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Scaling Theory of Quantum Ratchet

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The asymmetric responses of the system between the external force of right and left directions are called "nonreciprocal". There are many examples of nonreciprocal responses such as the rectification by p-n junction. However, the quantum mechanical wave does not distinguish between the right and left directions as long as the time-reversal symmetry is intact, and it is a highly nontrivial issue how the nonreciprocal nature originates in quantum systems. Here we demonstrate by the quantum ratchet model, i.e., a quantum particle in an asymmetric periodic potential, that the dissipation characterized by a dimensionless coupling constant α plays an essential role for nonlinear nonreciprocal response. The temperature (T) dependence of the second order nonlinear mobility μ_2 is found to be $\mu_2 \sim T^{6/\alpha-4}$ for $\alpha < 1$, and $\mu_2 \sim T^{2(\alpha-1)}$ for $\alpha > 1$, respectively, where $\alpha_c = 1$ is the critical point of the localization-delocalization transition, i.e., Schmid transition. On the other hand, μ_2 shows the behavior $\mu_2 \sim T^{-11/4}$ in the high temperature limit. Therefore, μ_2 shows the nonmonotonous temperature dependence corresponding to the classical-quantum crossover. The generic scaling form of the velocity v as a function of the external field F and temperature T is also discussed. These findings are relevant to the heavy atoms in metals, resistive superconductors with vortices and Josephson junction system, and will pave a way to control the nonreciprocal responses.

Chirality is one of the most basic subjects in whole sciences including physics, chemistry, and biology [1, 2]. Most of the focus is on the symmetry of the static structures of molecules and organs etc. However, once the motion or flow of particles is considered, the distinction between right and left directions of the quantum dynamics is a highly nontrivial issue even when the system lacks the inversion and mirror symmetries, i.e., chiral. Classical dynamics of particle under asymmetric potential has been a deeply studied topic in wide fields of science since Feynman conceived the idea of Brownian ratchet [3]. Researches range from molecular motor [4, 5], colloid dynamics [6], optically trapped molecule [7] to drop of mercury [8].

Quantum effects on the particle dynamics under the nonreciprocal periodic potential $V(x)$ is one of the most fundamental problems in condensed matter physics. Without the dissipation, the engenstates of this problem is given by the Bloch wavefunctions characterized by the crystal momentum k and the engenenergy $\varepsilon_n(k)$ with n being the band index. Neglecting the spin degrees of freedom, $\varepsilon_n(k)$ is symmetric between k and $-k$, i.e., $\varepsilon_n(k) = \varepsilon_n(-k)$ as far as $V(x)$ is real, i.e., Hermitian. Therefore, the transport phenomena are symmetric between right and left directions as long as the many-body interaction is neglected [9]. This is in sharp contrast to the daily experience, which is governed by classical mechanics, that it is more difficult to climb up the steeper slope compared with the gentle one. Especially, the role of friction is important; even at the classical dynamics, the time-reversal symmetry and energy conservation law prohibit the difference between the motions to the right and left directions. Therefore, an important question is how the dissipation brings about the nonreciprocal transport of a quantum particle.

Dynamics of a quantum Brownian particle in the periodic potential with dissipation has been the subject of intensive studies for a long term [10]. The formulation of the quantum dissipation in terms of the coupling to harmonic bath by Caldeira-Leggett gives a way to handle this problem in the path integral formalism [11, 12], and the real-time formalism to calculate the influence integral is often used to calculated the mobility [13]. Using these methods combined with the renormalization group analysis, the quantum phase transition is discovered at the critical value of the dimensionless friction α, which separates the extended ground state at $\alpha < \alpha_c = 1$ and the localized one at $\alpha > \alpha_c = 1$ [14–21]. As for the linear mobility μ_1 is concerned, it approaches to a finite value $\mu_1 \propto 1/\alpha$ when $\alpha < 1$, while μ_1 vanishes as $\mu_1 \sim T^{2(\alpha-1)}$ when $\alpha > 1$ in the limit $T \to 0$. This transition can be regarded as that from quantum to classical dynamics as the friction α increases. Therefore, it is interesting to see how this transition affects the nonreciprocal dynamics of the quantum particle in the asymmetric potential.

Experimentally, the quantum ratchet effects in semiconductor heterostructure with artificial asymmetric gating [22], Josephson junction array [23], and φ Josephson junction [24] are reported.

