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Symmetry Obstruction to Fermi Liquid Behavior in the Unitary Limit

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We show that a Fermi gas, in three dimensions, at temperatures above the superconducting phase transition (T_c) but below the Fermi temperature (T_F), can not be described by Fermi Liquid Theory (FLT) in the unitary limit where the scattering length diverges. The result follows by showing that the there are no effective field theory descriptions that both behave like a Fermi liquid and properly non-linearly realize the spontaneously broken boost and conformal invariance of the system. We have also derived an exact result for the beta function for the coupling function in the unitary limit.

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INTRODUCTION

Landau's Fermi liquid theory (FLT) is a theory of interacting fermions which describes the normal state of metals [1–3]. In modern parlance the ubiquity of Fermi liquid behavior is a consequence of the fact that under a certain set of generic assumptions the long distance or low energy behavior is governed by a universal fixed point. That is, Fermi liquids fall into a universality class. The starting assumption of the theory is that the interacting system can be reached by beginning with a free theory and adiabatically turning on the interactions such that the free fermion states evolve into interacting quasi-particles with the same charge and spin as an electron but not necessarily the same mass. Landau showed that, under these assumptions, the width (Γ) of these quasi-particles is suppressed due to Pauli blocking of the final states such that $\Gamma(E) \sim (E-\mu)^2$, where μ is the chemical potential. This *a posteriori* justifies the notion of a quasi-particle. The signature of FLT behavior in the normal phase of metals has generic features such as a resistivity scaling as T^2, the existence of zero sound and long lived gapless excitations.

If the theory is weakly coupled at energies well above E_F, there is good reason to believe that Fermi liquid behavior will arise at long distances, whereas for strongly interacting theories it need not be the case, and non-Fermi liquid behavior, as in high T_c compounds, arises such that the gap, which appears below T_c, leaves vestiges, so-called "pseudo-gaps" (for a discussion see [4]), as the temperature is raised above T_c. This has raised the question as to whether or not pseudo-gaps are generic features of strongly coupled systems of fermions.

A prime laboratory for such "strange metal" behavior is systems of cold atoms, which when sufficiently dilute, mimics fermionic many body systems. The utility of these systems is that the scattering length (a) can be tuned. When $a > 0$ the systems behaves like a Fermi liquid with a BCS like phase transition involving the condensation of weakly bound fermions (Cooper pairs). As the coupling is increased, a grows and eventually changes sign at which point the system behaves more like a Bose condensate (BEC) with pairs of atoms forming bosonic molecules and condensing. The in-between point where $1/a \to 0$ is called the "unitarity limit" ([11]).

There is no consensus in the literature as to whether such systems at unitarity behave like Fermi liquids. Ref.([5], [6]) found Fermi liquid behavior above T_c while ([7],[8],[12]) did not. It is important to take into account the fact these experiments were measuring different observables and the conclusions may assume that pseudo-gap behavior can not mimic Fermi liquid behavior for some sub-set [10] of these observables.

It is a theoretical challenge to understand such strongly coupled systems analytically as there is no small expansion parameter. Nonetheless, a tremendous amount can be learned about strongly coupled systems when symmetry considerations are taken into account. Given that the unitarity limit is a point of enhanced (conformal) symmetry one might hope for an increase in predictive power in this regime. In this letter we will show that this is indeed the case and that it can be shown from first principles that in three dimensions Fermi gases in the unitary limit can not be described by FLT when $T_F > T > T_c$, where T_c is the critical temperature for the superfluid phase transition and T_F is the Fermi temperature.

The starting point of our analysis is the effective field theory (EFT) of Fermi liquids developed in Ref. [13–15] which is based on an expansion around the Fermi surface and becomes exact in the infra red limit where $E/E_F \to 0$. This is the unique EFT that describes the universality class that is FLT. The founding assumptions is the existence of long lived quasi-particles with the quantum numbers of the electron. Once the theory has been defined, one shows that it does predict a quasi-particle width, $\Gamma(E) \sim (E-\mu)^2$, a self-consistency check and a defining characteristic of FLT. The crux of our argument is that at unitary this consistency check fails thus eliminating the possibility of standard FLT behavior in this limit.

