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Large- N_c analysis of two-nucleon neutrinoless double beta decay and charge-independence-breaking contact terms

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Abstract

The interpretation of experiments that search for neutrinoless double beta decay relies on input from nuclear theory. Cirigliano *et al.* recently showed that, for the light Majorana exchange formalism, effective field theory calculations require a $nn \rightarrow ppe^-e^-$ contact term at leading order. They estimated the size of this contribution by relating it to measured charge-independence-breaking (CIB) nucleon-nucleon interactions and making an assumption about the relative sizes of CIB operators. We show that the assumptions underlying this approximation are justified in the limit of the number of colors N_c being large. We also obtain a large- N_c hierarchy among CIB nucleon-nucleon interactions that is in agreement with phenomenological results.

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I. INTRODUCTION

Significant experimental efforts are underway to detect neutrinoless double beta ($0\nu\beta\beta$) decay [1–14], a process in which two neutrons are converted into two protons with the emission of two electrons but without the accompanying emission of neutrinos. Neutrinoless double beta decay is a highly sensitive probe of lepton number violation (LNV) and, if detected, would be a clear demonstration that neutrinos are Majorana particles [15]. If this process is observed, it would also shed light on the neutrino mass hierarchy [16, 17] and on the matter-antimatter asymmetry in the universe [18].

The inverse of the $0\nu\beta\beta$ half-life can be expressed as (see Refs. [19–21] for reviews)

$$[T_{1/2}^{0\nu}]^{-1} = G_{0\nu}|M_{0\nu}|^2 m_{\beta\beta}^2, \quad (1)$$

where $m_{\beta\beta}$ is the effective Majorana neutrino mass, $G_{0\nu}$ is a phase space factor, and $M_{0\nu}$ is the corresponding nuclear matrix element (NME). This sensitivity to the NME requires a deep understanding of the nuclear physics involved. One important component in the calculation of the NMEs are multi-nucleon operators that encode the underlying LNV mechanisms. While there are many models and methods that offer insight in this direction, effective field theory (EFT) offers a systematic, model-independent way to study LNV and the corresponding one- and two-nucleon operators that are required as input to many-body calculations. Each independent term in an effective Lagrangian comes with a low-energy coefficient (LEC), into which all unresolved short-distance details are subsumed. These LECs need to be determined from a fit to data or a nonperturbative quantum chromodynamics (QCD) calculation like those performed in lattice QCD (see, e.g., Refs. [22–26] for work related to double beta decays and Ref. [27] for a general review).

An initial step towards the application of EFT to $0\nu\beta\beta$ was taken in Ref. [28] in the context of chiral effective field theory. ChEFT refers to the generalization of chiral perturbation theory (χ PT) [29–34] (see e.g., Refs. [35–40] for reviews)—the EFT of pions and single nucleons based on the approximate chiral symmetry of QCD—to two and more nucleons. Recently, the EFT approach has received renewed attention focusing on the light-Majorana neutrino exchange mechanism [41–44], where it has been shown that a contact term with the undetermined LEC g_ν^{NN} is required at leading order (LO) [45, 46]. This term was absent in previous analyses. Additionally, Refs. [42, 45, 46] observed that isospin symmetry dictates

that g_ν^{NN} is related to the LEC \mathcal{C}_1 of an operator parameterizing charge independence breaking (CIB) in the two-nucleon system. The value of the LEC \mathcal{C}_1 is currently not determined by data. Only the linear combination $\mathcal{C}_1 + \mathcal{C}_2$, where \mathcal{C}_2 is the LEC of a second, independent CIB operator, has been extracted from experiment. To estimate the numerical impact of the contact term in nuclear matrix elements, Refs. [45, 46] assumed that $\mathcal{C}_1 \approx \mathcal{C}_2$ so that the value of g_ν^{NN} can be approximated by $g_\nu^{NN} \approx \frac{1}{2}(\mathcal{C}_1 + \mathcal{C}_2)$. Exploring this assumption from the large- N_c perspective is the main focus of this paper.

While some lattice QCD calculations of double- β decay matrix elements in the two-nucleon system [22, 23] and $0\nu\beta\beta$ calculations in the meson sector [23, 24] exist, a calculation of g_ν^{NN} is currently not available. In the absence of lattice QCD calculations and sufficient data to determine g_ν^{NN} , or equivalently the CIB LECs, the possibility of additional theoretical constraints is critical. Recently, Ref. [47] estimated the values of g_ν^{NN} and $\mathcal{C}_1 + \mathcal{C}_2$ using a method analogous to the Cottingham formula [48, 49]. Their results support the assumption of Refs. [45, 46]. Here, a complementary approach based on the large- N_c limit of QCD is explored. Constraints are obtained through the spin-flavor symmetry that arises in the large- N_c limit of QCD [50–53]. This method has been used to constrain nucleon-nucleon (NN) interactions [54–58], including parity-violating couplings [59, 60], time-reversal-invariance-violating couplings [61, 62], as well as magnetic and axial couplings in the context of pionless EFT (EFT $_\pi$) [63]. Similar work has been done in both the meson [64, 65] and single baryon sectors of chiral perturbation theory (χ PT) [66–70].

There are potential pitfalls to applying the large- N_c expansion to nuclei. For example, Skyrme models suggests that the binding energy per nucleon in nuclear matter predicted in the large- N_c limit is of order the nucleon mass $m_N \sim N_c$ [71, 72], while the observed binding energies are much smaller (of the order of a few MeVs.) For a more detailed discussion see, e.g., Refs. [73, 74] and references therein. Here, we use arguments based on the spin-flavor symmetry to compare the relative sizes of different terms in the NN Lagrangian. This approach gives results for the isospin-invariant NN interactions that are consistent with NN scattering data [54, 55, 58]. Moreover, the Wigner symmetry that was shown to emerge in the large- N_c limit yields agreement with some parity conserving experimental results for larger nuclei (see Ref. [54] and references therein).

This paper is structured as follows. Section II contains a discussion of the results from Refs. [45, 46] relevant for this work. A large- N_c analysis of one- and two-nucleon ma-

trix elements is given in Sec. III, and the spurion construction in ChEFT is discussed in Sec. IV. Complete but minimal sets of spurion operators for both the electromagnetic and weak interactions are derived along with the large- N_c scalings of the corresponding LECs in Sec. V. A large- N_c hierarchy of CIB interactions in comparison to phenomenological descriptions is discussed in Sec. VI. The large- N_c analyzed CIB Lagrangian is mapped onto the CIB Lagrangian of Ref. [46] in Sec. VII and the consistency of the LNV and CIB LECs demonstrated. Finally, Sec. VIII summarizes the results. The appendices contain a detailed discussion of an alternate large- N_c scaling of the quark and nucleon charges, as well as a summary of relevant Fierz identities.

II. BACKGROUND

In this section we introduce and discuss the relevant LNV and CIB Lagrangians. At leading order (LO) in the EFT power counting, there is a contribution to the two-nucleon LNV transition operator from tree-level neutrino exchange between the nucleons. At the same order, there exist contributions from dressing the tree-level diagram by iterations of the LO NN interactions, which include contact terms and one-pion exchange diagrams [46]. A careful analysis [45, 46] of the resulting amplitude using renormalization arguments shows that an LNV amplitude that consists of only the above contributions diverges logarithmically. Therefore, a leading-order contact operator must be included to obtain the correct amplitude at this order. The contact term in the LO Lagrangian is [45, 46]

$$\mathcal{L}_{|\Delta L=2|}^{NN} = \left(2\sqrt{2}G_F V_{ud}\right)^2 m_{\beta\beta} \bar{e}_L C \bar{e}_L^T \frac{g_\nu^{NN}}{4} \left[\left(\bar{N}u\tilde{Q}_L^w u^\dagger N\right)^2 - \frac{1}{6} \text{Tr}\left(\tilde{Q}_L^w \tilde{Q}_L^w\right) \left(\bar{N}\tau^a N\right)^2 \right] + \text{H.c.}, \quad (2)$$

where N represents the doublet of nucleon fields, e_L is the left-handed electron, the charge conjugation matrix is $C = i\gamma^2\gamma^0$, G_F is the Fermi constant, V_{ud} is an element of the Cabibbo-Kobayashi-Maskawa matrix, and

$$\tilde{Q}_L^w = \tau^+ = \frac{1}{2} (\tau^1 + i\tau^2) . \quad (3)$$

The matrix u is

$$u = \exp\left(\frac{i}{2F}\phi_a\tau^a\right), \quad (4)$$

where the ϕ_a ($a = 1, 2, 3$) are the pion fields in Cartesian coordinates, the τ^a are Pauli matrices in isospin space, and F is the pion decay constant in the chiral limit. The renormalization

group (RG) requirement to include a contact term at LO means that an additional unknown LEC, g_ν^{NN} , must be determined in order to analyze and interpret current and future measurements of $0\nu\beta\beta$ decay.

