Phase slips and parity jumps in quantum oscillations of inverted InAs/GaSb quantum wells
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Quantum oscillations arise in the electronic transport in metals and semiconductors in a magnetic field and provide insights into the physical properties of the system under investigation. At low fields, Shubnikov-de Haas (SdH) oscillations reveal the densities of the charge carriers present, as well as the degeneracy (spin, valley) of the Landau levels (LLs) that result from the quantization of motion in the magnetic field. At higher fields the quantum Hall (QH) effect may develop, leading to quantization in the Hall resistivity. Understanding the electronic structure in magnetic fields becomes an essential undertaking whenever a new material system is discovered. For instance, one of the major results accompanying the first successful exfoliation of graphene was the half-integer QH effect [1, 2]. Furthermore, quantum oscillations were used to infer the nontrivial Berry’s phase of massless Dirac fermions.

InAs/GaSb double quantum wells (QWs) are a composite system characterized by the hybridization of electrons and holes. Hybridization may result in a nontrivial band structure [3–8], making this system a platform for the realization of the quantum spin Hall insulator [9–14]. Here, we report on the LL structure in an inverted InAs/GaSb double QW and critically discuss the determination of Berry’s phase from quantum oscillations.

We uncover a periodic even and odd filling of LLs leading to a checkerboard pattern in the longitudinal resistivity, reminiscent of recent observations in transition metal dichalcogenides (TMDCs) [15–17]. We go on to explain how the combination of spin-orbit interaction (SOI) and Landau quantization can lead to an unconventional occupation sequence of LLs. Additionally, we discover an anomalous shift which violates the usual $1/B$-periodic sequence of LL filling. This shift is synonymous with discrete phase slips in the quantum oscillations. A fundamental consequence of the phase slips is that the intercept $\gamma$ for $1/B_{\perp} \rightarrow 0$ in an index plot $\nu(1/B_{\perp})$, where $\nu$ is the filling factor, is not meaningful for evaluating Berry’s phase. We compare this finding to experiments on other two- and three-dimensional (2D/3D) systems.

Measurements were performed on a gated Hall bar of 10$\mu$m width and 20$\mu$m length oriented along the [011] crystallographic direction on a heterostructure consisting of an 8 nm GaSb QW and a 13.5 nm InAs QW [Fig. 1(a)]. All measurements were conducted in a dilution refrigerator at a base temperature of 135 mK using low-frequency lock-in techniques with constant current bias.

The inverted band structure of our double QW is schematically depicted in Fig. 1(b). The InAs QW thickness is sufficiently large to drive the system into the semimetallic phase. This implies enhanced hybridization between electron and hole bands, while the anisotropy of the dispersion effectively quenches the hybridization gap. The Rashba-type SOI already present in the constituent QWs is amplified by the hybridization, and leads to significant splitting of valence and conduction bands [18–22]. The longitudinal resistivity $\rho_{xx}$, see Fig. 1(c), shows no local resistance maximum at or close to the charge neutrality point (CNP), indicating the lack of a true energy gap, as expected [23]. In the shaded area in Fig. 1(b) between the CNP and the former top of the GaSb valence band both electrons (majority) and holes (minority charge carriers) are present. This region is probed in the following.

Figure 1(d) shows a map of $\rho_{xx}$ as a function of top gate voltage $V_{tg}$ and perpendicular magnetic field $B_{\perp}$. The voltage $V_{tg}$ tunes the total charge carrier density in the system. Two sets of lines fanning outwards from the CNP follow minima in $\rho_{xx}$. Remarkably, we discern an atypical yet regular pattern in the distribution of min-
FIG. 1. (a) Conduction (valence) band edge energy $E_c$ ($E_v$) as a function of growth direction $z$ for the InAs/GaSb system. (b) Schematic band structure $E(k)$ of the double QW system. The shaded area is investigated in subsequent experiments. (c) $\rho_{xx}$ at $B_x = 0$ as function of $V_{tg}$ with the position of the CNP highlighted. (d) $\rho_{xx}$ as function of $V_{tg}$ and $B_{\perp}$ with the position of the CNP highlighted.

Minima in $\rho_{xx}$ of constant filling are modulated, moving towards and away from zero resistivity in a systematic fashion depending on their position in the $(V_{tg}, B_{\perp})$ space. We focus on the region to the right of the CNP where electrons are in the majority and discuss the properties and origins of the observed checkerboard pattern.

