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Isoscaling and Nuclear Reaction Dynamics

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Isoscaling parameters $\alpha$ and $\beta$ have been explored as a function of breakup angle in binary excited projectile-like fragment decays produced in collisions of $^{70}\text{Zn} + ^{70}\text{Zn}$ and $^{64}\text{Zn} + ^{64}\text{Zn}$ at 35 MeV per nucleon. In this analysis, focus was placed on isoscaling the second heaviest fragment with $4 \leq Z_1 \leq 8$ emitted from the excited projectile-like fragment in events that contained a heavy fragment with $Z_H \geq 12$. The breakup orientation $\theta_{\text{prox}}$ was defined as the angle between the heavy and light fragments’ center of mass velocity and the fragment pairs’ relative velocity. Breakups between $0^\circ < \theta_{\text{prox}} \leq 80^\circ$ have been shown to be dominated by dynamical contributions, while break-up angles of $\theta_{\text{prox}} > 100^\circ$ are predominantly statistical. Historically, isoscaling has often been understood and applied in a statistical context, assuming that the fragments are produced after statistical equilibrium is achieved. Studying isoscaling parameters as a function of $\theta_{\text{prox}}$ reveals the sensitivity of $\alpha$ and $\beta$ to the mechanism of fragment production.

\section{I. INTRODUCTION}

The asymmetry energy (typically referred to as the symmetry energy) term in the nuclear equation of state describes how isospin content contributes to the binding energy of nuclear matter. While the value of the asymmetry energy is well constrained at saturation density, its evolution away from saturation density is largely unknown. Multifragmentation in heavy ion collisions has been investigated to probe regions of low density nuclear matter in the search for asymmetry energy constraints [1, 2]. It is necessary to find observables that are sensitive to the asymmetry energy to allow constraints through experimental comparisons to theoretical models [3].

Isoscaling is a method of comparing integrated yields of isotopes from two reaction systems that differ only in their isospin make-up [4–6]. Molecular dynamics models and experiment both yield ratios of isotopes and isotones that can be described by the exponential scaling law

$$R_{\text{N}} = Y_2(\text{N,Z})/Y_1(\text{N,Z}) = C \exp(\alpha N + \beta Z).$$

$Y_2$ is the yield from the more neutron-rich system and $Y_1$ is the yield from the less neutron-rich system while $C$, $\alpha$, and $\beta$ are fit parameters. This phenomenon has been observed for multifragmentation [6–9], deep inelastic collisions [5, 10], evaporation [11] and fission [12, 13]. Constraints to the asymmetry energy have been applied through the proportionality between these fit parameters and the asymmetry energy term in a statistical decay framework [14, 15].

Due to the statistical and thermal equilibrium assumption used to infer information about the asymmetry energy from scaling parameters, it is essential to understand the contribution of any effects that may alter the initial fragment yield before experimental detection. Pre-equilibrium emission (including dynamical decay) and secondary decay modify the fragment yield distributions from the purely standard statistically produced primary products [3]. The popular method of invoking micro or grand canonical ensembles to interpret isoscaling parameters come under question when considering fragments that originate from varying temperature and density conditions throughout the reaction. To make use of the statistical framework it is essential to differentiate between fragment production mechanisms when determining experimental isoscaling parameters.

Various molecular dynamics models have successfully predicted isoscaling to accurately describe fragment yields very early in the reaction process, where the system is out of equilibrium and fragments are produced from dynamical processes [16–18]. These simulation studies have also shown that the isoscaling parameters vary wildly with the evolution of the reaction system. It has been shown that there is a disparity between isoscaling parameters obtained from excited projectile-like fragments (PLF*) and emitted fragments [9]. The present work makes use of the orientation of PLF* break-up that has proven to be an effective way of examining the equilibrium within a deformed nuclear system as a function of time [19, 20]. Importantly, this method uses the break-up orientation to distinguish regions corresponding primarily to statistical fragment production and to dynamical fragment production [19–24]. This work seeks to investigate how well isoscaling describes dynamically produced fragments in an experimental setting, and how the isoscaling properties change depending on the mechanism of fragment production. Further, the evolution of isoscaling parameters with the time scale of fragment production may yield an observable with improved sensitivity to the asymmetry energy.