It has been shown in Refs. [45, 46] that chiral symmetry relates the LEC g_ν^{NN} to an electromagnetic CIB isotensor LEC \mathcal{C}_1 . The CIB isotensor Lagrangian in ChEFT has received a significant amount of study [75–77]. In Ref. [46] it is written as

$$\begin{aligned} \mathcal{L}_{CIB}^{NN} = \frac{e^2}{4} \left\{ \mathcal{C}_1 \left[\left(\bar{N}u^\dagger \tilde{Q}_{Ru}N \right)^2 + \left(\bar{N}u \tilde{Q}_{Lu}^\dagger N \right)^2 - \frac{1}{6} \text{Tr} \left(\tilde{Q}_L^2 + \tilde{Q}_R^2 \right) \left(\bar{N}\tau^a N \right)^2 \right] \right. \\ \left. + \mathcal{C}_2 \left[2 \left(\bar{N}u^\dagger \tilde{Q}_{Ru}N \right) \left(\bar{N}u \tilde{Q}_{Lu}^\dagger N \right) - \frac{1}{3} \text{Tr} \left(U \tilde{Q}_L U^\dagger \tilde{Q}_R \right) \left(\bar{N}\tau^a N \right)^2 \right] \right\}, \end{aligned} \quad (5)$$

where $U = u^2$ and here

$$\tilde{Q}_L = \tilde{Q}_R = \frac{1}{2}\tau^3. \quad (6)$$

Additionally, many high-precision NN potentials, such as the Argonne v_{18} [78] and the CD-Bonn [79], as well as several interactions derived from ChEFT [80–83] include short-range CIB and charge-symmetry-breaking (CSB) operators to reproduce the observed scattering data. In principle, determination of \mathcal{C}_1 from data also fixes the value of the $0\nu\beta\beta$ LEC g_ν^{NN} . However, at present only the linear combination $\mathcal{C}_1 + \mathcal{C}_2$ is constrained by available data. The combination $\mathcal{C}_1 - \mathcal{C}_2$ is sensitive to two-nucleon-multi-pion interactions and is currently inaccessible. Reference [46] obtains an estimate of g_ν^{NN} by assuming that the two LECs \mathcal{C}_1 and \mathcal{C}_2 are of the same size and sign, which implies $g_\nu^{NN} \approx \frac{1}{2}(\mathcal{C}_1 + \mathcal{C}_2)$. In the next sections we examine this assumption using large- N_c scaling arguments. In particular, we show that the terms proportional to $\mathcal{C}_1 - \mathcal{C}_2$ are suppressed in the large- N_c limit compared to those proportional to $\mathcal{C}_1 + \mathcal{C}_2$, thereby adding support to the assumptions that \mathcal{C}_1 and \mathcal{C}_2 are of the same size and sign and that g_ν^{NN} can be approximated by the sum of the CIB LECs divided by two.

III. LARGE- N_c SCALING

In this section we outline the basic elements needed to perform large- N_c analyses. The large- N_c scaling of the single-nucleon matrix elements of an n -body operator $\mathcal{O}_{IS}^{(n)}$ with spin S and isospin I is provided by [54, 55]

$$\langle N | \frac{\mathcal{O}_{IS}^{(n)}}{N_c^n} | N \rangle \lesssim N_c^{-|I-S|}, \quad (7)$$

where n denotes the number of quarks involved in the operator. In the large- N_c limit, the Hamiltonian takes a Hartree form [55, 84],

$$H = N_c \sum_n \sum_{s,t} v_{stn} \left(\frac{S^i}{N_c} \right)^s \left(\frac{I^a}{N_c} \right)^t \left(\frac{G^{ia}}{N_c} \right)^{n-s-t}, \quad (8)$$

where the one-body operators are

$$S^i = q^\dagger \frac{\sigma^i}{2} q, \quad I^a = q^\dagger \frac{\tau^a}{2} q, \quad G^{ia} = q^\dagger \frac{\sigma^i \tau^a}{4} q. \quad (9)$$

The nucleon ground state is totally antisymmetric in the color degrees of freedom, and q is a colorless, bosonic quark field. The coefficients v_{stn} are functions of momentum and at most scale as $O(N_c^0)$ [55]. In addition to single-nucleon matrix elements (also see Ref. [85] and references therein), these results were used in the study of two-nucleon interactions via matrix elements of the form [54, 55]

$$V(\mathbf{p}_-, \mathbf{p}_+) = \langle N_\alpha(\mathbf{p}'_1) N_\beta(\mathbf{p}'_2) | H | N_\gamma(\mathbf{p}_1) N_\delta(\mathbf{p}_2) \rangle, \quad (10)$$

where the Greek subscripts indicate combined spin and isospin quantum numbers and

$$\mathbf{p}_\pm \equiv \mathbf{p}' \pm \mathbf{p}, \quad (11)$$

where $\mathbf{p}' = \mathbf{p}'_1 - \mathbf{p}'_2$ and $\mathbf{p} = \mathbf{p}_1 - \mathbf{p}_2$. The two-nucleon matrix elements factorize in the large- N_c limit [54],

$$\langle N_\gamma N_\delta | \mathcal{O}_1 \mathcal{O}_2 | N_\alpha N_\beta \rangle \xrightarrow{N_c \rightarrow \infty} \langle N_\gamma | \mathcal{O}_1 | N_\alpha \rangle \langle N_\delta | \mathcal{O}_2 | N_\beta \rangle + \text{crossed}, \quad (12)$$

and the large- N_c scaling of the two-nucleon matrix elements is determined by the large- N_c dependence of the operators \mathcal{O}_1 and $\mathcal{O}_2 \in \{S^i, I^a, G^{ia}, \mathbb{1}\}$,

$$\begin{aligned} \langle N' | S^i | N \rangle &\sim \langle N | I^a | N \rangle \lesssim 1, \\ \langle N' | G^{ia} | N \rangle &\sim \langle N | \mathbb{1} | N \rangle \lesssim N_c. \end{aligned} \quad (13)$$

In addition, there can be a hidden large- N_c suppression in the momentum dependence of the functions v_{stn} [55]. In t-channel diagrams, factors of \mathbf{p}_+ only enter through relativistic corrections and are therefore suppressed by the nucleon mass, which scales as N_c . Since the analysis in the t-channel is sufficient to establish the large- N_c scaling, momenta are counted as [55]

$$\mathbf{p}_- \sim 1, \quad \mathbf{p}_+ \sim N_c^{-1}. \quad (14)$$

Finally, large- N_c scaling is impacted by the number of pions involved in the process. In χ PT, pion fields are encoded in the exponential matrix u of Eq. (4). Expanding u in the number of pions, we see that each pion field is accompanied by a factor of $1/F$. In the large- N_c limit the decay constant F is $O(\sqrt{N_c})$ [84, 86]; each additional pion field in the expansion of Eq. (4) yields a suppression by $1/\sqrt{N_c}$.

In summary, the large- N_c scaling of the LECs is determined by the spin-isospin structure of the matrix elements of nucleon bilinear operators, the scaling of any relevant momentum factors, and additional suppressions from any pion fields. Finally, the overall factor of N_c in the Hamiltonian of Eq. (8) reduces the scaling of the LECs by one power of N_c .

