Bulk viscosity and cavitation in heavy ion collisions

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The Skinny on Bulk Viscosity and Cavitation in Heavy Ion Collisions

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Relativistic heavy ion collisions generate nuclear-sized droplets of quark-gluon plasma (QGP) that exhibit nearly inviscid hydrodynamic expansion. Smaller collision systems such as p+Au, d+Au, and ^{3}He+Au at the Relativistic Heavy Ion Collider, as well as p+Pb and high-multiplicity p+p at the Large Hadron Collider may create even smaller droplets of QGP. If so, the standard time evolution paradigm of heavy ion collisions may be extended to these smaller systems. These small systems present a unique opportunity to examine pre-hydrodynamic physics and extract properties of the QGP, such as the bulk viscosity, where the short lifetimes of the small droplets makes them more sensitive to these contributions. Here we focus on the influence of bulk viscosity, its temperature dependence, and the implications of negative pressure and potential cavitation effects on the dynamics in small and large systems using the publicly available hydrodynamic codes SONIC and MUSIC. We also discuss pre-hydrodynamic physics in different frameworks including AdS/CFT strong coupling, IP-GLASMA weak coupling, and free streaming.

I. INTRODUCTION

At temperatures exceeding $T > 150$ MeV, hadronic matter transitions to a state of deconfined quarks and gluons referred to as quark-gluon plasma (QGP). Some properties of QGP can be calculated via lattice QCD, for example the Equation of State (EOS) – for a recent review see Ref. [1]. In contrast other key properties such as the shear viscosity η and bulk viscosity ζ, typically normalized by the entropy density s, remain beyond current first principles calculations. There has been enormous attention paid to studies of the QGP shear viscosity to entropy density ratio η/s in part because there is a conjectured lower bound of $1/4\pi$ derived within the context of string theoretical AdS/CFT [2]. Extensive experimental measurements at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) indicate that the QGP value near the crossover temperature region $150 < T < 400$ MeV is close to the conjectured bound – for a useful review see Ref. [3, 4]. In contrast, less attention has been cast on studies and experimental constraints of bulk viscosity, though it is no less fundamental a property of QGP.

Modern fluid dynamics is defined as an effective field theory for long-wavelength degrees of freedom. For an uncharged relativistic fluid, these are the energy density ϵ and fluid velocity u^μ. Expectation values of operators such as the energy-momentum tensor $\langle T^{\mu\nu}\rangle$ in the underlying quantum field theory are expanded in powers of gradients of ϵ, u^μ, e.g.

$$\langle T^{\mu\nu}\rangle = T^{\mu\nu}_{(0)} + T^{\mu\nu}_{(1)} + \dots\,, \tag{1}$$

where $T^{\mu\nu}_{(0)}$ contains no gradients, $T^{\mu\nu}_{(1)}$ contains first-order gradients, and so on. One can show that for an uncharged fluid the first-order term contains only two different components,

$$T^{\mu\nu}_{(1)} = \eta\sigma^{\mu\nu} + \zeta\left(\nabla_\lambda u^\lambda\right)\Delta^{\mu\nu}\,, \tag{2}$$

where $\sigma^{\mu\nu}$ is the traceless shear-stress tensor that is first order in gradients, and $\Delta^{\mu\nu}$ is a symmetric projector that obeys $u_\mu\Delta^{\mu\nu} = 0$ and $\Delta^\mu_\mu = 3$. The form of (2) implies that the shear viscosity coefficient η couples to shear stresses, while bulk viscosity ζ couples to expansion or compression of the fluid.

Conservation of the energy-momentum tensor (which is exact) leads to the equations of motion for the fields ϵ, u^μ, defining the theory. To lowest order in the gradient expansion (ideal fluid dynamics), one finds for instance

$$u^\mu\nabla_\mu\epsilon + (\epsilon + P)\left(\nabla_\lambda u^\lambda\right) = 0 \tag{3}$$

Many non-relativistic fluids are well approximated by assuming incompressibility, e.g. $\epsilon =$ const., which from the above equation of motion therefore implies $\nabla_\lambda u^\lambda = 0$, hence the bulk-viscous term in (2) is absent even if $\zeta \neq 0$. Therefore, quite often bulk viscosity does not play an important role in non-relativistic fluid dynamics.

On the other hand, relativistic fluids are never incompressible since Lorentz contractions must be exactly preserved. Therefore, bulk viscous effects can potentially be of importance for relativistic systems.

Since $\mathrm{Tr}\,T^{\mu\nu}_{(1)} = 3\zeta\left(\nabla_\lambda u^\lambda\right)$, the bulk viscosity coefficient vanishes if the energy-momentum tensor is traceless, $\mathrm{Tr}\,\langle T^{\mu\nu}\rangle = 0$. This is the case for systems with conformal symmetry, such that for conformal fluids $\zeta = 0$. For non-conformal systems, $\zeta \neq 0$, and the value of ζ is related to the two-point correlation function of the trace of the energy-momentum tensor.