\section{II. EXPERIMENT AND DATA SELECTION}

The systems used in this study were symmetric collision of $^{70}\text{Zn} + ^{70}\text{Zn}$ and $^{64}\text{Zn} + ^{64}\text{Zn}$. In both cases, the beam was accelerated to 35 MeV/nucleon by the K500 Cyclotron at Texas A&M University and focused on a thin foil target [25–27]. The two systems have considerably different isospin content while retaining similar charge; they also display similar gross reaction charac-
teristics (collision dynamics, temperature, etc). The reaction products were measured in the Neutron Ion Multidetector for Reaction Oriented Dynamics (NIMROD) [28–31]. NIMROD provides excellent geometric angular coverage over the range 3.6° to 167° as well as typically affording charged particle isotopic identification up to at least Z = 17 and elemental identification up to the charge of the beam (Z = 30).

Events were selected that had at least two charged particles detected in NIMROD and had a total event charge detection of Z > 20, ensuring a majority of PLF* detection. Particles that are elementally identified but not isotopically identified are included in the charge detection requirement. The fragments used to calculate the break-up orientation angle are the two heaviest fragments detected in each event. These two fragments must be isotopically identified; however, there is no isotopic identification requirement on additional fragments that contribute to the charge detection requirement. The heaviest fragment (HF) is defined to be the one with the largest atomic number. The second heaviest fragment (LF) is defined to be the one with the second largest atomic number. Any ties are broken first with the mass number, and in the case of identical mass, randomly. In this study, focus is placed on isoscaling the LF with 4 ≤ Z_L ≤ 8 in events that have a corresponding HF with Z_H ≥ 12. The current analysis is performed using only yields of the LF in each event.

III. RESULTS AND DISCUSSION

Heavy ion collisions near the Fermi energy result in a dynamically deformed system where the competition between the strong force and the velocity gradient between the projectile-like and target-like fragment produces a low density neck region. This neck is neutron rich due to the action of the asymmetry energy. As the system becomes stretched and further deformed, the velocity gradient exceeds the attractive nuclear force and the neck ruptures as illustrated in Figure 1 [32]. The detached PLF* is likely to be deformed along the separation axis due to the collision dynamics. The evolution of this dynamic system leads to N-Z equilibration between the neutron-rich and neutron-poor regions of the PLF*. The subsequent breakup of the PLF* can develop into an HF and an LF in the exit channel. Angular momentum gained by the PLF* in mid-peripheral collisions can cause this breakup axis to have a nonzero angle (θ_prox) relative to the PLF*-TLF* (excited target-like fragment) separation axis and can be calculated by

\[ \theta_{\text{prox}} = \arccos \left( \frac{\vec{v}_{\text{CM}} \cdot \vec{v}_{\text{REL}}}{\| \vec{v}_{\text{CM}} \| \| \vec{v}_{\text{REL}} \|} \right) \]  

\[ \vec{v}_{\text{CM}} = \frac{m_{\text{HF}} \vec{v}_{\text{HF}} + m_{\text{LF}} \vec{v}_{\text{LF}}}{m_{\text{LF}} + m_{\text{HF}}} \]  

\[ \vec{v}_{\text{REL}} = \vec{v}_{\text{HF}} - \vec{v}_{\text{LF}} \]

\[ \theta_{\text{prox}} = \arccos \left( \frac{\vec{v}_{\text{CM}} \cdot \vec{v}_{\text{REL}}}{\| \vec{v}_{\text{CM}} \| \| \vec{v}_{\text{REL}} \|} \right) \]  

\[ \vec{v}_{\text{CM}} = \frac{m_{\text{HF}} \vec{v}_{\text{HF}} + m_{\text{LF}} \vec{v}_{\text{LF}}}{m_{\text{LF}} + m_{\text{HF}}} \]  

\[ \vec{v}_{\text{REL}} = \vec{v}_{\text{HF}} - \vec{v}_{\text{LF}} \]

Figure 1. Illustration depicting the dynamical interaction process and decay. (a) Deformation between the PLF* and TLF* (light gray) prior to the first break along the neck region. (b) At a later time, the PLF* will break a second time after rotating relative to the TLF* (measured by the angle θ_prox) forming the HF and LF in the exit channel. The blue (dark gray) region indicates neutron excess while the red (medium gray) region indicates relative neutron deficiency.

The velocity vectors used in the calculation of θ_prox (1) are the two-fragment center-of-mass velocity in the laboratory frame (2) and the relative velocity between the HF and LF (3). The time of interaction for N-Z equilibration to occur can be probed through θ_prox [19]. An in-depth study of N-Z equilibration in this framework is detailed in prior work [20]. Note that the naming convention of the breakup orientation used in this study (θ_prox) differs from prior work (α) to eliminate confusion with isoscaling parameter α.