This approach has been used to analyze the large- N_c behavior of NN interactions in the symmetry-even [54–58, 87, 88] and symmetry-odd sectors [59–62], three-nucleon forces [89], and the coupling of two nucleons to external magnetic and axial fields [63]. In this paper the large- N_c scaling is used to establish relationships among LECs associated with CIB NN operators.

We briefly comment on the role of the Δ in large- N_c ChEFT. The nucleon and Δ mass splitting is $O(1/N_c)$; therefore, the nucleon and the Δ resonance become degenerate in the large- N_c limit. The Δ was shown to play a crucial role in deriving the spin-flavor symmetry and obtaining consistent large- N_c scaling for pion-baryon scattering [90–93], as well as in understanding the meson-exchange picture of the NN interactions [56]. For the quantities of interest here, the Δ can only appear in intermediate states and effects of the virtual Δ degrees of freedom are not considered explicitly in the following. We thus obtain constraints on the LECs in a ChEFT that does not include explicit Δ s. Including the Δ resonance in χ PT changes the size of the LECs and leads to quantities that may depend on the ratio

$$\frac{m_\Delta - m_N}{m_\pi}, \quad (15)$$

which depends on the order in which the large- N_c and chiral limits are taken [34, 52, 94, 95]. While this apparent difference in the treatment of the Δ between the large- N_c and the EFT approaches is an important issue to be resolved, earlier work on the NN interaction that similarly excluded intermediate Δ states obtained results that did not contradict available data [54, 55, 58]. The role of intermediate Δ states in NN scattering in the 1S_0 channel and how they can be integrated out is discussed in Ref. [96].

IV. CHIRAL EFFECTIVE FIELD THEORY AND SPURION FIELDS

The CIB ChEFT Lagrangian is constructed using the spurion technique, the same technique used to construct the mass term in the LO pion Lagrangian and to include the effects of virtual photons and leptons [97–105]. The CIB Lagrangian of interest here contains terms with two insertions of the quark (or equivalently nucleon) charge matrix. The QCD Lagrangian for two flavors in terms of left- and right-handed quark fields with minimal coupling to an electromagnetic potential is

$$\mathcal{L} = i\bar{q}_L \not{\partial} q_L + i\bar{q}_R \not{\partial} q_R - \bar{q}_L M^\dagger q_R - \bar{q}_R M q_L + ieA_\mu [\bar{q}_L \gamma^\mu Q_q q_L + \bar{q}_R \gamma^\mu Q_q q_R], \quad (16)$$

where $Q_q = \text{diag}(\frac{2}{3}, -\frac{1}{3})$ is the quark charge matrix and the unit of charge e is factored out of Q_q (see the last terms in Eq. (16)). The quark mass and the charge matrix terms break chiral symmetry explicitly. For the mass terms, the pattern of symmetry breaking can be mapped onto the effective Lagrangian by (*i*) assuming that the constant matrix M transforms under the chiral symmetry group as

$$M \mapsto RML^\dagger, \quad (17)$$

where R and L are $SU(2)$ matrices transforming the right- and left-handed components of the quark fields, respectively, and (*ii*) constructing all allowed terms that are chirally invariant with the assumed transformation behavior of the quark mass matrix. The same approach can be adopted for terms containing the charge matrix. First, Q is separated into two matrices, Q_L and Q_R , such that the electromagnetic part of the Lagrangian can be written as

$$\mathcal{L}_{EM} = ieA_\mu [\bar{q}_L \gamma^\mu Q_L q_L + \bar{q}_R \gamma^\mu Q_R q_R]. \quad (18)$$

Next, the charge matrices are required to transform under the chiral symmetry group as

$$Q_R \mapsto RQ_R R^\dagger, \quad (19)$$

$$Q_L \mapsto LQ_L L^\dagger. \quad (20)$$

At the nucleonic level, the Lagrangian can be written in terms of the nucleon doublet $N = (p, n)^T$, which transforms under chiral symmetry as

$$N \mapsto K(L, R, u)N, \quad K \in SU(2), \quad (21)$$

while the pion matrix

$$u \mapsto u' = RuK^\dagger = KuL^\dagger. \quad (22)$$

In Ref. [46], the matrix containing the pion fields transforms as $U \mapsto LUR^\dagger$, where $u^2 = U$, while the transformation used here is $U \mapsto RUL^\dagger$ in accord with Ref. [30]. However, this difference does not impact the results. The construction of all possible nucleon operators with two spurion insertions that are invariant under chiral transformations is simplified by using the combinations

$$Q_\pm = \frac{1}{2} [u^\dagger Q_R u \pm u Q_L u^\dagger], \quad (23)$$

which transform under the chiral symmetry group as

$$Q_\pm \rightarrow KQ_\pm K^\dagger. \quad (24)$$

It is useful to separate these spurions into isoscalar and isovector components,

$$Q_\pm = \frac{1}{2} \text{Tr}(Q_\pm) \mathbb{1} + \tilde{Q}_\pm, \quad (25)$$

where

$$\tilde{Q}_\pm = \frac{1}{2} \text{Tr}(Q_\pm \tau^a) \tau^a, \quad (26)$$

so that the operators are written in terms of $\text{Tr}(Q_\pm)$ and \tilde{Q}_\pm .

V. LARGE- N_c SCALING OF NN INTERACTIONS WITH TWO SPURION FIELDS

The large- N_c analysis discussed in Sec. III can be extended to include the spurion operators when an explicit form for the spurion is chosen. The greatest possible large- N_c scaling of a given CIB operator can be deduced from its spin-flavor structure, which is used to guide the elimination of redundant operators when Eq. (26) is inserted in the relevant nucleon bilinears (see Appendix B). However, as discussed above, some operators may receive additional $1/N_c$ suppressions when the leading term contains pion fields from the expansion of u . We will point out an explicit example of this in the next subsection.

One might attempt to obtain the large- N_c scaling of g_ν^{NN} , \mathcal{C}_1 , and \mathcal{C}_2 directly from Eqs. (2) and (5). However, the forms of the Lagrangians in Eqs. (2) and (5) are obtained by using Fierz identities to eliminate redundant operators. This procedure can obscure the correct large- N_c scalings [59, 106]. Therefore, we present an alternative minimal basis in which the

large- N_c scaling of the LECs is manifest. The relationships between these LECs and the ones in Eqs. (2) and (5) are given in Sec. VII.

Instead of working in the basis of Ref. [41], we will use the spurions defined in Eq. (23) and then translate between the two bases after the LO-in- N_c Lagrangian has been derived. When determining the large- N_c scaling of general operator forms we will leave out electromagnetic or weak factors such as e^2 or $(G_F V_{ud})^2$. While these factors impact the overall size of an operator, they will not be relevant for understanding the relative large- N_c rankings among operators that have the same overall multiplicative factor. The most general set of operators for this analysis is given by

$$\begin{aligned}
B_1 &= \text{Tr}(Q_+)^2 (N^\dagger \Gamma N)^2, \\
B_2 &= \text{Tr}(Q_+) (N^\dagger \Gamma N) (N^\dagger \tilde{Q}_+ \Gamma N), \\
B_3 &= (N^\dagger \tilde{Q}_+ \Gamma N)^2, \\
B_4 &= \text{Tr}(Q_-)^2 (N^\dagger \Gamma N)^2, \\
B_5 &= \text{Tr}(Q_-) (N^\dagger \Gamma N) (N^\dagger \tilde{Q}_- \Gamma N), \\
B_6 &= (N^\dagger \tilde{Q}_- \Gamma N)^2, \\
B_7 &= \text{Tr}(Q_+) \text{Tr}(Q_-) (N^\dagger \Gamma N) (N^\dagger \Gamma N), \\
B_8 &= \text{Tr}(Q_-) (N^\dagger \tilde{Q}_+ \Gamma N) (N^\dagger \Gamma N), \\
B_9 &= \text{Tr}(Q_+) (N^\dagger \tilde{Q}_- \Gamma N) (N^\dagger \Gamma N), \\
B_{10} &= (N^\dagger \tilde{Q}_+ \Gamma N) (N^\dagger \tilde{Q}_- \Gamma N), \\
B_{11} &= \text{Tr}(\tilde{Q}_+^2 + \tilde{Q}_-^2) (N^\dagger \Gamma N)^2 = \frac{1}{2} \text{Tr}(\tilde{Q}_R^2 + \tilde{Q}_L^2) (N^\dagger \Gamma N)^2, \\
B_{12} &= \text{Tr}(\tilde{Q}_+^2 - \tilde{Q}_-^2) (N^\dagger \Gamma N)^2 = \text{Tr}(U \tilde{Q}_L U^\dagger \tilde{Q}_R) (N^\dagger \Gamma N)^2, \\
B_{13} &= \text{Tr}(\tilde{Q}_+ \tilde{Q}_-) (N^\dagger \Gamma N)^2 = \text{Tr}(\tilde{Q}_R^2 - \tilde{Q}_L^2) (N^\dagger \Gamma N)^2, \tag{27}
\end{aligned}$$