Using power counting arguments, it can be shown that the only relevant interactions near the Fermi surface are, the forward scattering (FS) of quasi-particles or the superconducting (BCS) back to back interaction between quasi-particles[13–15]. The same conclusion can also be reached by using only the RG invariance and the Galilean boost symmetry [18]. As such, the Fermi liquid action can be written as

$$S = \int dt d^d p \; \psi^\dagger_{\vec{p}}(t) \left(i\partial_t - \varepsilon(\vec{p}) + \mu \right) \psi_{\vec{p}}(t) - \frac{1}{2} \prod_{i=1}^{4} \int dt d^d p_i \; \delta^d \left(\sum_{i=1}^{4} \vec{p}_i \right) g(\vec{p}_1, \vec{p}_2, \vec{p}_3, \vec{p}_4) \psi^\dagger_{\vec{p}_1}(t) \psi^\dagger_{\vec{p}_2}(t) \psi_{\vec{p}_3}(t) \psi_{\vec{p}_4}(t) \tag{1}$$

where the coupling function g is restricted to forward scattering (g_{FS}) or a BCS back-to-back (g_{BCS}) kinematic configurations. The energy functional $\varepsilon(p)$ leads to a generalized dispersion relation $E = \epsilon(p)$ which when expanded around a point ($\vec{p}_F(\theta)$) on the Fermi surface, $\vec{p} = \vec{p}_F(\theta) + \delta\vec{k}$, gives $\epsilon(p_F(\theta)) \approx \vec{v}_F(\theta) \cdot \delta\vec{k}$, with v_F being the fermi velocity. The BCS coupling (in some channel) grows in the IR and leads to condensation of Cooper pairs, whereas the forward scattering interaction is RG invariant by power counting arguments (see e.g. [14]).

A crucial part of our analysis relies on insisting that the low energy theory properly realizes the space-time and internal symmetries of the short distance physics which are Galilean invariance and particle number conservation, the latter of which, along with the translational sub-group [37], are explicitly realized in Eq. (1). Rotational invariance implies that the Fermi velocity is a constant and the coupling function $g(p_i)$, is only a function of the relative angle between the 3-momentum vectors. The only spontaneously broken symmetry[38] is the Galilean boost invariance which is not manifest in Eq. (1).

THREE PATHS TO SYMMETRY REALIZATION

Naively the spontaneous breaking of the Galilean boost symmetry should, by Goldstone's theorem [19, 20], lead to the existence of a massless scalar boson called the *framon* [21]. However, as is well known, when space-time symmetries are broken there need not be a one-to-one map between broken generators and Goldstone bosons. This is usually explained as being due to an "inverse Higgs constraint" (IHC)[22–24]. These constraints arise as a consequence of the fact that it is often possible that only one Goldstone boson is needed to assure invariance under multiple symmetry transformations [26]. The conditions for the existence of an IHC are [22–24]

$$[P_\nu, X'] \;\supseteq\; X, \tag{2}$$

where (P_ν) is an unbroken translation (which may include internal translation (see for instance [21]) and X' and X are broken generators. When this condition is met one can eliminate the Goldstone boson for X' (π') in favor of X (π). This is accomplished by setting, $\nabla_\nu \pi$ (to be defined below) to zero, which results in an algebraic relation between π and π'. A classic example of this is a crystal where there are no independent Goldstone bosons for the broken rotations, as phonons suffice to saturate all the Ward identities.

We would like to point out that it is not necessary to impose an IHC. A theory involving all the Goldstone modes is perfectly acceptable although in practice the theory might be cumbersome to use. This however would be the most straight forward way of realizing the broken symmetries. The second option, which is usually the one used, is to impose all possible IHCs and work with a minimal set of Goldstones. Finally, there is a third possibility which we call the Dynamical Inverse Higgs Constraints (DIHC) [18], whereby an operator constraint ensures the symmetry is realized. The canonical example of DIHC arises in the case of FLT, where only boost invariance is broken but there is no possibility for an IHC.

Of the three paths to symmetry, the first two may not be compatible with FLT behavior. To see this one first notes that when a spacetime symmetry is broken the Goldstone mode may be non-derivatively coupled (for a proof of this see [16, 18]), and thus will not necessarily be irrelevant in the low energy limit. A simple one loop calculation shows that framons fluctuations generate a quasi-particle width which scales as

$$\Gamma(E) \sim (E - \mu)^{d/3}, \tag{3}$$

which leads to Non-Fermi liquid behavior in d=2 and Marginal Fermi liquids in d=3.