Recently, the vortex flow resistance in a noncentrosymmetric superconductor is shown to express a large directional dichroism at the low temperature [25]. The classical dissipative dynamics of a point particle in the presence the asymmetric pinning potential is investigated as a candidate model [26], however the low temperature behavior is not addressed where the quantum tunneling plays a vital role.

In this paper, we study the quantum dynamics of the particle in an asymmetric periodic potential with Ohmic dissipation. The form of the potential is for exam-

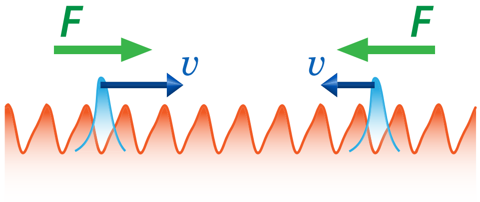

FIG. 1. **Schematic picture of the present system.** The particle wave packet under the ratchet potential is driven by the external force F resulting in a nonreciprocal velocity; $|v(-F)| \neq |v(F)|$.

ple taken as $V(x) = V_1 \cos\left(2\pi \frac{x}{a}\right) + V_2 \sin\left(4\pi \frac{x}{a}\right)$, which breaks the inversion symmetry $x \to -x$. This model describes the quantum ratchet, and several earlier works addressed this problem [27–32, 34, 35]. The instanton approach in the strong coupling limit has been employed in [28–30], where the non-monotonous temperature dependence of the nonlinear mobility μ_2 has been obtained due to the crossover from temperature assisted transition to quantum tunneling. Here, the coherence between the tunneling events has been neglected, which eventually becomes important in the low temperature limit. Scheidl-Vinokur [32] and Peguiron-Grifoni [34, 35] employed the weak coupling perturbation theory with respect to the potential $V(x)$ and obtained the lowest order expression for the second order mobility $\mu_2 \propto V_1^2 V_2$, and the rectified velocity $v(F) + v(-F) \propto V_1^2 V_2$, respectively, in terms of the integral over the two time variables t_1 and t_2. However they have not carefully examined the detailed temperature dependence especially at low temperature.

Here, we rederive the general expression of the steady state velocity as a function of external force F in the presence of the dissipation and the general form of asymmetric corrugation $V(x)$ in a perturbative way. This perturbation theory is justified for $\alpha < 1$, where the potential is irrelevant. We will discuss the other case $\alpha > 1$ later. The general formula for steady velocity $v(F)$ enable us to investigate the detailed temperature scaling for arbitrary order mobility μ_n. The dissipation is handled in terms of the Feynman-Vernon's influence functional technique [13] where the infinite set of harmonic oscillators with Ohmic spectral density $J(\omega) = \eta\omega$ are coupled bilinearly to the quantum mechanical point particle and integrated out. The lowest order perturbative expansion with respect to $V(x)$ allows us to compute the velocity and the mobility in the long time limit in the real time expression for the general strength of the dissipation, temperature T, and the external force F. Since the derivation is tedious and just a straightforward generalization of earlier works [17, 32–35], the detail is given in the Supplemental Material (SM) [36] and we here show only the final expression. Another approach to derive the same expression is also given in SM [36]. Throughout this paper, we set $\hbar = k_B = 1$.

The zeroth order in V gives $v^{(0)} = F/\eta$ and the first order correction is zero. In the order of V^2, the modification to velocity is [17, 32–35]

$$v^{(2)} = -\frac{2}{\eta}\int_0^\infty \mathrm{d}t \sum_k k\,|V_k|^2 \sin\left[\frac{F}{\eta}kt\right] \times \sin\left[\frac{1}{\pi\eta}k^2 Q_1(t)\right] \exp\left[-\frac{1}{\pi\eta}k^2 Q_2(t)\right]. \quad (1)$$

Here V_k is the Fourier component of the periodic potential $V(x)$ with k being the integer multiple of $2\pi/a$. Q_1 and Q_2 are [12]

$$Q_1(t) = \int_0^\infty \mathrm{d}\omega \frac{J(\omega)}{\eta\omega^2} \sin(\omega t) f(\omega/\gamma) \quad (2)$$

$$Q_2(t) = \int_0^\infty \mathrm{d}\omega \frac{J(\omega)}{\eta\omega^2} (1 - \cos(\omega t)) \coth\left(\frac{\omega}{2T}\right) f(\omega/\gamma). \quad (3)$$