II. BULK VISCOSITY, NEGATIVE PRESSURE, AND CAVITATION

There are numerous works that have attempted to understand the bulk viscosity as a function of temperature motivated by various theoretical calculations – for a comprehensive discussion see Refs. [5–13]. General arguments follow the reasoning that the QGP becomes more conformal at very high temperature, as indicated by both perturbative QCD and lattice QCD calculations of the trace anomaly. This leads to the expectation that the bulk viscosity to entropy density goes to zero at high temperature, though this gives little guidance as to how quickly. Conversely, deep in the hadronic phase (T << 150 MeV), bulk viscosity is expected to become exponentially large, $\zeta \propto \frac{f_\pi^8}{m_\pi^5} e^{2m_\pi/T}$ with m_π and f_π are the pion mass and decay constant, respectively [8]. However, note that this result for the bulk viscosity depends on the time-scale one is interested in. In equilibrium (extremely long times), bulk viscosity is dominated by the rate of number changing processes, which are very slow (hence the large bulk viscosity coefficient). If one is interested in slightly out-of-equilibrium situations, the effective bulk viscosity coefficient is smaller [9]. In a real-world heavy-ion collision, the relevant time-scale is much shorter, so number changing processes are not sufficiently fast to equilibrate particle species. As a result, the effective bulk viscosity in this low-temperature regime is extremely small (negligible). However, since particle species can no longer be exchanged, the system has frozen out chemically [8], such that in lieu of a bulk viscosity there is a chemical potential for each frozen-out species. In Ref. [14], it has been suggested that bulk viscosity may have some relation with hadronization in heavy-ion collisions.

Using a phenomenological model to relate lattice gauge theory results for $\mathrm{Tr}\,\langle T^{\mu\nu}\rangle$ in QCD to the bulk viscosity coefficient, Ref. [15] reported a decline by an order of magnitude for ζ/s from T_c to $1.2T_c$, where T_c is the critical temperature in their calculation. Unfortunately, some of the bulk viscosity values reported in Ref. [15] were *negative*, which would have indicated a thermodynamic instability of QCD. In Ref. [16], it was later shown that the ad-hoc model used in Ref. [15] was based on an incorrect sum-rule. As a result, there currently is no reliable constraint on the temperature dependence in the range accessible by present-day colliders.

Phenomenological studies at temperatures below the QCD transition, i.e. in the hadron gas regime including multiple resonance states, yield a bulk viscosity with a rather flat temperature dependence. Inclusion of an exponential continuum of resonance states (a so-called "Hagedorn State" (HS) continuum [18]) yields ζ/s values that modestly increase with temperature [19]. However, this work [19] again uses the incorrect sum-rule model proposed in Ref. [15]. Regardless, using results from Refs. [15, 19] as input, the authors of Ref. [20] in turn introduced the following parameterization for ζ/s for use

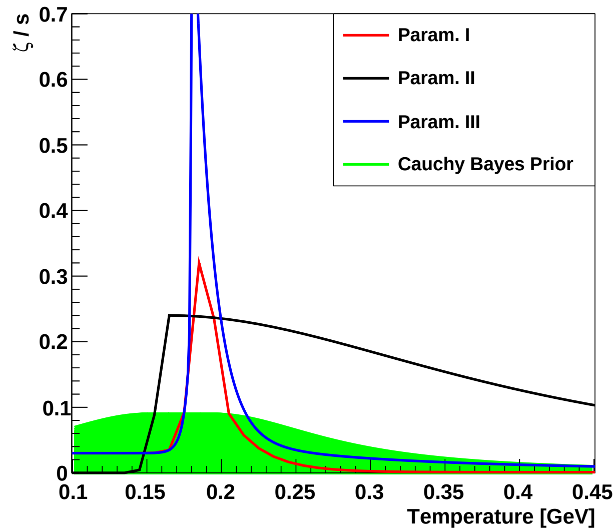

FIG. 1: Various parameterizations for the temperature dependence of ζ/s. The green band represents the family of parameters of the Cauchy function utilized at the Bayesian Prior as detailed in Ref. [17].

in hydrodynamic calculations:

$$\zeta/s = \begin{cases} 0.03 + 0.9e^{\frac{T/T_p-1}{0.0025}} + \\ \qquad 0.22e^{\frac{T/T_p-1}{0.022}} & T < 180 \text{ MeV} \\ \\ 27.55(T/T_p) - 13.45 \\ \qquad -13.77(T/T_p)^2 & 180 < T < 200 \\ \\ 0.001 + 0.9e^{\frac{-(T/T_p-1)}{0.0025}} + \\ \qquad 0.25e^{\frac{-(T/T_p-1)}{0.13}} & T > 200 \text{ MeV} \end{cases} \qquad (4)$$

where $T_p = 180$ MeV. This parameterization, labeled as "Param. I" peaks with a maximum $\zeta/s \approx 0.3$ and is shown in Figure 1.

Recently, an alternative parameterization has been put forth [21] that has a large value for ζ/s over a very broad temperature range, also shown in Figure 1 and labeled as "Param. II".

$$\zeta/s = \begin{cases} B_{norm} exp\left(-\frac{T-T_{peak}}{T_{width}}^2\right) & T < T_{peak} \\ \\ B_{norm}\frac{B_{width}^2}{(T/T_{peak}-1)^2+B_{width}^2} & T > T_{peak} \end{cases} \qquad (5)$$

The parameters are set with $T_{peak} = 165$ MeV, B_{norm}=0.24, B_{width} =1.5, and T_{width}= 10 MeV, where we note an error in Ref. [21] misquoted T_{width}=50 MeV. This new parameterization is determined within their hybrid (IP-GLASMA + hydrodynamic MUSIC + hadronic scattering URQMD) calculation primarily by the mean transverse momentum of midrapidity hadrons $\langle p_T \rangle$ in heavy-ion $A+A$ collisions, highlighting the significant constraint from peripheral $A+A$ collisions. The IP-GLASMA [22] early stage of evolution follows weakly-coupled Yang-Mills dynamics and as such is close to free streaming. Thus, large radial flow quickly develops, i.e. where the radial position and radial velocities are highly correlated. This is particularly true in smaller collision volumes where the length scale of pre-hydrodynamic expansion $c\tau$ is on the same scale as the overall size of the system. Thus, a larger bulk viscosity is needed to temper the large radial flow that the hydrodynamics is initialized with – as noted in Ref. [23].