A representative yield distribution for Z_L = 6 as a function of θ_prox is shown in Figure 2. The distribution shows significant yield at 0° < θ_prox < 80° and levels off at larger angles. The dynamical process described would favor PLF* breakup aligned with the PLF*-TLF* separa-
Figure 3. $R_{21}$ values for $4 \leq Z_L \leq 8$. Left panels (a) and (c): Exponential fits are performed on isotopes of the same Z. Right panels (b) and (d): Exponential fits are performed on isotones of the same N. Closed markers and solid fits correspond to an orientation that is predominantly dynamical while open markers and dashed fits correspond to an orientation that is predominantly statistical. Top panels (a) and (b): Each isotopic and isotonic series are fit independently. Bottom panels (c) and (d): For each orientation, the yield ratios are fit for all measured nuclides to obtain $\alpha$ and $\beta$ simultaneously. Statistical error bars are smaller than the size of the markers.

Figure 4. Isoscaling Parameters $\alpha$ and $\beta$ as a function of $\theta_{\text{prox}}$ for the total yield of $4 \leq Z_L \leq 8$. Error bars contain statistical error from the fitting of $R_{21}$ values.
with a value of 0.563 ± 0.009 and quickly decreases in magnitude until it levels off to an average value of 0.460 ± 0.008 over the statistical range of 80° < θ_{prox} < 160°. The smooth decrease in α and |β| over the dynamic range of θ_{prox} may be understood from the dynamics of the reaction mechanism. With θ_{prox} functioning as a clock for time of interaction for N-Z equilibration to occur, smaller angles of θ_{prox} correspond to fragments originating from a system significantly out of equilibrium. A small value of α indicates that the difference in the mean of the mass distribution between the two systems is small compared to the width of the mass distribution (for either system). A large value of α indicates that the difference in the mean of the mass distribution between the two systems is large compared to the width of the mass distribution. This implies that the LFs produced statistically for the neutron rich system - and neutron poor systems are more similar in their mass distributions than the LFs produced dynamically from the two systems. This implies the argument that the excess neutrons are initially attracted to the low-density neck - and more so for the neutron rich system - and that this neutron enhancement relaxes over time as the density gradient allows it to. The capability of θ_{prox} distinguishing between statistical and dynamical fragments in this class of events may benefit the extraction of isoscaling parameters as statistical assumptions are often invoked in the analysis of isoscaling [5].

The dynamical yield contribution over θ_{prox} can be described by \( Y_{\text{dyn}} = Y_{\text{tot}} - Y_{\text{stat}} \). An average isotopic composition weighted by the yield over the statistical range 100° < θ_{prox} < 180° can be subtracted through the reflection about θ_{prox} = 90°. The statistical yield from 80° to 100° is approximated to be constant and equal to the yield at 100°. This correction estimates the composition of dynamically produced fragments in order to better understand their sensitivity to the isoscaling analysis. Figure 5 shows the resulting R_{21} values with global exponential fits over the θ_{prox} range containing parameter values that differ most from statistical fragment values. Note that \(^{15}\text{O}\) is excluded due to a lack of statistics after the subtraction. Exponential fitting describes well the isolated dynamical fragment yields that result from the correction.

Figure 6 shows α and |β| values as a function of θ_{prox} for the dynamical region with the statistical contribution removed. The α and |β| values obtained from the total yield as seen in Figure 4 are included for comparison. The dynamical parameters only extend to θ_{prox} = 80° at which point the statistically meaningful dynamical range ends. The α value at θ_{prox} = 25° after statistical subtraction is 0.75 ± 0.01 compared to 0.563 ± 0.009 in the combined yields. The α and |β| values for dynamic fragments follows a similar trend as that for the total yield but with greatly increased sensitivity. The isoscaling parameters for the combined dynamical and statistical yields are significantly lower than that of the purely dynamical yield precisely because of the contribution of the statistical yield in that region. Thus far, isoscaling

![Figure 5. R_{21} values for 4 ≤ Z_L ≤ 8 after statistical subtraction correction for 20° < θ_{prox} < 30°. Global exponential fits are performed to extract α and β simultaneously shown as a function of neutron number (panel (a)) and as a function of proton number (panel (b)) similar to the bottom panels of Figure 3.](image)