γ, being η divided by the particle mass M, is the characteristic frequency scale in the present system. f is appropriate soft cutoff function. Here we take $f(\omega/\gamma) = e^{-\omega/\gamma}$. This result is the same as Peguiron-Grifoni's one [34, 35] and reduces to the Scheidl-Vinokur's result [32] in the small F limit and to Fisher-Zwerger's result [17] if we take only $k = \pm\frac{2\pi}{a}$. Note here that as the effect of the asymmetry of the potential $V(x)$ is missing in this formula, this result in nothing to do with the ratchet effect therefore $v^{(2)}$ is the odd function of F. To clarify the low temperature behavior of $v^{(2)}$, the asymptotic forms of Q_1 and Q_2 for $t, T^{-1} \gg \gamma^{-1}$ are important;

$$Q_1(t) = \tan^{-1}(\gamma t) \to const. \quad (4)$$

$$Q_2(t) = \log\left(\left[1 + (\gamma t)^2\right]^{1/2} \left|\frac{\Gamma(1 + \frac{T}{\gamma})}{\Gamma(1 + \frac{T}{\gamma} + iTt)}\right|^2\right) \to \log(\gamma t) + \log\left(\frac{\sinh(\pi T t)}{\pi T t}\right) \quad (5)$$

with $\Gamma(\cdot)$ being the Gamma function. From these asymptotic behaviors, when expanded in F, the n-th order term of $v^{(2)}$ scales in the leading order as

$$v^{(2)} \sim T^{\frac{2}{\alpha} - 1 - n} F^n \quad (6)$$

in the order of F^n with n being odd integers. Here widely used dimensionless dissipation strength is

$$\alpha = \frac{\eta a^2}{2\pi}. \quad (7)$$

In the third order of V's, where the quantum ratchet effect appears, we similarly have [17, 32–35]

$$v^{(3)} = \frac{4}{\eta}\int_0^\infty \mathrm{d}t_1 \int_0^\infty \mathrm{d}t_2 \sum_{\substack{k_1,k_2,k_3 \\ k_1+k_2+k_3=0}} k_1$$
$$\times \left(\mathrm{Re}\left[V_{k_1}V_{k_2}V_{k_3}\right] \sin\left[\frac{F}{\eta}(k_1t_1 - k_3t_2)\right] + \mathrm{Im}\left[V_{k_1}V_{k_2}V_{k_3}\right]\left(\cos\left[\frac{F}{\eta}(k_1t_1-k_3t_2)\right]-1\right)\right)$$
$$\times \exp\left[\frac{1}{\pi\eta}\left(k_1k_2Q_2(t_1)+k_2k_3Q_2(t_2)+k_3k_1Q_2(t_1+t_2)\right)\right]\sin\left[\frac{1}{\pi\eta}k_1k_2Q_1(t_1)\right]\sin\left[\frac{1}{\pi\eta}\left(k_2k_3Q_1(t_2)+k_3k_1Q_1(t_1+t_2)\right)\right]. \tag{8}$$

This result reduces to the Scheidl-Vinokur's result [32] in the order of F^2 and reproduces the Peguiron-Grifoni's result for the rectified velocity $v(F)+v(-F)$ in the presence of up to the second harmonic potential; $k = \pm\frac{2\pi}{a}, \pm\frac{4\pi}{a}$ [34, 35]. Although the expression is rather complex, we can see the behavior in the low temperature limit by the power-counting of the integrand using the asymptotic forms as follows. We see from Eq.(5) that the exponential of $-Q_2(t)$ function gives us a power of t and the large t cutoff of the form $\exp[-\pi T t]$ at finite temperature. Thus we are allowed to count the power at zero temperature and cutoff the integral domain $[0, T^{-1}]$ to see the T dependence at low temperature.