It is notable that IP-GLASMA evolution may have even larger radial expansion than free streaming since in the latter the longitudinal pressure $p_L = 0$ while in the former one can have negative longitudinal pressure leading to larger transverse pressure. However, a more critical consideration may be how the matching between pre-hydrodynamic and hydrodynamic stages is handled where there is a mismatch in pressure between the two stages. In Ref. [24], they deal with the mismatch by introducing an effective bulk viscous term $\Pi = P_{CYM} - P_{Lat}$, where $P_{CYM} = \varepsilon/3$ and P_{Lat} are the pressure from the Yang-Mills IP-GLASMA side and the QCD lattice equation of state side, respectively. They note that this leads to an additional outward push. Other approaches simply set $\Pi = 0$ (for example in Ref. [17]); however, both by necessity invoke a bulk relaxation time that may play a very important role – also see the discussion in Ref. [12].

In peripheral $A+A$ collisions, experimental data indicates a decrease in the $\langle p_{\mathrm{T}} \rangle$, whereas without the large bulk viscosity the hybrid calculation would have a modest increase. We highlight that using peripheral $A+A$ data for $\langle p_{\mathrm{T}} \rangle$ is likely incorrect as recent studies indicate that a centrality selection bias works to anti-select against jets in these events [25]. Jets increase the multiplicity and move events to slightly more central categories, and thus more peripheral categories see a suppression of higher p_T particles [26]. We also note that in this hybrid approach, the transition temperature between hydrodynamics and hadron cascade is set at 145 MeV, corresponding to a point where ζ/s is actually quite small. This parameterization has also been applied to hydrodynamic calculations for small collision systems [24] – which is very relevant to this paper as the interplay of pre-hydrodynamic physics and QGP properties becomes critical.

We highlight two other parameterizations considered in the literature for comparison. There is an additional parameterization similar to Param. I in that it is sharply peaked near the transition temperature. This parameterization, referred to as "Param. III" is shown in Figure 1 and peaks with an even larger value of ζ/s [27].

Lastly, in a Bayesian inference analysis of p+Pb and Pb+Pb flow patterns [17], the authors include a parameterization of ζ/s using a Cauchy function:

$$(\zeta/s)(T) = \frac{(\zeta/s)_{max}}{1 + ((T - (\zeta/s)_{T_0})/(\zeta/s)_{width})^2}. \quad (6)$$

In their Bayesian prior, they consider these parameters over the following ranges: $(\zeta/s)_{max} = 0.0 - 0.1$, $(\zeta/s)_{T_0} = 150 - 200$ MeV, and $(\zeta/s)_{width} = 0 - 100$ MeV. The full range of functional forms with these parameter ranges is shown as a green band in Figure 1. It is striking that the Bayesian prior is rather restrictive and completely excludes the three other parameterizations used elsewhere in the current literature. In part, the authors [17] simply avoid values of ζ/s where significant negative pressures occur in the hydrodynamic simulations. The calculations incorporate a free-streaming pre-hydrodynamic evolution and are able to match the mean transverse momentum with lower bulk viscosity, in part due to a mismatch in the "effective" pressures between the pre-hydrodynamic and hydrodynamic stages – see the earlier discussion.

Both shear and bulk viscosity represent out-of-equilibrium corrections to ideal hydrodynamics. At the freeze-out point in hydrodynamics, the standard approach is to transition to particles via the Cooper-Frye formalism [28] which explicitly utilizes the equilibrium condition. In the case of large shear or bulk viscosity, the hadronization formalism requires non-equilibrium corrections, which are parametrized by a δf term. For small departures from equilibrium, the δf term may be obtained by matching to kinetic theory, cf. Refs. [3, 29]

$$\delta f \propto f_{\mathrm{eq}} \frac{p^\mu p^\nu}{T p^\rho u_\rho} \left(\frac{\sigma_{\mu\nu}}{2} + \left(\frac{\Delta_{\mu\nu}}{3} - c_s^2 u_\mu u_\nu \right) \nabla_\lambda u^\lambda \right). \quad (7)$$

For conformal systems where the bulk correction vanishes, a model for δf can be derived that correctly reproduces the hydrodynamic energy-momentum tensor and is well behaved even for large shear stresses (cf. [3], Section 3.1.5). Similarly, models for δf within the framework of anisotropic hydrodynamics have been proposed that correctly reproduce a non-interacting expanding gas [30].

However, in the case of bulk viscosity, the δf correction is not well known [31]. Thus, in some works with significant non-zero ζ/s, they simply apply no δf correction at all. Ref. [32] addresses this as follows: "Given this uncertainty and the small ζ/s at particlization, we assume that bulk corrections will be small and neglect them for the present study, i.e. δf bulk $= 0$. This precludes any quantitative conclusions on bulk viscosity, since we are only allowing bulk viscosity to affect the hydrodynamic evolution, not particlization." A major issue here is that there should be a systematic uncertainty in the calculations for the temperature at which one transitions from hydrodynamics to hadron cascade. Of course, by not varying this temperature, one only considers cases where hadronization occurs after the ζ/s value has fallen by more than a factor of ten from its peak at the nominal transition temperature. As we explore in this paper, this