where Γ can be $\mathbb{1}$, σ^i , τ^a , or $\sigma^i \tau^a$. However, several of the operators that arise once all four of the possibilities for Γ are inserted into the general forms of Eq. (27) will be redundant. The nucleon bilinears contained in operators from B_1 , B_4 , B_7 , B_{11} , B_{12} , and B_{13} have the same structure as the operators from nucleon-nucleon scattering, so operators with $\Gamma = \mathbb{1}$ and $\sigma^i \tau^a$ may start to contribute at LO in N_c , while those with $\Gamma = \sigma^i$ and τ^a are $1/N_c^2$ suppressed. The Fierz identity

$$(N^\dagger \sigma^i \tau^a N)^2 = -3 (N^\dagger N)^2 \tag{28}$$

can be used to eliminate $\sigma^i \tau^a$ in favor of $\mathbb{1}$, and since the corresponding LECs are of the same order there is no change in the scaling obtained. Similarly, the identity

$$(N^\dagger \tau^a N)^2 = -2 (N^\dagger N)^2 - (N^\dagger \sigma^i N)^2 \quad (29)$$

shows that the bilinear with τ^a is not independent of those containing $\mathbb{1}$ and σ^i in the operators of the form $B_1, B_4, B_7, B_{11}, B_{12}$, and B_{13} .

For operators of the form $B_2, B_3, B_5, B_6, B_8, B_9$, and B_{10} , on the other hand, the insertion of $\Gamma = \tau^a$ or $\sigma^i \tau^a$ creates terms containing products of Pauli matrices in isospin space in a single nucleon bilinear. The structure of these terms does not match directly onto the Hartree Hamiltonian of Eq. (8). But the terms can be rewritten using

$$\tau^a \tau^b = \delta^{ab} \mathbb{1} + i \epsilon^{abc} \tau^c, \quad (30)$$

which generates structures that contain at most a single isospin Pauli matrix. Again, the large- N_c scaling of these terms can be determined from Eq. (13), and the forms with $\Gamma = \tau^a$ and $\sigma^i \tau^a$ can be eliminated for this set of operators. There is one more redundancy. Operators with $\Gamma = \mathbb{1}$ or σ^i can be removed through the use of Eq. (26) and Fierz transformations. For B_3, B_6 , and B_{10} , $\Gamma = \mathbb{1}$ can be eliminated, while either choice is suitable for B_2, B_5, B_8 , and B_9 since both choices scale with N_c in the same way. Again, Appendix B contains greater detail about this procedure. In the next two subsections, the explicit forms of the spurion fields for the electromagnetic and the weak cases, respectively, are considered.

A. Electromagnetic Spurions

For the electromagnetic case, it is useful to write the Lagrangian in terms of the nucleon charge matrix,

$$Q = \frac{1}{2} (\mathbb{1} + \tau^3). \quad (31)$$

The difference between using the nucleon charge matrix and using the quark charge matrix amounts to a shift by an unobservable constant [104]. Here, the nucleon charge matrix is independent of N_c , which implies that the up and down quark charges are N_c -dependent. The alternative choice that the quark charges are constant and the nucleon charge depends on N_c is discussed in Appendix A.

Using Eq. (31) as the spurion field in Eq. (27) yields operators with a clear spin-flavor structure. Setting $Q_R = Q_L = Q$ gives

$$\tilde{Q}_\pm = \frac{1}{4} [u^\dagger \tau^3 u \pm u \tau^3 u^\dagger]. \quad (32)$$

The corresponding traces of operators are [105]

$$\text{Tr}(Q_+) = 1, \quad (33)$$

$$\text{Tr}(Q_-) = 0, \quad (34)$$

$$\text{Tr}(\tilde{Q}_+^2 + \tilde{Q}_-^2) = \text{Tr}(\tilde{Q}^2) = 1/2, \quad (35)$$

$$\text{Tr}(\tilde{Q}_+^2 - \tilde{Q}_-^2) = \text{Tr}(U\tilde{Q}U^\dagger\tilde{Q}), \quad (36)$$

$$\text{Tr}(\tilde{Q}_+\tilde{Q}_-) = 0. \quad (37)$$

Since

$$\text{Tr}(\tilde{Q}_+^2 + \tilde{Q}_-^2) (N^\dagger \mathcal{O} N)^2 \sim \text{Tr}(Q_+)^2 (N^\dagger \mathcal{O} N)^2, \quad (38)$$

the operators from B_{11} can be absorbed into those from B_1 . Therefore, the only independent operators are those from $B_1, B_2, B_3, B_6, B_9, B_{10}$, and B_{12} . The operators from B_{10} vanish at least through $O(\phi^2)$ when u is expanded and can be neglected at this order. After eliminating redundancies (see Appendix B) the remaining operators are

$$\mathcal{O}_{1,1} = (N^\dagger N)^2, \quad (39)$$

$$\mathcal{O}_{1,2} = (N^\dagger \sigma^i N)^2, \quad (40)$$

$$\mathcal{O}_2 = (N^\dagger N) (N^\dagger \tilde{Q}_+ N), \quad (41)$$

$$\mathcal{O}_3 = (N^\dagger \sigma^i \tilde{Q}_+ N) (N^\dagger \sigma^i \tilde{Q}_+ N), \quad (42)$$

$$\mathcal{O}_6 = (N^\dagger \sigma^i \tilde{Q}_- N) (N^\dagger \sigma^i \tilde{Q}_- N), \quad (43)$$

$$\mathcal{O}_9 = (N^\dagger \tilde{Q}_- N) (N^\dagger N), \quad (44)$$

$$\mathcal{O}_{12,1} = \text{Tr}(U\tilde{Q}U^\dagger\tilde{Q}) (N^\dagger N)^2, \quad (45)$$

$$\mathcal{O}_{12,2} = \text{Tr}(U\tilde{Q}U^\dagger\tilde{Q}) (N^\dagger \sigma^i N)^2, \quad (46)$$

where the first subscript i in $\mathcal{O}_{i,j}$ indicates the B_i from which each operator originates, and the second index j , where necessary, refers to a specific operator within the B_i , $j = 1, 2, 3, 4$ for $\Gamma = \mathbb{1}, \sigma^i, \tau^a, \sigma^i \tau^a$, respectively.