![Figure 6. Isoscaling Parameters α and β as a function of θ_{prox} of 4 ≤ Z_L ≤ 8. Black circular markers contain dynamical and statistical contributions while red (gray) square markers are after statistical subtraction correction. The horizontal lines are α values obtained from isoscaling fragments that span the visual range of θ_{prox}. The color (shade) of the line corresponds to whether the fragments are from the total yield or statistical subtraction correction. Error bars contain statistical error from the fitting of R_{21} values. Note the differing y-axis range from Figure 4.](image)
parameters have been obtained by fitting yields confined to small ranges of breakup orientation to observe how they evolve with $\theta_{\text{prox}}$. The relative amounts of dynamical and statistical yields can be seen directly in Figure 2. The horizontal lines in Figure 6 are isoscaling parameter values obtained using fragment yields over larger ranges of $\theta_{\text{prox}}$ that correspond to different fragment types. The value of $\alpha$ for the dynamical and statistical orientation of the total yield (black, long dashed line) from $20^\circ < \theta_{\text{prox}} < 160^\circ$ is $0.497 \pm 0.003$. The value for the statistical yield (black, short dashed line) from $100^\circ < \theta_{\text{prox}} < 160^\circ$ is $0.458 \pm 0.005$. The value for the dynamical orientation of the total yield (black, solid line) from $20^\circ < \theta_{\text{prox}} < 80^\circ$ is $0.522 \pm 0.003$. The value for the dynamical orientation with the statistical contribution removed (red (gray), long dashed line) from $20^\circ < \theta_{\text{prox}} < 80^\circ$ is $0.591 \pm 0.004$. These differences make clear the importance of understanding isoscaling parameter dependencies of fragments produced in different regions of phase space which can have different production mechanisms, especially when applying statistical equilibrium assumptions. Moreover, by characterizing the isoscaling for fragments produced in different ways, a stronger isoscaling signature is obtained.

It is possible that this observable may allow for tighter constraints to be placed on the asymmetry energy. Within a statistical description of isoscaling, the isoscaling parameter $\alpha$ is related to the asymmetry energy through the temperature and composition of the fragmenting sources in the two reaction systems: $\alpha = 4C_{\text{asy}}[(Z_1/A_1)^2-(Z_2/A_2)^2]/T$ where $C_{\text{asy}}$ is the asymmetry energy coefficient used for calculating the asymmetry energy. In principle, information on the asymmetry energy could be constrained if temperature and density are properly evaluated. For example, in references [34, 35], the authors describe their method of evaluating $T$, $\rho$, and therefore $C_{\text{asy}}$ in the context of antisymmetricated molecular dynamics simulations. This method can be applied to the $\text{Zn} + \text{Zn}$ systems studied in the present work, which could then yield a constraint on the asymmetry energy. Of course, verifying the applicability of the method to the present data set, in which a moderately rare class of peripheral collisions is treated, is beyond the scope of this article. Clearly, care must be taken since the fragmenting source is dynamically evolving and has significant density gradients as it equilibrates. Therefore, it is unlikely that constraints extracted using this simple formula correspond to the true density dependent asymmetry energy. However, dynamical transport model calculations include reaction dynamics as emergent from the microscopic interactions. These model calculations can predict fragment yields and momentum distributions which can be treated in the same way as experimental data. By varying the asymmetry energy within the model calculation, multiple data sets can be generated and compared to the measurement to determine which asymmetry energy is closest to reality. Of course, stronger constraints can be placed when multiple observables are reproduced simultaneously and multiple models arrive at similar conclusions.

**IV. SUMMARY**

Isoscaling was performed on fragments with $4 \leq Z_L \leq 8$ that had a measured $Z_H \geq 12$ in reactions of $^{70,70}\text{Zn} + ^{64,64}\text{Zn}$ at $35$ MeV/nucleon. The isoscaling parameters $\alpha$ and $|\beta|$ were studied as a function of the breakup orientation $\theta_{\text{prox}}$. Distinct regions of dynamical and statistical production over $\theta_{\text{prox}}$ allowed for the properties of isoscaling dynamical and statistical fragments in this framework to be distinguished and studied. The results show that $\alpha$ and $|\beta|$ vary significantly with $\theta_{\text{prox}}$. Dynamical fragments have larger $\alpha$ and $|\beta|$ values, with a decreasing trend spanning the dynamical range of $\theta_{\text{prox}}$. This is consistent with the degree of equilibration within the PLF* increasing with increasing $\theta_{\text{prox}}$. Further, this reflects the argument that excess neutrons are attracted to the low-density neck formed early in the dynamic reaction mechanism. The study suggests that selections on breakup orientation for this class of events plays a key role in ensuring the experimental yield contributions are in line with the statistical theoretical assumptions used for the extraction of the asymmetry energy. The validity of the isoscaling model for dynamical fragments in itself is a non trivial result that cannot be fully explained with a statistical model. Moreover, the results agree with transport model studies in other work that have shown similar parameter characteristics explicitly as a function of reaction time [16]. The trend in isoscaling parameter values as a function of breakup orientation may be a sensitive observable for extracting asymmetry energy constraints through transport model calculations.

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