The region of interest in Fig. 1(d) is reproduced in Fig. 2(a) for clarity. Figure 2(b) is a cut at constant total density at $V_{tg} = 3$ V, showing both longitudinal and transverse resistivities $\rho_{xx}$ and $\rho_{xy}$ versus $1/B_{\perp}$. Well-developed plateaus are described by $\rho_{xy} = h/e^2$ with integer $\nu$ and occur concomitantly with minima in $\rho_{xx}$. Because we probe a composite system with multiple charge carrier species, $\nu$ corresponds to a total filling factor, taking both electron- and hole-like LLs into account [24]. While minima typically do not reach zero, the quantization in $\rho_{xy}$ suffices for an unambiguous assignment of $\nu$. Looking at the sequence of plateaus in $\rho_{xy}$, we deduce that $\nu$ decreases in increments of two with $1/B_{\perp}$, with the exception of selected transitions indicated by the dotted lines. There, $\nu$ changes by one only. This way, the parity of $\nu$ switches between even (denoted by stars) and odd (circles) as a function of magnetic field. The positions of the minima in $\rho_{xx}$ at $V_{tg} = 3$ V are also marked by symbols in Fig. 2(a). The minima in $\rho_{xx}$ corresponding to the missing plateaus are suppressed. Using the quantization of $\rho_{xy}$, we assign a filling factor to each minimum and highlight minima of even and odd $\nu$ in Fig. 2(a) with differently colored contour lines. We observe that neighboring minima of the same parity seemingly lie on lines of negative slope, as illustrated by the dotted line. A cut at
fixed $V_{tg}$ such as in Fig. 2(b) typically intersects multiple such lines, so that minima correspond to $\nu$ being piece-wise even or odd. The pattern apparently changes upon approaching the CNP, becoming more complex. Similar even-odd behavior in TMDCs was attributed to interplay between cyclotron and Zeeman energies and to a density dependent $g$-factor [15–17]. We also observed faint signs of said behavior in weakly inverted InAs/GaSb double QWs, erroneously attributing it only to avoided crossings between LLs mediated by ordinary and spin-orbit interband coupling effects [7].

In addition to the unconventional filling sequence, there exists another peculiarity in the form of discrete phase slips occurring whenever the parity switches. Figure 2(c) depicts $\rho_{xx}(1/B_{1})$, again at $V_{tg} = 3$ V. Starting from low $1/B_{1}$, we see that the minima corresponding to $\nu = 7, 9, 11$ are equidistantly spaced in $1/B_{1}$. However, the subsequent $\nu = 12$ minimum is not located at the expected position, but instead halfway between where the $\nu = 12$ and $\nu = 13$ would lie according to the periodicity set by the $\nu = 7, 9, 11$ minima. The same phenomenon repeats itself at higher $1/B_{1}$ at the transition from even to odd $\nu$. There, the $\nu = 21$ minimum lies halfway between the nonexistent $\nu = 21$ and $\nu = 22$ minima which would follow the periodicity determined by the observed $\nu = 12–20$ minima. The period in $1/B_{1}$ remains approximately constant regardless of the shifts. We have verified that the shifts occur generically for all $(V_{tg}, B_{1})$ shown in Fig. 2(a) whenever the parity of $\nu$ changes.

We may describe the shifts in terms of discrete phase slips. To quantify the phase slips, we extract an average total density $n_{\text{QHE}}$ of charge carriers in the QH state by piecewise fitting of $\nu(1/B_{1})$ in an index plot for fixed $V_{tg}$ [25]. Then, we calculate the phase slip $\Delta \Phi_{\nu}/\pi$ using $\Delta \Phi_{\nu}/\pi = 2\hbar n_{\text{QHE}}(\Delta(1/B_{1}))/e$, where $\Delta(1/B_{1})$ is the difference in $1/B_{1}$ between the expected position of the minimum corresponding to $\nu$, $\nu e\hbar/n_{\text{QHE}}$, and the position where it actually occurs. Figure 2(d) presents the evolution of $\Delta \Phi_{\nu}/\pi$ for several $V_{tg}$. The phase is seen to jump downwards by around $\pi$ whenever the parity switches.