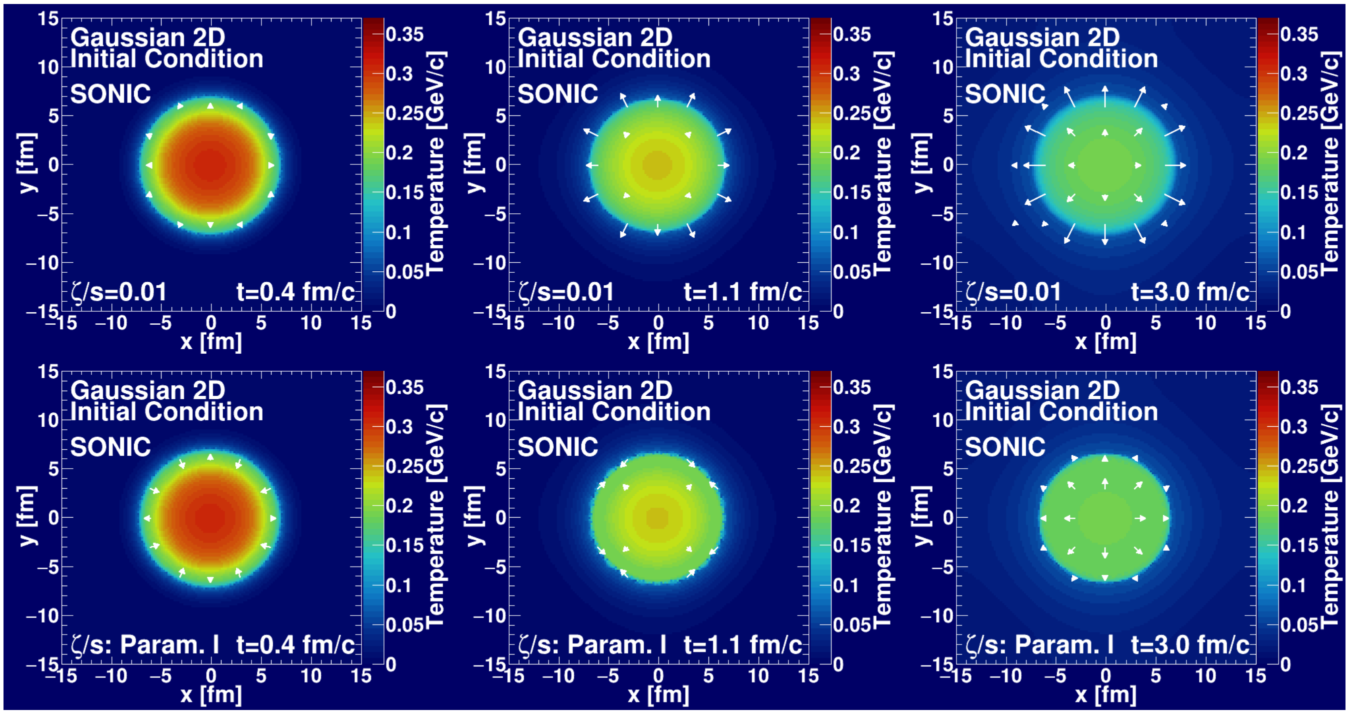

FIG. 2: SONIC hydrodynamic simulation with single Gaussian source and $\eta/s = 1/4\pi$ and $\zeta/s = 0.01$ (left column) or ζ/s from Param. I (right column). Arrows represent the fluid cell velocities.

leads to enormous uncertainties, particularly for small collision systems.

In contrast, other calculations utilize specific examples of possible bulk δf parameterizations – see for example Ref. [20]. In general they also only consider variations in hadronization temperatures in the lower temperature region when ζ/s is very small. In the SONIC calculations [33], they only consider very small values of $\zeta/s = 0.01$ (discussed more below) and thus neglect any δf correction. We highlight here that they implement a strongly-coupled AdS/CFT inspired pre-hydrodynamic evolution. As such, in contrast to free streaming or IP-GLASMA evolution, no extreme radial flow develops early and hence no large bulk viscosity is required to temper the growth in $\langle p_T \rangle$, as was explicitly shown Figure 5 of Ref. [34].

A critical issue with implementations of large bulk viscosity relate to cavitation. Cavitation is the formation of bubbles or cavities within a fluid, appearing in areas of low relative pressure. In case of QGP droplets, cavitation implies regions of hadronic gas inside the fluid at temperatures well above $\mathrm{T_c}$. These bubbles can grow, shrink, collide and otherwise interacted in a highly non-trivial manner, and the physics of cavitation is not described in current hydrodynamic codes – for example MUSIC, SONIC, or any other heavy-ion implementation.

We highlight that there is no full dynamical solution for the QGP going into the cavitation stage. Thus, one can only say for certain that there are cases with large negative pressure and thus instabilities may arise that are definitely outside the domain of current hydrodynamic codes. Whether full bubbles form from cavitation will also depend on the interplay of the expansion rate for a given geometry QGP and the rate of cavitation.

Cavitation in heavy-ion collisions was first discussed in detail in Ref. [35]. The exploratory study concluded that ζ/s with a significant enough peak near the transition temperature will trigger cavitation. A SONIC calculation detailed in Ref. [36] calculates LHC energy $p{+}p$ collisions and a bulk viscosity $\zeta/s \approx 0.01 - 0.02$ is required to moderate the growth in $\langle p_T \rangle$ in high-multiplicity events. These small values of ζ/s yield small values of the bulk pressure Π and thus cavitation does not occur.

In contrast, in Ref. [27], the authors consider large ζ/s scenarios corresponding to Param. I and Param. III, and find that for $A + A$ central collisions, Param. I leads to modest regions of negative pressure at large radius at early times and Param. III leads to significant negative pressure over a very large portion of the space-time volume of QGP. They plot the ratio of bulk pressure Π divided by the standard pressure $\mathrm{P_0}$, $\Pi/\mathrm{P_0}$. If $\Pi/\mathrm{P_0}$ falls below -1.0, then the effective pressure is negative and cavitation is possible. We highlight that this regime of large negative pressure can also result in violations of causality within specific implementations of hydrodynamics [37], and that these considerations warrant further concern about the reliability of any results.

