resonance at 3.3 MeV were possible, but not necessary, to reproduce their data.

Aoyama *et al.* argued that the structure of <sup>10</sup>N can be used to shed light on the much-debated structure of its mirror <sup>10</sup>Li. A number of experimental studies have concluded that there is a virtual or antibound  $n+{}^{9}$ Li state [7–10]. A virtual state is where the  $s_{1/2}$  attractive po-



FIG. 2. Decay-energy distributions obtained from p+C coincidences in the two experiments. (a) is the backgroundsubtracted distribution produced in the knockout reaction while (b) is from the proton-pickup experiment. The solid red curve in (a) shows a fit assuming a  $\ell=0$  ground state and  $\ell=1$  excited states using *R*-matrix line shapes. The individual contributions are shown as the dotted curves. The fit also allowed for an additional linear background which is indicated by the blue-dashed curve which is only significant is the high-energy tail of the distribution. The same line shapes were used to fit the distribution in (b) where only their magnitudes were varied.

Fig. 2(b). No background subtraction was made in this case as the production of heavier C and O isotopes was negligible. The much smaller statistics reflect the expected small cross sections for proton pickup. This distribution has been fit with the same two line shapes as in Fig. 2(a) allowing only their magnitudes to vary. This fit shows that this distribution is dominated by the contribution from the excited state. The disappearance of the ground-state  $\ell=0$  resonance is not unexpected as such pickups are greatly suppressed due to the linear and angular moment mismatch. The same logic implies that the significant population of the excited state results from a  $\ell=1$  resonance. These data then tend to support the argument of Tilley *et al.* that the peak at  $E_r^{max} \approx 2.8$  MeV is an  $\ell=1$  resonance, possibly the analog of the 1<sup>+</sup> first excited state in <sup>10</sup>Li [18].

Some caution must be taken in interpreting our fits. We have assumed only two levels contribute to the observed yields yet theoretical calculations predicts more peaks in this region of  $E_T$  which are listed in Table II. Two studies with the complex scaling method by Aoyama

tential is just not deep enough to produce a bound state and it is manifested by large  $s_{1/2}$  scattering cross section as the bombarding energy approaches zero. In contrast to this, a more recent study by Cavallaro *et al.* found no evidence of  $s_{1/2}$  strength near threshold, but instead found evidence for such strength 1.5 MeV above threshold [11]. This raises doubts as to whether the  $s_{1/2}$  and  $p_{1/2}$  single-particle orbitals are inverted, as in <sup>11</sup>Be, or have the regular energy ordering. Subsequently a number of theoretical works suggested that the experiment of Cavallaro *et al.* has little sensitivity to the proposed virtual state [36, 37].

The three theoretical studies in Table II also predicted <sup>10</sup>Li levels and, in all three cases, the  $p+{}^{9}C$  potential was assumed identical to that for  $n+{}^{9}Li$  apart from the addition of a Coulomb component. With a Coulomb barrier, a  $J^{\pi}=2^{-}$  virtual state in <sup>10</sup>Li can be the mirror of a resonance in <sup>10</sup>N. Within their theoretical framework, Aoyama et al. noted "that an s-wave resonance might exist at  $\approx 1.5$  MeV below the *p*-wave resonance in <sup>10</sup>N, if the s-wave neutron in is not bound but near zero energy in <sup>10</sup>Li". The  $n+{}^{9}$ Li potential used by Garrido et al. from Ref. [36] was constructed to produce a  $J^{\pi}=2^{-1}$ virtual state and the predicted location of the mirror  $2^$ state in  $^{10}$ N is similar to that found by Aoyama *et al.* The centroids we find for the fitted  $\ell=0$  and  $\ell=1$  <sup>10</sup>N components, which differ by 1.3 to 1.5 MeV depending on the definition of the reference energy definition (see Table I), are consistent with both complex scaling predictions of a virtual  $2^{-}$  state in  ${}^{10}$ Li.

However the GSM calculations paint a different picture. The predicted  $1^-$  and  $2^- \ell = 0$  resonances in  ${}^{10}\text{Li}$  are above the  $1^+$  and  $2^+ \ell = 1$  resonances, i.e. the opposite order to that predicted for  ${}^{10}\text{N}$ . Indeed the locations of these  $\ell=0$  resonances is close to that inferred experimentally by Cavallaro *et al.* [11]. Our fitted  $\ell=0$  component in Fig. 2(a) could be associated with the  $1^{-10}$  N resonance in the GSM, however the GSM predicts a second  $\ell=0$  resonance which would lie underneath our fitted  $\ell=1$ component. The location of this second  $\ell=0$  resonance in the GSM is close to the second  $\ell=0$  resonance measured by Hooker *et al.* [6] (see Table I). Thus, the results of Hooker *et al.* support the structure of  $^{10}$ Li as predicted by the GSM. Unfortunately the resonances extracted in the present work can be interpreted within both the GSM and the complex-scaling calculations but these intrepretations lead to conflicting conclusions about structure of  $^{10}$ Li.

In conclusion, evidence for <sup>10</sup>N states were found in the invariant-mass distributions of  $p+{}^{9}C$  pairs produced following the knockout of one proton and two neutrons from an E/A=65.4-MeV <sup>13</sup>O beam and from proton pickup by an E/A=68-MeV <sup>9</sup>C beam, in both cases, using the same <sup>9</sup>Be target. Both invariant-mass distributions are dominated by broad strength with similar shape centered at  $E_T=2.8$  MeV. Theoretical calculations predict both  $\ell=0$  and  $\ell=1$  resonances at low energy, but based on the expected bias of the pickup reaction for higher  $\ell$  values, we attribute this strength to  $\ell=1$  resonance(s). This strength is consistent with previous measurement by Lépine-Szily et al. in a multi-nucleon exchange reaction which Tillev *et al.* argued is  $\ell=1$ . Theoretical models which are cognizant of the continuum also predict a doublet of  $\ell=1$  resonances in this energy region.

The experimental distribution from the knockout reaction has extra strength at lower energies with is attributed to contributions from one or more  $\ell=0$  states. This is roughly consistent with the resonance  $p+{}^{9}C$  elastic scattering data of Hooker *et al.* which was fit with two  $\ell=0$  resonances. The higher-energy of these two  $\ell=0$ resonances is also roughly centered at our assigned  $\ell=1$ strength. This generates some tension between the analysis of these different data sets, but perhaps both  $\ell=0$ and  $\ell=1$  strength exist at this energy.

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