To understand the origin of the even-odd periodicity and the phase slips we investigate the densities of all charge carriers, as displayed in Fig. 3 [25]. The low-field SdH oscillations exhibit a single frequency $f_{2}$ for $V_{tg} > 2.5$ V which decreases upon decreasing $V_{tg}$ and therefore corresponds to electron-like states. The frequency $f_{2}$ is related to the Hall density $n_{\text{tot}}$ obtained from fitting $\rho_{xy}$ above a certain $B_{1}$-value where $\rho_{xy}$ is linear in $B_{1}$ through $n_{\text{tot}} \approx 2f_{2} \times e/\hbar$. Because $n_{\text{tot}}$ determined in this way measures the total density of free charge carriers, the factor of two implies that the population imbalance due to spin-splitting is negligible and that carriers populate a twofold degenerate electron-like band. For $V_{tg} \leq 2.5$ V, an additional frequency $f_{1}$ appears, see Fig. 3(a). The frequency $f_{1}$ increases upon decreasing $V_{tg}$, implying hole-like states. Close to $V_{tg} = 2.5$ V we have $n_{\text{tot}} \approx (2f_{2} - f_{1}) \times e/\hbar$, signifying that $f_{1}$ describes a single spin-orbit split hole subband. Upon decreasing $V_{tg}$ further a systematic deviation between $n_{\text{tot}}$ and $(2f_{2} - f_{1}) \times e/\hbar$ appears, as observed in Fig. 3(b). The frequencies $f_{1}$ and $f_{2}$ cannot account for all charge carriers, increasingly overestimating the density, $n_{\text{tot}} < (2f_{2} - f_{1}) \times e/\hbar$. This motivates us to look for the second spin-orbit split hole subband containing the missing holes that does not partake in SdH oscillations.

We therefore fit $\rho_{xx}$ and $\rho_{xy}$ simultaneously with a three-band model by inverting $\sigma_{xx} = \sum_{i} \sigma_{xx}^{i}$, and $\sigma_{xy} = \sum_{i} \sigma_{xy}^{i}$, with $\sigma_{xx}^{i}$, $\sigma_{xy}^{i}$ being the conductivities of the individual bands ($i = 1, 2, 3$). We fix two of the three densities to $p_{1} = f_{1} \times e/\hbar$ and $n = 2f_{2} \times e/\hbar$, respectively, leaving four fitting parameters, namely $p_{2}$, the missing density, and three mobilities $\mu_{i}$ [25]. The complete result for all $V_{tg}$ is shown in Fig. 3(c). Three-band fitting works for $V_{tg} \leq 1$ V. For $V_{tg} > 1$ V $p_{2}$ is too small compared to $p_{1}$ and $n$ to be determined reliably and a two-band model sufficiently describes the data, resulting in densities that agree with $p_{1}$ and $n$ as defined above for $1 < V_{tg} < 2.5$ V.
For $V_{tg} > 2.5\text{ V}$ a two-band fit allows us to determine the continuation of $p_1$ where $f_1$ disappears from the SdH oscillations.

We found that the spin-splitting of the electron-like band of density $n$ cannot be experimentally resolved. The same is true for the conventional Zeeman splitting of this band [26]. Hole-like states exist in two subbands and have different dispersions due to the SOI, and therefore Landau quantization of each results in nondegenerate levels. The hole-like subband of density $p_1$ enters the QH state, whereas the subband of density $p_2$ does not. Above the dashed line in Fig. 2(a), $p_2 < eB_\perp/h$, being insufficient to change the total filling factor by one, and we may think of these holes as forming a background density.

The dispersion of LLs close to a (anti-) crossing point between twofold degenerate electron-like and nondegenerate hole-like levels is schematically depicted by lines in Fig. 4(a) together with $\nu_e$ and $\nu_h$, the filling factors of the electron- and hole-like levels, respectively ($\nu = \nu_e - \nu_h$). Filling factors $\nu_e$ are even and change in increments of two, whereas $\nu_h$ is even or odd and changes in increments of one. Converting from $(E_\perp, B_\perp)$ to $(V_{tg}, B_\perp)$, we obtain the diagram in Fig. 4(b), recognizing the even-odd pattern. The hole mass being larger than the electron mass explains why typically several electron-like LLs are depopulated before a hole-like LL is depopulated for constant $V_{tg}$. A simple density of states model confirms this picture [25].