In the calculations with Param. II that include large bulk viscosities over a wide range of temperatures, the issue of cavitation is not discussed [21, 24]. A focus of

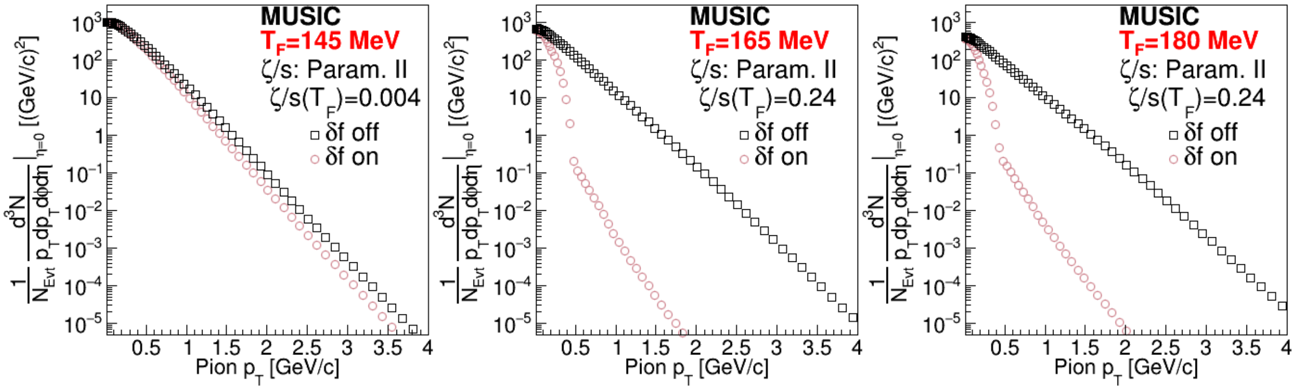

FIG. 3: MUSIC results for a single Gaussian initial geometry run with ζ/s Param. II with three different freeze-out temperatures. Shown are pion p_T distributions with and without δf corrections applied.

Test case	Bulk Param.	δf corr.	$T_{hadronize}$	$< p_{\rm T} >$ pion	$< p_{\rm T} >$ proton
0	Param. I	yes	145 MeV	413 MeV	790 MeV
1	Param. I	no	145 MeV	494 MeV	853 MeV
2	Param. I	yes	165 MeV	382 MeV	698 MeV
3	Param. I	no	165 MeV	496 MeV	810 MeV
4	Param. I	yes	180 MeV	145 MeV	617 MeV
5	Param. I	no	180 MeV	545 MeV	889 MeV
6	Param. II	yes	145 MeV	406 MeV	706 MeV
7	Param. II	no	145 MeV	450 MeV	735 MeV
8	Param. II	yes	165 MeV	170 MeV	534 MeV
9	Param. II	no	165 MeV	466 MeV	742 MeV
10	Param. II	yes	180 MeV	151 MeV	532 MeV
11	Param. II	no	180 MeV	499 MeV	788 MeV

TABLE I: MUSIC results for the single Gaussian geometry for the pion and proton $\langle p_{\rm T}\rangle$ for different ζ/s parameterizations, turning on and off δf corrections, and different freeze-out temperatures.

this paper is to explore the issue of negative pressure and discuss implications of cavitation in these various scenarios.

Noting that large ζ/s is likely to lead to unreliable hydrodynamic results as well as very large systematic uncertainties from variations in the hadronization temperature and δf corrections, that does not logically lead to the conclusion that these large ζ/s values are incorrect. Mother nature does not have to be kind.

However, if cavitation does indeed correspond to the onset of hadronization as advocated for in Ref. [3], then its occurrence should be observable. According to the argument outlined in Ref. [38], cavitation would lead to the creation of hadrons at temperatures well above the QCD phase transition, which would be detected. Since experimentally measured hadron spectra are inconsistent with freeze-out temperatures much in excess of T_c, this implies that significant cavitation does not occur in heavy-ion collisions. As a consequence, bulk viscosity values that lead to strong cavitation events in hydrodynamic simulations of ion collisions are likely disallowed by experiment.

III. DEFINITION OF METHODS

In this study, we are not interested in exact matching of experimental data and rather understanding the basic consequences of different bulk viscosity implementations. To that end, we consider a simple geometry in our studies. We utilize a geometry defined by two-dimensional Gaussian distribution for the energy density

$$\epsilon(x,y) = \epsilon_0 e^{-(x^2+y^2)/2\sigma^2} \tag{8}$$

where the width is set $\sigma = 1.0$ fm and the normalization ϵ_0 is set such that the corresponding central temperature $T \approx 370$ MeV. This initial energy density is a proxy for small collision systems, and one where we have chosen an azimuthally symmetric parameterization to focus on radial expansion.

We have utilized two publicly available hydrodynamic codes, namely SONIC [3] and MUSIC [39]. Both codes provide numerical solutions to relativistic viscous hydrodynamics and include as inputs temperature-dependent pa-