The dominant contribution to the integral originates from $(k_1, k_2, k_3) = \pm\frac{2\pi}{a}(1, 1, -2)$ and its permutations. By means of the polar coordinate (r, θ), the integral is $F^n \int r\mathrm{d}r\ r^{n-\frac{6}{\alpha}} \sim T^{\frac{6}{\alpha}-2-n}F^n$. On the other hand, if we fix one of the variables, say t_1, the integral behaves as $F^n \int \mathrm{d}t_2\ t_2^{n-\frac{2}{\alpha}} \sim T^{\frac{2}{\alpha}-1-n}F^n$. Although the latter contribution seems to dominate the former one at low temperature for $\alpha < 4$, the closer inspection shows that the summation over k_1, k_2, k_3 causes an exact cancellation of these leading order contributions. The proof of this cancellation is given in SM [36] and numerical calculations support this cancellation up to 12 digits in double precision calculations. Thus, the low temperature exponent is governed by the sub leading contributions;

$$v^{(3)} \sim T^{\frac{6}{\alpha}-2-n}F^n \tag{9}$$

in the order of F^n with n being a positive integer.

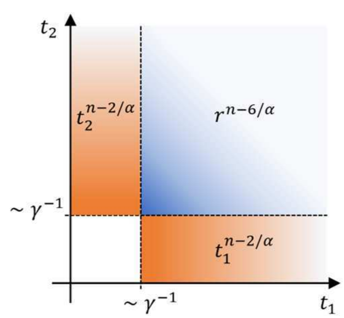

FIG. 2. **Asymptotic behavior of the integrand.** Asymptotic behavior of the integrand of n-th order expansion with respect to F of Eq.(8) in each region in the t_1-t_2 plane. As the leading order contributions from the orange regions cancel out among the terms, the blue region determines the temperature behaviors.

The numerical evaluation of second order mobility $\mu_2^{(3)}$ which is given by the expansion of Eq.(8) with respect to F depicted in Figs.3(a) and (b) clearly show temperature dependence as described by Eq.(9) at low temperature. For $0 < \alpha < 3/2$, μ_2 turn to decrease as decreasing temperature around $T = T^* \sim \gamma$. This is a peculiar behavior of the present system which can be captured in real experiments. For $\alpha > 1$, the potential is a relevant operator, and therefore the pertubative expansion with respect to the potential diverges towards the low temperature. In this case, the system is in the localized phase, and therefore $\mu_2^{(3)}$ must vanish at the zero temperature. This indicates the existence of another crossover temperature T^{**}, which can be lower than T^* when the potential is weak enough. In the view point of renormalization group (RG) analysis, the potential V scales as $V(\Lambda) = V(\Lambda_0)(\Lambda/\Lambda_0)^{1/\alpha-1}$ for the high energy cutoff Λ [17]. The cutoff is truncated at $\Lambda \sim T$ at finite temperature therefore we can estimate the crossover temperature as $V(\Lambda_0)(T^{**}/\Lambda_0)^{1/\alpha-1} \sim T^{**}$.

The higher crossover temperature deduced from the peaks of Fig.3(a) is shown in Fig.3(c) together with that for the linear mobility $\mu_1^{(2)}$ evaluated from Eq.(1). The crossover temperature for $\mu_2^{(3)}$ is always larger than that for $\mu_1^{(2)}$ but is comparable. Thus we can conclude that the crossover observed in $\mu_2^{(3)}$ is the quantum to classical crossover as known for $\mu_1^{(2)}$. Note that the peaks in $\mu_2^{(3)}$ for small α is not clear due to many sign changes in the crossover region.

This low temperature dependence is in contrast to the saturating behavior discussed in ref.[32] where a nontrivial approximation is made in the evaluation of Q_2, which fails to capture the quantitative behavior of $\mu_2^{(3)}$. For the higher temperature, $\mu_2^{(3)}$ decreases equally irrespective of α as $\mu_2^{(3)} \sim T^{-11/4}$ whose derivation is given in SM [36]. This value is slightly different from $T^{-17/6}$ obtained in