Finally, as discussed in Sec. III, each additional pion field introduces a factor of $1/F \sim 1/\sqrt{N_c}$. Expanding each operator to second order in the pion fields to determine the maximum N_c scaling of the corresponding LECs yields

$$\mathcal{O}_{1,1} = (N^\dagger N)^2 + \dots, \quad (47)$$

$$\mathcal{O}_{1,2} = (N^\dagger \sigma^i N)^2 + \dots, \quad (48)$$

$$\mathcal{O}_2 = \frac{1}{2} \left(1 - \frac{1}{2F^2} \phi_a \phi_a \right) (N^\dagger N) (N^\dagger \tau^3 N) + \frac{1}{4F^2} \phi_3 \phi_a (N^\dagger N) (N^\dagger \tau^a N) + \dots, \quad (49)$$

$$\mathcal{O}_3 = \frac{1}{4} \left(1 - \frac{1}{F^2} \phi_a \phi_a \right) (N^\dagger \sigma^i \tau^3 N)^2 + \frac{1}{4F^2} \phi_3 \phi_a (N^\dagger \sigma^i \tau^3 N) (N^\dagger \sigma^i \tau^a N) + \dots, \quad (50)$$

$$\mathcal{O}_6 = \frac{1}{4F^2} \epsilon^{3ab} \epsilon^{3cd} \phi_a \phi_c (N^\dagger \sigma^i \tau^b N) (N^\dagger \sigma^i \tau^d N) + \dots, \quad (51)$$

$$\mathcal{O}_9 = -\frac{1}{2F} \epsilon^{3ab} \phi_a (N^\dagger \tau^b N) (N^\dagger N) + \dots, \quad (52)$$

$$\mathcal{O}_{12,1} = \left[\frac{1}{2} - \frac{4}{F^2} (\phi_a \phi_a - \phi_3 \phi_3) \right] (N^\dagger N)^2 + \dots, \quad (53)$$

$$\mathcal{O}_{12,2} = \left[\frac{1}{2} - \frac{4}{F^2} (\phi_a \phi_a - \phi_3 \phi_3) \right] (N^\dagger \sigma^i N)^2 + \dots \quad (54)$$

where the ellipses indicate additional pion fields. The scaling of the LECs $\bar{\mathcal{C}}_{i,j}$ multiplying $\mathcal{O}_{i,j}$ in the Lagrange density is given by

$$\bar{\mathcal{C}}_{1,1} \sim N_c, \quad (55)$$

$$\bar{\mathcal{C}}_{1,2} \sim N_c^{-1}, \quad (56)$$

$$\bar{\mathcal{C}}_2 \sim 1, \quad (57)$$

$$\bar{\mathcal{C}}_3 \sim N_c, \quad (58)$$

$$\bar{\mathcal{C}}_6 \sim 1, \quad (59)$$

$$\bar{\mathcal{C}}_9 \sim N_c^{-1/2}, \quad (60)$$

$$\bar{\mathcal{C}}_{12,1} \sim N_c, \quad (61)$$

$$\bar{\mathcal{C}}_{12,2} \sim N_c^{-1}. \quad (62)$$

The operators $\mathcal{O}_{1,1}$ and $\mathcal{O}_{12,1}$ differ only at the multi-pion level. Therefore, differences between the two will be $1/N_c$ suppressed. The same holds for the operators $\mathcal{O}_{1,2}$ and $\mathcal{O}_{12,2}$. The operator \mathcal{O}_6 provides a concrete example of an earlier point: the generic spin-flavor structure of the operator, before expanding u in the number of pion fields, indicates that it could be $O(N_c)$, but the first nonzero term has two pion fields and is thus suppressed by an additional factor of $1/N_c$.

The Lagrangian at LO and next-to-leading order (NLO) in the large- N_c expansion is

$$\mathcal{L}_{\text{LO-in-}N_c} = e^2 \left\{ \left[\bar{\mathcal{C}}_{1,1} + \bar{\mathcal{C}}_{12,1} \text{Tr} \left(U \tilde{Q} U^\dagger \tilde{Q} \right) \right] (N^\dagger N)^2 + \bar{\mathcal{C}}_3 \left(N^\dagger \sigma^i \tilde{Q}_+ N \right)^2 \right\}, \quad (63)$$

$$\mathcal{L}_{\text{NLO-in-}N_c} = e^2 \left\{ \bar{\mathcal{C}}_2 (N^\dagger N) \left(N^\dagger \tilde{Q}_+ N \right) + \bar{\mathcal{C}}_6 \left(N^\dagger \sigma^i \tilde{Q}_- N \right)^2 \right\}. \quad (64)$$

The $\bar{\mathcal{C}}_i$ and $\bar{\mathcal{C}}_{i,j}$ are LECs that have to be determined from comparison to data or from a calculation in terms of the underlying QCD degrees of freedom. Expanding the matrices u and U in the number of pion fields also creates terms at higher order in the large- N_c counting than indicated by the subscript on the left side; see the discussion in Sec. III. In Sec. VII, we will map the form of the Lagrangian in Eqs. (63) and (64) to the one used in Eq. (5) to determine the large- N_c scaling of the LECs in Eq. (5).

B. Weak Spurions

For weak interactions, Q_L is given by Eq. (3) while $Q_R = 0$, which gives

$$Q_\pm = \tilde{Q}_\pm = \pm \frac{1}{2} u Q_L u^\dagger = \pm \frac{1}{2} u \tau^+ u^\dagger. \quad (65)$$

As a result, all traces in Eqs. (27) vanish and therefore operators from $B_1, B_2, B_4, B_5, B_7, B_8, B_9, B_{11}, B_{12}$, and B_{13} do not contribute. Since $\tilde{Q}_+ = -\tilde{Q}_-$, the only nonvanishing term is

$$\left(N^\dagger u \tau^+ u^\dagger \Gamma N \right)^2, \quad (66)$$

and the structures B_3, B_6 , and B_{10} become identical. As pointed out in Ref. [41], the two operators corresponding to $\Gamma = \mathbb{1}$ and σ^i in this term are related through a Fierz identity and are not independent at $O(\phi^0)$. The authors of Ref. [41] choose to retain $\Gamma = \mathbb{1}$; that is, the operator $(N^\dagger \tau^+ N)^2$. According to Eq. (13), this operator does not appear at LO in the large- N_c expansion. However, eliminating the operator $(N^\dagger \sigma^i \tau^+ N)^2$ through the Fierz transformation

$$\left(N^\dagger \sigma^i \tau^+ N \right)^2 = -3 \left(N^\dagger \tau^+ N \right)^2 \quad (67)$$

introduces a hidden LO-in- N_c contribution in the term proportional to $(N^\dagger \tau^+ N)^2$. As a result, after removing the overall factor of N_c from the Hartree Hamiltonian as discussed in Sec. III, g_ν^{NN} is of LO in the large- N_c expansion, $g_\nu^{NN} \sim O(N_c)$. This result by itself does not justify the assumptions underlying the approximation $g_\nu^{NN} \approx \frac{1}{2}(\mathcal{C}_1 + \mathcal{C}_2)$ proposed in

Refs. [45, 46]. However, an inconsistency in the large- N_c scaling of g_ν^{NN} versus $(\mathcal{C}_1 + \mathcal{C}_2)$ would cast doubt on the approximation. As will be shown in Sec. VII, $(\mathcal{C}_1 + \mathcal{C}_2) \sim O(N_c)$, consistent with the LO scaling of g_ν^{NN} found here.

VI. LARGE- N_c HIERARCHY OF CHARGE-INDEPENDENCE-BREAKING INTERACTIONS

Before focusing on the isotensor terms and their relation to g_ν^{NN} , we will analyze the large- N_c scaling of general CIB NN interactions using the results of Sec. V A. In the absence of external pions, the operators in Eqs. (47) - (54) that contain pions only contribute through pion-loop diagrams that are of higher order in the chiral power counting than is considered in this analysis. Adopting the conventions in Ref. [107] (also see Ref. [108]), the NN interactions, including the effects of virtual photons, are divided into four classes characterized by the following isospin structures:

- (I) isospin invariant and charge symmetric: $\mathbb{1}_1 \mathbb{1}_2, \vec{\tau}_1 \cdot \vec{\tau}_2,$
- (II) CIB but not charge-symmetry-breaking (CSB), which have the isotensor form: $\tau_1^3 \tau_2^3 - \frac{1}{3} \vec{\tau}_1 \cdot \vec{\tau}_2,$
- (III) CSB (and thus CIB) terms that are symmetric in spin and isospin indices: $\tau_1^3 + \tau_2^3,$
- (IV) CSB with isospin mixing (these vanish on nn and pp systems, but not np , and only occur in $L \neq 0$ partial waves): $\tau_1^3 - \tau_2^3, (\vec{\tau}_1 \times \vec{\tau}_2)^3.$

The subscripts in the expressions above denote nucleon bilinears one and two. Refs. [77, 108, 109] use dimensional analysis to argue that the size of these interactions is such that Class (I) > Class (II) > Class (III) > Class (IV).