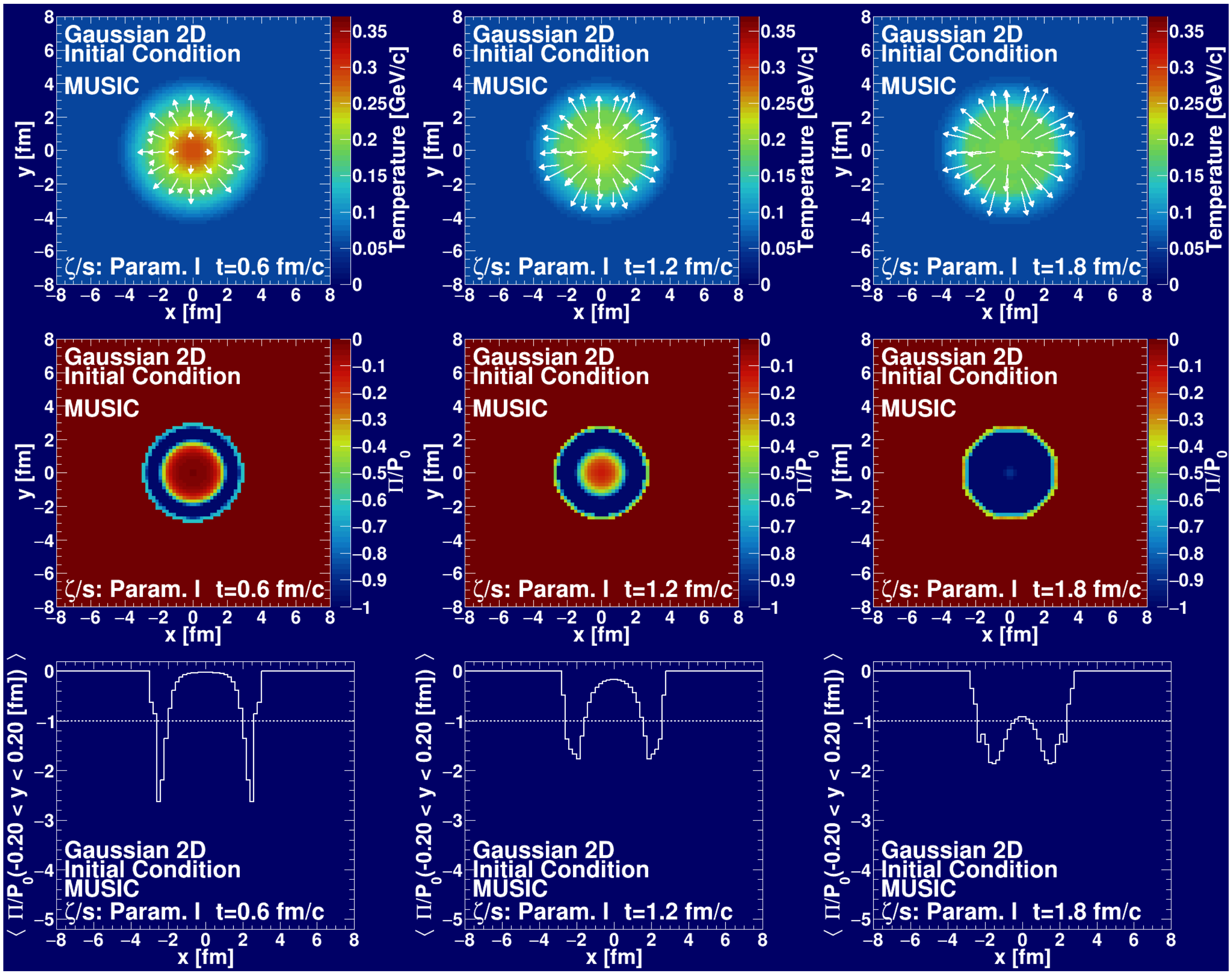

FIG. 4: MUSIC hydrodynamic simulation with single Gaussian source and $\eta/s = 1/4\pi$ and ζ/s as Param. I. Left to right is a time progression from 0.6 to 1.2 to 1.8 fm/c, while top panels are temperature profile, middle panels are Π/P_0, and bottom panels are a 1-D slice of Π/P_0 across middle region -0.2 < y < 0.2 fm/c. In middle panels, darkest blue cells indicate regions of cavitation, also indicated in bottom panels by values of Π/P_0 < -1.

rameterizations for the shear η/s and bulk ζ/s to entropy density ratios. In the case of MUSIC, we have run the code in 2+1 dimension mode, i.e. not invoking the 3+1 dimension option. Also, MUSIC has a δf correction option that can be turned on and off for shear and bulk viscosity. In contrast there is no bulk δf correction option in SONIC. In both cases, we are simply running the hydrodynamics starting at $\tau_0 = 0.2$ fm/c with the initial energy density $\epsilon(x, y)$, i.e. no pre-hydrodynamic evolution, and then with no post-hydrodynamic hadronic re-scattering.

IV. RESULTS

Utilizing the SONIC code, time snapshots of the two-dimensional temperature profile from SONIC for the simple Gaussian geometry run with $\eta/s = 1/4\pi$ (temperature independent) and $\zeta/s = 0.01$ (also temperature independent) are shown in Figure 2 (left column). The arrows represent a sampling of the fluid cell velocities. A significant radial outward flow quickly develops before the system reaches the $T = 145$ MeV user-defined freeze-out temperature. In contrast, shown in Figure 2 (right column) for the same times are SONIC results with the same simple Gaussian initial geometry with a significant bulk viscosity given by Param. I, with peak $\zeta/s \approx 0.3$. The result is a substantial reduction in the transverse expansion as much of the system simply cools via longitudinal expansion. The velocity arrows are significantly smaller and the overall radial extent of the system is correspondingly smaller at later times.