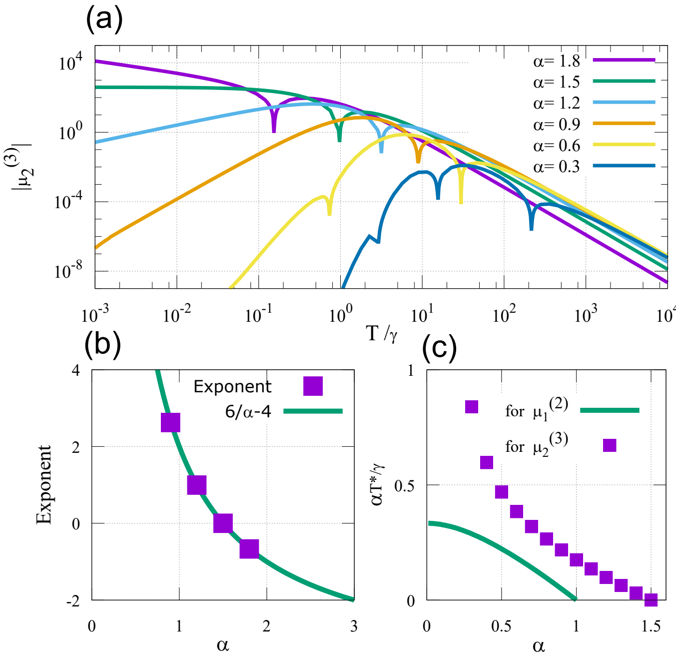

FIG. 3. Temperature dependence of the second order mobility $\mu_2^{(3)}$. (a) The second order mobility $\mu_2^{(3)}$ is evaluated from Eq.(8) for the asymmetric potential $V(x) = V_1\cos(2\pi x/a) + V_2\sin(4\pi x/a)$ with $V_2 = V_1/4$ for each value of α. There are two power law regions with different exponents; $6/\alpha-4$ for low temperature region and $-11/4$ for high temperature region. The quantum-to-classical crossover region with sigh changes (cusps) in between is also seen. For $\alpha > 1$, as the pertubative expansion with respect to the potential fails and the system goes to the localized phase, $\mu_2^{(3)}$ vanishes at zero temperature. Therefore there must be another crossover point T^{**} at low temperature where the perturbative treatment breaks down. (b) The low temperature power low exponent of $\mu_2^{(3)}$ which clearly follows the asymptotic form $\mu_2^{(3)} \propto T^{6/\alpha-4}$. (c) The crossover temperature T^* defined by peak positions in (a). The green line is that for $\mu_1^{(2)}$ evaluated from Eq.(1).

ref.[32]. This discrepancy is due to the difference of the choice of cutoff function $f(\omega/\gamma)$ as discussed in SM. In the intermediate temperature, the crossover-like behavior and some sign changes are observed as pointed out by ref.[32].

For $\alpha < 1$, the perturbative treatment of the potential V's is appropriate. And the leading order terms leads to the scaling form in the low temperature limit as

$$v = \frac{F}{\eta} - F^{2/\alpha-1} f_o^<(F/T) - F^{6/\alpha-2} f_e^<(F/T)$$
$$= \frac{F}{\eta} - T^{2/\alpha-1} g_o^<(F/T) - T^{6/\alpha-2} g_e^<(F/T) \qquad (10)$$

where $f_o^<, g_o^<$ are odd functions while $f_e^<, g_e^<$ are even. The basis of this scaling is that the velocity vanished in the limit $F \to 0$, which is given by the integral region of large time variable $t \gtrsim 1/F$. Note that only the asymptotic behavior of the integrand at large time variable determines the scaling behavior for the velocity v itself, while the expression for the coefficient of F_n for the velocity v does not appear so. Therefore, the divergence of the nonlinear mobility as $T \to 0$ does not mean the divergence of v, but the functional form becomes non-analytic at the zero temperature $T = 0$. In Eq.(10), the functions $g_o^<, g_e^<$ are an analytic functions of their argument F/T since the perturbative expansion is always possible when $F \ll T$, while $f_o^<, f_e^<$ are not. Trivially, they are related by $f_i^<(\eta) = \eta^{1-2/\alpha} g_i^<(\eta)$ with $i \in \{e, o\}$. The role of the nonreciprocal potential, i.e., V_2, is to introduce the even component $g_e^<$. One can easily see that the second order nonlinear mobility μ_2 scales as $\mu_2 \sim T^{6/\alpha-4}$. Furthermore, the generic odd (even) nonlinear mobility of n-th order scales as $\mu_n \sim T^{2/\alpha-n-1}$ ($\mu_n \sim T^{6/\alpha-2-n}$) for $\alpha < 1$, and it diverges when $2/(n+1) < \alpha < 1$ ($6/(n+2) < \alpha < 1$) while it vanished otherwise in the limit $T \to 0$. Note here that the I-V relation of the Tomonaga-Luttinger liquid (TLL) system under weak asymmetric potential $I \sim V^{6g-2}$ with g being the Tomonaga-Luttinger's interaction parameter, is shown in ref. [37] which is analogous to the $f_e^<$ term in eq.(10). There are many well-known similarities between the present system and the TLL system [19, 20] and some of them are exemplified in SM [36].