Neglecting the operators \mathcal{O}_6 and \mathcal{O}_9 because they contain at least one pion field, the independent contact operators generated by the spurion formalism fall into the categories

$$(I) \mathcal{O}_{1,1}, \mathcal{O}_{1,2}, \quad (68)$$

$$(II) \mathcal{O}_3, \quad (69)$$

$$(III) \mathcal{O}_2. \quad (70)$$

As discussed in Sec. VA, the pionless parts of the operators $\mathcal{O}_{12,1}$ and $\mathcal{O}_{12,2}$ are identical to $\mathcal{O}_{1,1}$ and $\mathcal{O}_{1,2}$, respectively. Class (I) and (II) interactions appear at the same order in the large- N_c expansion, while Class (III) terms are suppressed by $1/N_c$. It may be unexpected that the large- N_c analysis suggests that the isospin-invariant Class (I) interactions appear at the same order as CIB terms. Recall, though, that the operators considered here are accompanied by factors of e^2 in the Lagrangian. The Class (I) terms derived here are therefore $O(e^2)$ -suppressed corrections to the dominant isospin-invariant interactions. Taking into account the additional e^2 suppression of the isospin-violating terms, our results are not in contradiction with the expectations of Refs. [77, 108, 109] that some Class (I) terms are larger than Class (II) terms. Contact operators leading to Class (IV) CIB contain two derivatives and are of higher order in the EFT expansion. Taking into account the scaling of the momenta in Eq. (14), these terms are at most $O(N_c^0)$. Additionally, at the level of the NN Lagrangian, the two operators that lead to the Class (IV) potential given in [108] are related by Fierz transformations and are not independent at the two-derivative order in the EFT expansion. But previous work [110–112] has shown that formally Fierz-equivalent operators can lead to ambiguities when used in deriving potentials with local regulators.

VII. LARGE- N_c JUSTIFICATION FOR $g_\nu^{NN} \approx \frac{1}{2}(C_1 + C_2)$ [46]

To connect Eq. (63) to Eq. (5), it is helpful to rearrange the LO-in- N_c Lagrangian (Eq. (63)) as

$$\begin{aligned} \mathcal{L}_{\text{LO-in-}N_c} = e^2 \left\{ \frac{1}{2} [2\bar{\mathcal{C}}_{1,1} + \bar{\mathcal{C}}_{12,1} - \bar{\mathcal{C}}_3] \text{Tr}(\tilde{Q}_+^2) (N^\dagger N)^2 \right. \\ \left. + \bar{\mathcal{C}}_3 \left[(N^\dagger \sigma^i \tilde{Q}_+ N)^2 - \frac{1}{6} \text{Tr}(\tilde{Q}_+^2) (N^\dagger \sigma^i \tau^a N)^2 \right] \right\}, \end{aligned} \quad (71)$$

where the second term proportional to $\bar{\mathcal{C}}_3$ is now a symmetric traceless isotensor. The included trace term appears at the same order in the large- N_c expansion. This rearrangement also produces a NLO-in- N_c contribution such that Eq. (64) becomes

$$\begin{aligned} \mathcal{L}_{\text{NLO-in-}N_c} = e^2 \left\{ \frac{1}{2} [2\bar{\mathcal{C}}_{1,1} - \bar{\mathcal{C}}_{12,1} - \bar{\mathcal{C}}_6] \text{Tr}(\tilde{Q}_-^2) (N^\dagger N)^2 + \bar{\mathcal{C}}_2 (N^\dagger N) (N^\dagger \tilde{Q}_+ N) \right. \\ \left. + \bar{\mathcal{C}}_6 \left[(N^\dagger \sigma^i \tilde{Q}_- N)^2 - \frac{1}{6} \text{Tr}(\tilde{Q}_-^2) (N^\dagger \sigma^i \tau^a N)^2 \right] \right\}. \end{aligned} \quad (72)$$

We now consider the isotensor CIB term proportional to $\bar{\mathcal{C}}_3$ in more detail, and relate it to the terms used in Ref. [46], see Eq. (5). Fierz transformations are used to rewrite the

leading terms (see Eq. (B8)). This uncovers LO-in- N_c scaling in terms that naively appear to be of higher order. The resulting Lagrangian is

$$\mathcal{L}_{\text{LO-in-}N_c}^{\Delta I=2} = -3e^2\bar{\mathcal{C}}_3 \left[\left(N^\dagger \tilde{Q}_+ N \right)^2 - \frac{1}{6} \text{Tr} \left(\tilde{Q}_+^2 \right) \left(N^\dagger \tau^a N \right)^2 \right]. \quad (73)$$

Using the definition of the spurion fields in Eqs. (23), the Lagrangian of Eq. (5) can be written as

$$\begin{aligned} \mathcal{L}_{\text{CIB}}^{\text{NN}} = \frac{e^2}{2} \left\{ (\mathcal{C}_1 + \mathcal{C}_2) \left[\left(N^\dagger \tilde{Q}_+ N \right)^2 - \frac{1}{6} \text{Tr} \left(\tilde{Q}_+^2 \right) \left(N^\dagger \tau^a N \right)^2 \right] \right. \\ \left. + (\mathcal{C}_1 - \mathcal{C}_2) \left[\left(N^\dagger \tilde{Q}_- N \right)^2 - \frac{1}{6} \text{Tr} \left(\tilde{Q}_-^2 \right) \left(N^\dagger \tau^a N \right)^2 \right] \right\}. \quad (74) \end{aligned}$$

Comparison with Eq. (73) shows that

$$\frac{1}{2} (\mathcal{C}_1 + \mathcal{C}_2) = -3\bar{\mathcal{C}}_3, \quad (75)$$

which demonstrates that $\mathcal{C}_1 + \mathcal{C}_2 \sim N_c$. A similar transformation for the isotensor term in Eq. (72) shows that

$$\frac{1}{2} (\mathcal{C}_1 - \mathcal{C}_2) = -3\bar{\mathcal{C}}_6, \quad (76)$$

demonstrating that $\mathcal{C}_1 - \mathcal{C}_2$ is $1/N_c$ suppressed relative to $\mathcal{C}_1 + \mathcal{C}_2$. Inverting these equations gives

$$\mathcal{C}_1 = -3\bar{\mathcal{C}}_3 - 3\bar{\mathcal{C}}_6 = -3\bar{\mathcal{C}}_3 [1 + O(1/N_c)], \quad (77)$$

$$\mathcal{C}_2 = -3\bar{\mathcal{C}}_3 + 3\bar{\mathcal{C}}_6 = -3\bar{\mathcal{C}}_3 [1 + O(1/N_c)]. \quad (78)$$

These results support the assumption of Ref.[46] that the LECs in the CIB Lagrangian are of the same size and sign, and that therefore the neutrinoless LEC can be approximated as $g_\nu^{\text{NN}} \approx \frac{1}{2}(\mathcal{C}_1 + \mathcal{C}_2)$.

VIII. CONCLUSION

The renormalization group analysis of Refs. [45, 46] showed that, for light-Majorana exchange, an LNV contact term is required at leading order in ChEFT. The presence of this term impacts the calculation of nuclear matrix elements relevant for $0\nu\beta\beta$ decay. Neither sufficient data nor lattice QCD results are currently available to determine the size of the corresponding LEC, g_ν^{NN} . To estimate the contribution of the LNV contact term to nuclear

matrix elements, Refs. [45, 46] assumed that the two CIB LECs \mathcal{C}_1 and \mathcal{C}_2 are of the same size and sign, which allowed them to approximate $g_\nu^{NN} \approx (\mathcal{C}_1 + \mathcal{C}_2)/2$.

Here, we performed large- N_c analyses of the LNV and CIB NN operators appearing at the first nonvanishing order in ChEFT power counting. Our results show that the assumptions underlying the approximations of g_ν^{NN} used in Refs. [45, 46] are consistent with ordering based upon the large- N_c limit, lending additional support to the numerical estimates for matrix elements found there. They are also in line with the recent results of Ref. [47].