In the case of large ζ/s the mean transverse momentum $\langle p_T \rangle$ for resulting hadrons is significantly reduced, as discussed in detail later. Thus the bulk viscosity has a substantial effect on the overall space-time evolution as well as the resulting distribution of hadrons. In contrast,

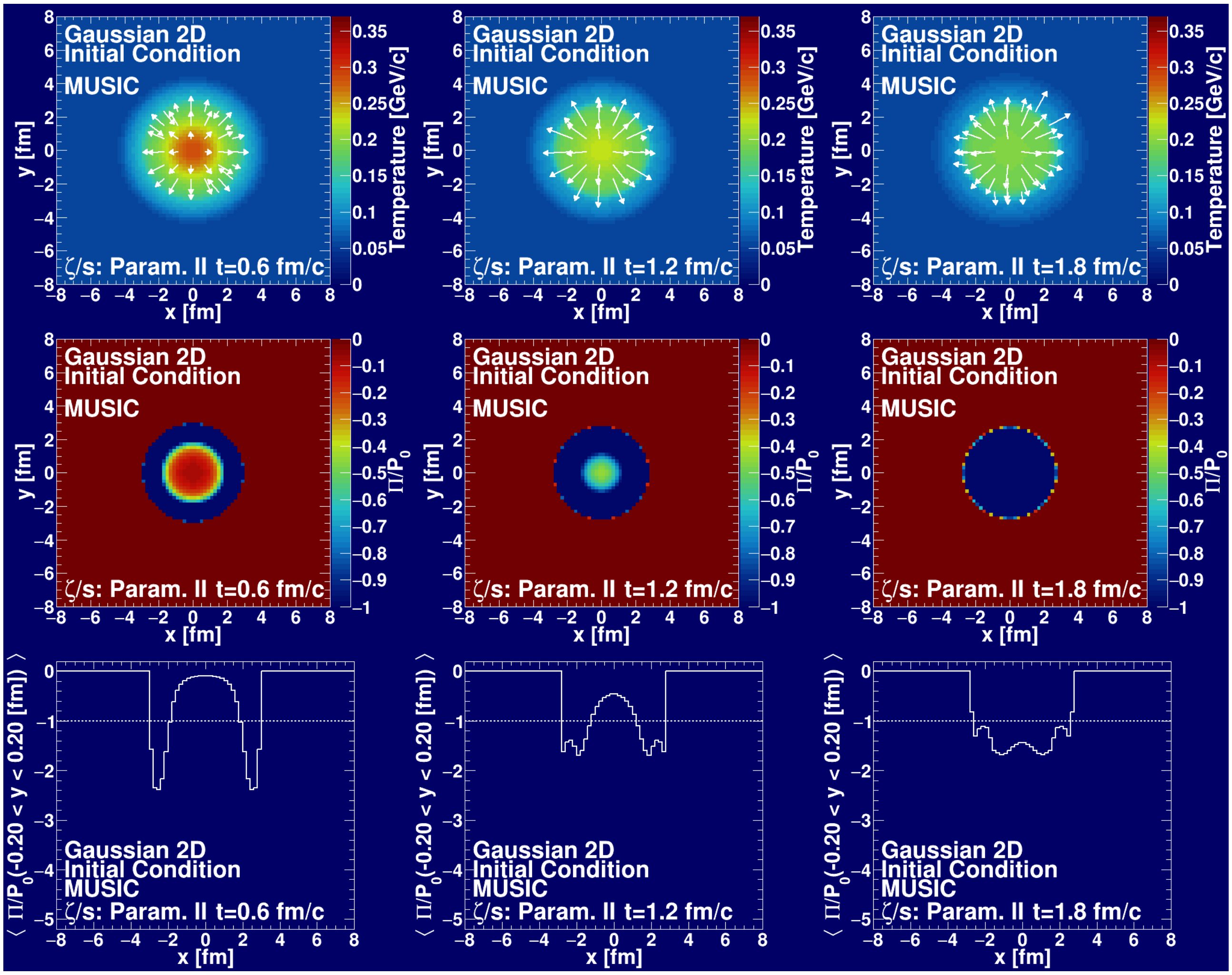

FIG. 5: MUSIC hydrodynamic simulation with single Gaussian source and $\eta/s = 1/4\pi$ and ζ/s as Param. II. Left to right is a time progression from 0.6 to 1.2 to 1.8 fm/c, while top panels are temperature profile, middle panels are Π/P_0, and bottom panels are a 1-D slice of Π/P_0 across middle region -0.2 < y < 0.2 fm/c. In middle panels, darkest blue cells indicate regions of cavitation, also indicated in bottom panels by values of Π/P_0 < -1.

SONIC studies with AdS/CFT pre-hydrodynamic evolution for p+p collision geometries, a modest $\zeta/s = 0.01$ (implemented as temperature independent) was found to slightly reduce the $\langle p_\mathrm{T} \rangle$ from initial radial flow in agreement with experimental data [33].

We have confirmed that running SONIC and MUSIC with the same initial geometry, the same ζ/s parameterization, and bulk δf correction turned off (since SONIC does not have an implementation for bulk viscosity related δf) gives good agreement for the pion and proton p_T distributions. Hence, using MUSIC we can further explore the dependencies of the different ζ/s parameterizations in conjunction with turning on and off the δf correction. Shown in Figure 3 are the pion p_T distributions at three different freeze-out temperatures ($T_f = 145, 165, 180$ MeV from left to right) for ζ/s parameterization Param. II with and without δf corrections. The change in the pion p_T with and without δf corrections is modest when $T_f = 145$ MeV since as seen in Figure 1 the value at that temperature for $\zeta/s < 0.01$. However, as soon as one changes $T_f = 160$ MeV, there is an enormous change in the p_T distribution since the bulk viscosity is already very large. Table I includes a full summary of pion and proton mean p_T values corresponding to ζ/s Param. I and II, three different freeze-out temperatures, and with and without δf corrections. The systematic uncertainty based on bulk-viscous non-equilibrium corrections via δf are very large, often of order 100% on the mean p_T, unless applied where the ζ/s parameterization yields very small values on the low temperature side (i.e. $T < 170$ MeV for Param. I and $T < 150$ MeV for Param. II).