From the viewpoint of the RG, V_1 is irrelevant for $\alpha < 1$ while becomes relevant for $\alpha > 1$. Similarly V_2 is irrelevant for $\alpha < 4$, and becomes relevant for $\alpha > 4$. Naively, this might lead to the critical α being 4 for the nonreciprocal mobility. However, the RG procedure generates the composite operator V_1V_2, which includes $\sin\left(2\pi\frac{x}{a}\right)$, which has the same scaling dimension as V_1. This fact is reflected in each term of the double time integral where the dominant contribution comes from the region where one of t_1 and t_2 is finite, and the asymptotic behavior is basically given by the one-dimensional integral over time. However, the combination of $\cos(2\pi x/a)$ and $\sin(2\pi x/a)$ simply shifts the potential leaving the inversion symmetry intact. This is the reason why the cancellation occurs for the leading order terms $\propto T^{2/\alpha-1-n}$ in $v^{(3)}$

Now we turn to the case of $\alpha > 1$, where V's are relevant and scale to larger values [14]. In this case, the tunneling t between the potential barrier is the irrelevant operator, and the perturbation theory in t should be employed [19, 20]. The question is how the asymmetry of the potential enters the problem. For this purpose, let us consider the tilting of the potential under the external field F. Due to the asymmetry of the potential, the change in the potential barrier linear in F exists, which results in the F-dependence of t, i.e., $t(F) = t + \gamma F$. This $t(F)$ is used for the calculation of v in the lowest perturbation, which results in

$$\begin{aligned} v &= t(F)^2 F^{2\alpha-1} f_o^{>}(F/T) \\ &= t(F)^2 T^{2\alpha-1} g_o^{>}(F/T), \end{aligned} \tag{11}$$

where $g_o^{>}(F/T)$ is the odd function of its argument, i.e., it contains only the odd order term in the Tailor expansion. Therefore, the second order nonlinear mobility μ_2 scales with $T^{2(\alpha-1)}$ similarly to the linear mobility μ_1, and goes to 0 as $T \to 0$.

For the check of the scaling form Eq.(11) also in the strong coupling regime where potential terms are relevant operators, we calculated a temperature dependence of the linear and the third order mobility in the perturbation in t. As shown in detail in SM [36], by the perturbation with respect to the tunneling amplitude, they precisely follow the expected power law as Eq.(11).

Lastly, we comment on the array of resistively shunted josephson juntion model, which is a direct generalization of the present system to higher dimensions. This model, composed of the superconducting islands connected by Josephson couplings with symmetric cosine potential and the shunting Ohmic dissipation, is a promising candidate to explain the low temperature behavior of the thin film of granular superconductors [38, 39]. It is shown that the model shows a quantum phase transition between coherent (superconducting) and disordered (normal) states at $\alpha = h/(4e^2R) = 1/z_0$ where R is the shunting resistance and z_0 is the half of the coordination number of the lattice of islands [38]. If we introduce a asymmetric potential to the Josephson phase, the nonlinear transport coefficients of the system should follow the present scaling form. One difference is that the current in the Josephson array acts as a tilting to the potential while the resulting time derivative of the Josephson phase is the voltage drop, therefore nonlinear resistivity, instead of mobility, follows the scaling given in the present paper. Another difference is the absence of the voltage drop for $z_0\alpha > 1$ due to the superconductivity. Thus we can conclude that n-th order resistivity with odd (even) n scales as $R_n \sim T^{2/(z_0\alpha)-n-1}$ ($R_n \sim T^{6/(z_0\alpha)-n-2}$) and diverges when $2/(n+1) < z_0\alpha < 1$ ($6/(n+2) < z_0\alpha < 1$) at zero temperature.

In summary, we have studied the role of dissipation in the nonreciprocal transport of quantum particle in the asymmetric periodic potential, i.e., quantum Ratchet model. We have derived the general expression of the steady state velocity v for the general value of the dissipation α, force F, temperature T, and shape of the periodic potential $V(x)$ and found different scalings behavior at low temperature depending on the even and odd powers of F. This results can be applied to various situations such as asymmetric Josephson junction array, motion of heavy atoms in noncentrosymmetric crystal, and vortex motion in noncentrosymmetric superconductors.

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