Our analysis also shows a hierarchy of the different classes of CIB NN interactions as defined in Refs. [107]. The ordering obtained does not contradict phenomenological expectations [77, 108, 109]. However, as is generally the case, the large- N_c results should not be treated as precise predictions. The ordering of LECs is based on expansions in $1/N_c$ and the assumption that other numerical factors are of natural size. For example, symmetries not captured by the large- N_c expansion may lead to unnaturally small parameters. In particular, lattice QCD calculations of baryon-baryon interactions suggest that there is an accidental SU(16) symmetry beyond the SU(6) symmetry in three-flavor large- N_c QCD [113, 114]. Two additional caveats to the results in this paper are that there are unresolved open questions involving the application of large- N_c scaling of operators within heavy nuclei, and the potential impact of Δ intermediate states. So far these issues have not exposed any practical flaws to the procedure used in this paper, but they should be kept in mind. We hope that this work will help guide many-body studies of LNV in heavier elements, as well as the interpretation of neutrinoless double beta decay experiments.

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Appendix A: Alternative Electric Charge Scaling

The choice of keeping the nucleon charge independent of N_c , while the quark charges scale with N_c , has the advantage that anomaly cancellations persist in a large- N_c extended standard model [115, 116]. The up and down quark charges in units of e are then given by

$$q_u = \frac{N_c + 1}{2N_c}, \quad q_d = \frac{1 - N_c}{2N_c}, \quad (\text{A1})$$

where N_c is odd but arbitrary. This choice leads to a proton with electric charge of one in units of e when it is taken to consist of $\frac{1}{2}(N_c + 1)$ up quarks and $\frac{1}{2}(N_c - 1)$ down quarks. Similarly, the neutron has electric charge 0 when the numbers of quark flavors are switched.

In the meson sector of χ PT, it is customary to use the quark charge matrix when constructing the spurion counterterms. However, when nucleons are included it is conventional to use the nucleon charge matrix. The terms in the pion Lagrangian are then replaced accordingly, but this only amounts to the addition of an unobservable constant term. When going to large- N_c , it is reasonable to ask if this is still the case when the charge matrices with different large- N_c scalings are interchanged. To answer this question, the quark and nucleon charge matrices are generalized [105],

$$Q = \alpha \mathbb{1} + \beta \tau^3. \quad (\text{A2})$$

The leading order operator in the pion Lagrangian is

$$e^2 C \text{Tr}(QUQU^\dagger) = e^2 C \text{Tr}(\alpha^2 \mathbb{1} + \beta^2 \tau^3 U \tau^3 U^\dagger). \quad (\text{A3})$$

The first term is indeed an unobservable constant shift, and the second term leads to the electromagnetic pion mass splitting when U is expanded to $O(\phi^2)$, i.e.

$$\delta m_\pi^2 = \frac{2e^2}{F_0^2} C. \quad (\text{A4})$$

For quark charges that scale as Eq. (A1), the quark and nucleon charge matrices become

$$Q_q^{\text{q-scaling}} = \frac{1}{2N_c} [\mathbb{1} + N_c \tau^3], \quad (\text{A5})$$

$$Q_N^{\text{q-scaling}} = \frac{1}{2} [\mathbb{1} + \tau^3], \quad (\text{A6})$$

where the superscript indicates that the quark $q = u, d$ charges scale with N_c .

Alternatively, it was argued that for baryons containing $O(N_c^0)$ strange quarks, quantization conditions require the quark charges be fixed to their physical values and independent of N_c [117]. However, for this choice, anomalies in an $SU(N_c)$ -extended standard model do not cancel [115, 116] and the nucleon charge becomes N_c -dependent and unbounded as $N_c \rightarrow \infty$. Nevertheless, as shown in the following, such a choice does not change our conclusions. The quark and nucleon charge matrices are then

$$Q_q^{\text{q-fixed}} = \frac{1}{6} [\mathbb{1} + 3\tau^3] , \quad (\text{A7})$$

$$Q_N^{\text{q-fixed}} = \frac{1}{6} [N_c \mathbb{1} + 3\tau^3] , \quad (\text{A8})$$

where the superscript indicates that the quark charges q are fixed as N_c changes. Regardless of whether the quark or nucleon charge matrices are chosen to scale with N_c , the coefficient $\beta = \frac{1}{2}$. Therefore, both choices lead to the same pion mass splitting.

Based on the argument that a single flavor trace operator in the meson sector of χPT corresponds to a single closed loop in large- N_c QCD, it might be expected that the LEC C scales at most as $C \sim N_c$. Using the typical diagrammatic arguments in Fig. 1, adding a photon in the loop does not modify the color structure, so it still consists of a single sum over all colors but it does pick up a factor of e^2 . Therefore, the pion mass splitting in Eq. (A4) is at most $O(N_c^0)$ when e is taken to be fixed and after accounting for the suppression due to F_0 . However, it was shown (see, e.g., Ref. [115, 116]) that for electroweak effects to be finite, the electromagnetic coupling can be rescaled like the strong coupling, i.e. $e \sim N_c^{-1/2}$. In this case, the mass splitting will be $O(1/N_c)$.

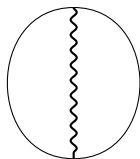


FIG. 1. Leading order diagram in large- N_c QCD which is $O(e^2 N_c)$.

When the nucleon charge is chosen to have the N_c dependence given by Eq. (A8), the large- N_c behavior of the operators in Eqs. (27) needs to be reexamined for possible changes. The operators that contain $\text{Tr}(Q_{\pm})$ are now multiplied by an overall factor of N_c for each

insertion of the trace. This leads to

$$\bar{\mathcal{C}}_{1,1} \sim N_c^3, \quad (\text{A9})$$

$$\bar{\mathcal{C}}_{1,2} \sim N_c, \quad (\text{A10})$$

$$\bar{\mathcal{C}}_2 \sim N_c, \quad (\text{A11})$$

$$\bar{\mathcal{C}}_9 \sim N_c^{1/2}, \quad (\text{A12})$$

while the large- N_c scaling of the other LECs remains unchanged. The operators relevant for the classification of the CIB terms are still $\mathcal{O}_{1,1}$, $\mathcal{O}_{1,2}$, \mathcal{O}_2 , and \mathcal{O}_3 . To obtain the traceless form of the Class (II) interactions, the term in the Lagrangian containing \mathcal{O}_3 can be rewritten as

$$(N^\dagger \sigma^i \tau^3 N)^2 = (N^\dagger \sigma^i \tau^3 N)^2 - \frac{1}{3} (N^\dagger \sigma^i \tau^a N)^2 + \frac{1}{3} (N^\dagger \sigma^i \tau^a N)^2. \quad (\text{A13})$$

The first two terms on the right-hand side combine to form the Class (II) interaction. The last term is absorbed as an $O(N_c)$ contribution into the Class (I) interaction. With the alternative large- N_c scaling of the charges, the classes of charge dependence are then altered such that (II) and (III) are the same order in N_c while they are both suppressed by N_c^{-2} relative to (I). This also indicates that the correspondence between the LNV operator and the CIB contact term remains intact regardless of the choice taken for the scaling of the nucleon charge with N_c .

Appendix B: Fierz identities and the elimination of redundant operators

The operators that contain only traces of the spurions have the form $(N^\dagger \Gamma N)^2$, where Γ can be $\mathbb{1}$, σ^i , τ^a , or $\sigma^i \tau^a$. The Fierz identities in Eqs. (28) and (29) from Sec. V,

$$(N^\dagger \sigma^i \tau^a N)^2 = -3 (N^\dagger N)^2, \quad (\text{28})$$

$$(N^\dagger \tau^a N)^2 = -2 (N^\dagger N)^2 - (N^\dagger \sigma^i N)^2, \quad (\text{29})$$

reduce the number of independent operators from four to two;

$$(N^\dagger N)^2, \quad (\text{B1})$$

$$(N^\dagger \sigma^i N)^2, \quad (\text{B2})$$

where the first operator is LO-in- N_c , and the second is $1/N_c^2$ suppressed.