These results highlight that if one exclusively restricts the freeze-out temperature to $T = 145$ MeV, one can quote a rather small systematic uncertainty due to the lack of complete knowledge of the δf correction, i.e. by comparing with and without its inclusion. However, that is only because the parameterizations Param. I, II, and III all have rather small values of ζ/s at that temperature, specifically $\zeta/s = 0.03, 0.0044, 0.03$, respectively. It is reasonable to simultaneously systematically vary the freeze-out temperature, as that is physically allowed, and then from Table I one observes enormous systematic variations in the $\langle p_T \rangle$ values. One cannot avoid these systematic uncertainties in a full evaluation.

Given these large values for ζ/s it is critical to assess whether the total effective pressure becomes negative and hence cavitation is unavoidable. Figures 4 and 5 show for three different time snapshots the temperature distributions along with fluid velocity vectors (upper row), the Π/P_0 distributions (middle row), and a one-dimensional slice along $y = 0$ of the Π/P_0 values (lower row). Figure 4 corresponds to the sharply peaked ζ/s Param. I and Figure 5 to the wide ζ/s Param. II. We highlight that the same negative pressures are confirmed when running SONIC with ζ/s Param. I since the results are not specific to the numerical implementations in MUSIC versus SONIC.

First focusing on the results as used in Refs. [21, 24], the vast majority of the space-time volume has $\Pi/\mathrm{P}_0 < -1$ and hence the entire volume will undergo cavitation. The issue of cavitation is not mentioned in Refs. [21, 24], though we have confirmed with the authors that for their specific geometry (e.g. $p + A$) most of the space-time volume is in fact in the large negative pressure regime. This means that the results of the hydrodynamic evolution are unreliable as they are well outside the equations domain of validity. For the Param. I case, the possible cavitation effect is slightly less, but still placing the results outside the domain of hydrodynamics.

In an earlier check on cavitation in Pb+Pb collision geometries [27], the authors find that with the most sharply peaked Param. III, cavitation dominates the space-time volume. We have confirmed these results with our simulations. However, using Param. I they find a smaller space-time volume in the cavitation regime and then conclude that "cavitation will not happen." Here caution is warranted since even if only a minority, e.g. 25% of the space-time volume has $\Pi/\mathrm{P}_0 < -1$, is in the cavitation regime, it is not possible to reliably conclude that the overall hydrodynamic results are robust.

V. DISCUSSION

There are many recent cases where hydrodynamics appears to be applied outside its "domain of validity." For example, there are often large δf corrections even for shear viscosity. However, the case of cavitation is worse because it is a known mechanical instability of the system, whereas large δf merely implies that one did not use a good "model" for the transition from hydrodynamics to particles. Cavitation says "STOP: you are out of bounds." Why does hydrodynamics not handle such regions of low pressure, i.e. cavities? The reason is because these cavities are regions of a gas. One can argue that one sometimes models the hadronic gas using hydrodynamics, and that is true, but here the error one is making is of order one. A better approach would be to treat the cells that cavitate as "frozen out" with whatever their local temperature would be. For massive cavitation for cells with $T > 250$ MeV, the corresponding spectra will look unlike anything measured. That may simply be indicating that your chosen bulk viscosity is inconsistent with experiment.

Recent developments have argued that hydrodynamics is valid over a much larger domain than previously assumed. However, the argument is that one loses the hydrodynamic attractor if the system cavitates. One can also break hydrodynamics with just shear stress, but the point is that cavitation is the reaction to a massive bulk stress, not a small perturbation. The instability occurs whenever the pressure drops below the vapor pressure. Negative pressure is much more extreme, meaning that once the pressure is negative, the system is essentially guaranteed to have cavitation.

We recall again that mother nature does not have to be kind. In the case of SONIC with AdS/CFT prehydrodynamic evolution, i.e. strong coupling, there is no very strong build up of initial radial flow. Hence, only a modest, temperature independent bulk viscosity $\zeta/s = 0.01$ is needed for matching small system $\langle p_T \rangle$ [33]. In this case, there is almost no sensitivity to inclusion of a δf correction or not, as well as little sensitivity to the transition temperature for ending hydrodynamics and starting the hadronic re-scattering.

In contrast, the IP-GLASMA initial conditions and prehydrodynamic evolution appears to require a large bulk viscosity. This raises some crisp questions or set of scenarios. (1) If such a large bulk viscosity can be ruled out as put forward in Ref. [38], then IP-GLASMA initial conditions would be ruled out. (2) If we consider large bulk viscosity as possible, then how can one arrive at reliable space-time evolution since the currently solved hydrodynamic equations do not include critical cavitation, (3) Is there a way to reconcile IP-GLASMA initial conditions without a large bulk viscosity?

In order to evaluate these scenarios a set of apples-to-apples comparisons would be most fruitful. This means having identical initial conditions and matching conditions with a code module interchange for IP-GLASMA, free streaming, and AdS/CFT evolution prior to hydrodynamics. That way the key features in the pre-hydrodynamic evolution necessitating a large bulk viscosity can be discerned. Such tests also require consistency checks between the parameterization of relaxation times and the matching conditions between pre-hydrodynamic and hydrodynamic stages. Lastly, a significant tool would be a consistent checking of how negative pressures are handled in the different hydrodynamic codes, highlighting that setting negative pressures to zero values is not a systematic check on such effects.

VI. CONCLUSIONS

In conclusion, we find utilizing the publicly available SONIC and MUSIC hydrodynamic codes that currently used parameterizations of ζ/s result in large space-time volumes with negative effective pressure. Thus, the systems will undergo cavitation, which is outside the domain of validity for hydrodynamics. All implementations of ζ/s with different geometries need to test for cavitation. Currently it is not possible to assess the full systematic uncertainty arising from ignoring these effects. Future work to isolate the key initial conditions and pre-hydrodynamic evolution that necessitates in some models large bulk viscosity are critical to further progress.

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