The electrochemical potential $\mu_{elch}$ oscillates as a function of $B_\perp$ at constant total density for fixed $V_{tg}$. However, the densities of the individual subsystems also oscillate together with $\mu_{elch}$. The phase slips occur because the density of charge carriers in the QH state, $n_{QHE}$, is not constant due to the self-consistent charge transfer between the electron- ($n$) and hole-like states ($p_1$) in the QH state and the hole-like states constituting the background charge reservoir ($p_2$). Because the charge transfer is particularly favorable whenever $\mu_{elch}$ jumps between hole LLs, that is where the phase slips occur. The phase slips reflect the fact that the points in the index plot $\nu(1/B_\perp)$ do not all lie on a single line. The index plot is only piecewise linear, as highlighted in the Supplemental Material [25], with shifts in $1/B_\perp$ wherever the parity jumps. Each horizontal shift in $1/B_\perp$ is equivalent to a phase slip of around $\pi$. While the deviation of the points from a single line is easily overlooked [Fig. 4(c), it unveils itself dramatically when looking at $\rho_{xx}$ traces [Fig. 2(c)] or at the phases of the oscillations [Fig. 2(d)].

In 2D systems in which the total density $n_{QHE}$ of charge carriers quantized in a magnetic field is constant, $\mu_{elch}$ is a function of $B_\perp$. Then, the intercept $\gamma$ for $1/B_\perp \to 0$ in an index plot must always be either 0 or 1/2 [27]. A nontrivial intercept $\gamma = 1/2$ reflects the existence of a LL at zero energy, as in graphene, whereas a trivial $\gamma = 0$ reflects the lack thereof, independent of the zero-field spectrum. Since a zero energy LL may be viewed as a consequence of a nontrivial Berry’s phase of $\pi$, $\gamma = 1/2$ was used in graphene to infer the nontrivial Berry’s phase of massless Dirac fermions [1, 2]. In 3D topological insulators (TIs), the situation is different. The electrochemical potential is pinned by localized states in the bulk (whose number scales with the volume), so the 2D density at the conducting surfaces may vary, while $\mu_{elch}$ is fixed. Then, $\gamma$ may take on nontrivial values between 0 and 1/2, reflecting Berry’s phase [27–31].

There are experimental reports in the literature on 2D systems where $n_{QHE} \cong \text{const}$ should hold, yet a nontrivial $\gamma$ was seen and linked to Berry’s phase. Examples include HgTe [32] and InAs/GaSb double QWs [21]. In the latter Ref. 21, a nontrivial intercept $0 < \gamma < 1/2$ was reported close to the CNP and interpreted as originating from a nonzero Berry’s phase. We also find a nontrivial $\gamma$ close to the CNP ($\gamma = 0.30 \pm 0.05$ at $V_{tg} = -2\text{ V}$) and a trivial $\gamma \approx 0$ away from the CNP ($\gamma = 0.18 \pm 0.17$ at $V_{tg} = 2\text{ V}$), see Fig. 4(c). However, taking a single, common intercept is not a meaningful approach since the index plot is piecewise linear. Away from the CNP, $\gamma \approx 0$ due to averaging over the piecewise linear segments in the index plot. This is exemplified in the Supplemental Material [25]. Therefore, attributing nonzero intercepts to a nontrivial Berry’s phase is not justified here. When both $n_{QHE}$ and $\mu_{elch}$ depend on $B_\perp$, utmost care is required in interpreting $\gamma$.

In summary, electron-hole hybridization and the SOI lead to an even-odd periodicity upon Landau quantization. Electron- and hole-like LLs have different degenera-
cies, leading to parity jumps. Clandestine hole-like states that do not directly appear in quantum oscillations nevertheless have a profound impact, causing abrupt phase slips in the usual 1/\(B_\perp\) -periodic sequence of LLs due to intersubband charge transfer. This phenomenon can lead to nontrivial intercepts obtained from index plots which are not associated with a nontrivial Berry’s phase. Our findings are not specific to InAs/GaSb double QWs, but also apply to other 2D and quasi-2D systems which show appreciable SOI or are composite (multi-band) in nature, so that they may not fully quantize in a magnetic field, such as HgTe QWs [32, 33], TMDCs [34, 35], layered pnictides [36], few-layer black phosphorous [37] and 3D TI thin films and nanoribbons [38–40].

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[23] A resistance maximum gradually evolves at the CNP upon applying a perpendicular magnetic field \(B_\perp\).
[25] For more details refer to the Supplemental Material at [URL will be inserted by publisher].
[26] For pure InAs, the ratio of Zeeman and cyclotron energies is \(\sim 0.26\) for \(g = 15\) and \(m^* = 0.036 \times m_0\). Due to the presence of hole-like Landau levels, the spectrum is denser than expected for pure InAs electron systems.
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