For the operators involving the traceless part of the spurion field in the bilinears, the spurions in Eq. (26) are expanded and the products of Pauli matrices reduced using Eq. (30). All of the suppressions arising from the presence of pion fields are contained in coefficients defined by $c_{a,\pm} = \frac{1}{2} \text{Tr}(\tilde{Q}_{\pm}\tau^a)$. Therefore, $\Gamma = 1$ and σ^i , respectively, lead to

$$\left(N^\dagger\tilde{Q}_{\pm}N\right)\left(N^\dagger\tilde{Q}_{\pm}N\right) = c_{a,\pm}c_{b,\pm}\left(N^\dagger\tau^aN\right)\left(N^\dagger\tau^bN\right), \quad (\text{B3})$$

$$\left(N^\dagger\tilde{Q}_{\pm}\sigma^iN\right)\left(N^\dagger\tilde{Q}_{\pm}\sigma^iN\right) = c_{a,\pm}c_{b,\pm}\left(N^\dagger\tau^a\sigma^iN\right)\left(N^\dagger\tau^b\sigma^iN\right). \quad (\text{B4})$$

For $\Gamma = \tau^c$,

$$\begin{aligned} \left(N^\dagger\tilde{Q}_{\pm}\tau^cN\right)\left(N^\dagger\tilde{Q}_{\pm}\tau^cN\right) &= c_{a,\pm}c_{a,\pm}\left(N^\dagger N\right)^2 - c_{a,\pm}c_{a,\pm}\left(N^\dagger\tau^bN\right)^2 \\ &\quad + c_{a,\pm}c_{b,\pm}\left(N^\dagger\tau^aN\right)\left(N^\dagger\tau^bN\right), \end{aligned} \quad (\text{B5})$$

and the operator $\left(N^\dagger\tau^bN\right)^2$ can be removed using the Fierz identity of Eq. (29) to obtain

$$\begin{aligned} \left(N^\dagger\tilde{Q}_{\pm}\tau^cN\right)\left(N^\dagger\tilde{Q}_{\pm}\tau^cN\right) &= 3c_{a,\pm}c_{a,\pm}\left(N^\dagger N\right)^2 + c_{a,\pm}c_{a,\pm}\left(N^\dagger\sigma^iN\right)^2 \\ &\quad + c_{a,\pm}c_{b,\pm}\left(N^\dagger\tau^aN\right)\left(N^\dagger\tau^bN\right). \end{aligned} \quad (\text{B6})$$

The first two terms in Eq. (B6) have the same bilinear structure as the operators $\mathcal{O}_{1,1}$ and $\mathcal{O}_{1,2}$, respectively. Their contributions can be absorbed into a redefinition of the LECs of these operators. The third term in Eq. (B6) is Eq. (B3). For $\Gamma = \sigma^i\tau^c$, Eq. (B6) appears again except that the last term is Eq. (B4) instead of Eq. (B3). This shows that $\Gamma = \tau^c$ and $\sigma^i\tau^c$ do not yield additional independent operators and can be neglected.

Additional relationships exist among some of the operators corresponding to $\Gamma = \mathbb{1}$ and $\Gamma = \sigma^i$. For B_3 , B_6 , and B_{10} , $\Gamma = \mathbb{1}$ can be eliminated by applying Fierz transformations to Eq. (B3) along with the decomposition in Eq. (26). Using

$$\text{Tr}\left(\tilde{Q}_{\pm}^2\right) = 2c_{a,\pm}c_{a,\pm}, \quad (\text{B7})$$

the Fierz transformation for Eq. (B3) leads to

$$\begin{aligned} -3\left[\left(N^\dagger\tilde{Q}_{\pm}N\right)\left(N^\dagger\tilde{Q}_{\pm}N\right) - \frac{1}{6}\text{Tr}\left(\tilde{Q}_{\pm}\tilde{Q}_{\pm}\right)\left(N^\dagger\tau^aN\right)^2\right] \\ = \left(N^\dagger\tilde{Q}_{\pm}N\right)\left(N^\dagger\sigma^i\tilde{Q}_{\pm}N\right) - \frac{1}{6}\text{Tr}\left(\tilde{Q}_{\pm}\tilde{Q}_{\pm}\right)\left(N^\dagger\sigma^i\tau^aN\right)^2, \end{aligned} \quad (\text{B8})$$

which can be arranged, with the help of additional Fierz transformations, to be

$$\begin{aligned} \left(N^\dagger\tilde{Q}_{\pm}N\right)\left(N^\dagger\tilde{Q}_{\pm}N\right) &= -\frac{1}{3}\left(N^\dagger\tilde{Q}_{\pm}N\right)\left(N^\dagger\sigma^i\tilde{Q}_{\pm}N\right) - \frac{1}{2}\text{Tr}\left(\tilde{Q}_{\pm}\tilde{Q}_{\pm}\right)\left(N^\dagger N\right)^2 \\ &\quad - \frac{1}{6}\text{Tr}\left(\tilde{Q}_{\pm}\tilde{Q}_{\pm}\right)\left(N^\dagger\sigma^iN\right)^2. \end{aligned} \quad (\text{B9})$$

Therefore, the choice of $\Gamma = \mathbb{1}$ can be eliminated from B_3 , B_6 , and B_{10} in favor of combinations of $\Gamma = \sigma^i$ and operators from B_{11} , B_{12} , and B_{13} .

Following the same procedure for operators from B_2 , B_5 , B_8 , and B_9 results in

$$\text{Tr}(Q_{\pm}) \left(N^{\dagger} \tilde{Q}_{\pm} N \right) (N^{\dagger} N) = c_{a,\pm} \text{Tr}(Q_{\pm}) (N^{\dagger} N)^2, \quad (\text{B10})$$

$$\text{Tr}(Q_{\pm}) \left(N^{\dagger} \tilde{Q}_{\pm} \sigma^i N \right) (N^{\dagger} \sigma^i N) = c_{a,\pm} \text{Tr}(Q_{\pm}) (N^{\dagger} N)^2, \quad (\text{B11})$$

for $\Gamma = \mathbb{1}$ and σ^i , respectively. When $\Gamma = \tau^b$, Fierz transformations yield

$$\text{Tr}(Q_{\pm}) \left(N^{\dagger} \tilde{Q}_{\pm} \tau^b N \right) (N^{\dagger} \tau^b N) = -c_{a,\pm} \text{Tr}(Q_{\pm}) \left[2 (N^{\dagger} N)^2 + (N^{\dagger} \sigma^i N)^2 \right]. \quad (\text{B12})$$

Similarly, $\Gamma = \sigma^i \tau^b$ leads to

$$\text{Tr}(Q_{\pm}) \left(N^{\dagger} \tilde{Q}_{\pm} \sigma^i \tau^b N \right) (N^{\dagger} \sigma^i \tau^b N) = -3c_{a,\pm} \text{Tr}(Q_{\pm}) (N^{\dagger} N)^2. \quad (\text{B13})$$

This shows that again the operators with $\Gamma = \tau^a$ and $\Gamma = \sigma^i \tau^a$ are redundant for creating a complete leading-in- N_c description. As before, additional relationships exist between the $\Gamma = \mathbb{1}$ and $\Gamma = \sigma^i$ operators. Eliminating the remaining redundancy through Fierz transformations leads to

$$-3 (N^{\dagger} N) \left(N^{\dagger} \tilde{Q}_{\pm} N \right) = (N^{\dagger} \sigma^i N) \left(N^{\dagger} \sigma^i \tilde{Q}_{\pm} N \right). \quad (\text{B14})$$

Therefore, either $\Gamma = \mathbb{1}$ or $\Gamma = \sigma^i$ may be retained, and both choices give the same large- N_c counting.

This process eliminates the operators that possess a subleading spin-flavor structure. Any additional factors of N_c that might be present will arise from pion fields in the expansion of u ; however, these factors will not change the spin-flavor structure of the nucleon bilinears, and thus only lead to additional $1/\sqrt{N_c}$ suppressions arising from factors of $1/F$.

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