As such it is natural to ask how FLT is consistent with boost invariance? The answer follows from the process of elimination, as the only remaining possibility is the existence of a DIHC, the third alternative discussed above. In fact, in his seminal work [1], Landau showed that boost invariance implies that the following relation must hold

$$\frac{1}{m^\star} = \frac{1}{m} + \frac{G_1}{3}. \tag{4}$$

Here $m^\star$ is the effective mass of the quasi-particles defined by $m^\star v_F = k_F$, m is the free electron mass and $G_1 = g_1 D(\mu)$, where the (FS) coupling function is expanded in Legendre polynomials, $g(\theta) = \sum_l g_l P_l(\cos(\theta))$ and $D(\mu)$ is the density of states on the Fermi surface. Below we will show how this relation arises due to a DIHC.

It is interesting to note that a similar situation arises when rotational invariance is spontaneously broken by the Fermi surface but translations are unbroken as in case of a nematic Fermi fluid. Again there are two possible realizations, since there is no IHC. Either a condition similar to Eq.(4) is obeyed such that there exists a DIHC for the rotational symmetry, or a collective mode, the "*angulon*", must arise in the spectrum which couples non-derivatively and leads to non-FLT behavior [16]. In

Ref. [25], it was shown that this theory does generate a collective gapless mode that couples non-derivatively to the quasi-particles. This theory does not have any additional Goldstones (i.e. no framon) and so the boost invariance is only manifest once the Landau relation is obeyed. That is, the rotational symmetry is realized via a Goldstone mode, but the boost is realized via the existence of a DIHC.

FERMIONS AT UNITARITY

Here we are interested in fermions at unitarity whose short distance effective action is invariant under the larger Schrodinger group, which has two added symmetries beyond the standard Fermi liquid namely, dilations and special conformal transformations (SCT). These additional symmetries are also spontaneously broken by the Fermi surface. Our goal is to determine whether or not nature can realize these broken symmetries and still retain Fermi liquid behavior, and we shall now see that the answer is no in three spatial dimensions.

In [29] it was pointed out that there is no boost invariant kinetic term for the *dilaton*, the Goldstone associated with the breaking of dilations, which might explain its apparent absence in nature. However, the existence of *framon* resolves this issue [18], as the non-invariance of the naive kinetic term is compensated by a shift in the *framon* field. Hence at unitarity, one possible realization of the symmetries involves three gapless modes, corresponding to the three (two of which are conformal) broken symmetries.

The other possibilities are that the systems realizes the symmetry with fewer Goldstone bosons due to an IHC and/or a DIHC. We will explore both the possibilities in two and three dimensions. While it is well known that the two dimensional case should behave as a free Fermi gas [36], it is instructive to see how this result follows from symmetry requirements.

NON-LINEAR REALIZATIONS OF SPACETIME SYMMETRIES

To understand how Goldstones realize the spontaneously broken space-time symmetries, we will utilize the coset construction [27] as applied to space-time symmetries [22–24]. A Fermi liquid tuned to unitarity spontaneously breaks the full Schrdinger group, $\mathcal{G}$ into an unbroken sub-group, $\mathcal{H}$ consisting of the symmetry generators for translations $(H, \vec{P})$, rotations $(\vec{J})$ and $U(1)$ particle number (M). The broken generators are boosts $(\vec{K})$, dilations (D) and SCT (C). An element of the coset space, $\mathcal{G}/\mathcal{H}$ can be parametrized as [22–24],

$$\Omega = e^{iHt}e^{-i\vec{P}\cdot\vec{x}}e^{-i\vec{K}\cdot\vec{\eta}(x)}e^{-iC\Lambda(x)}e^{-iD\phi(x)}, \qquad (5)$$

where $\vec{\eta}(x)$ is the *framon*, $\phi(x)$ is the *dilaton* and $\Lambda(x)$ is the Goldstone boson for SCT. Building blocks for writing down $\mathcal{G}$-invariant actions can be obtained from the Maurer-Cartan form via the identification,

$$\Omega^{-1}\partial_\mu\Omega = iE^\nu_\mu\left(P_\nu + \nabla_\nu\vec{\eta}\cdot\vec{K} + D\nabla_\nu\phi + C\nabla_\nu\Lambda + A_\nu M\right). \qquad (6)$$

We can use the covariant derivatives $(\nabla_\nu\phi,\ \nabla_\nu\vec{\eta},\ \nabla_\nu\Lambda)$, the vielbein (E^ν_μ) and the gauge field (A_ν), to write down terms which are linearly $\mathcal{H}$-invariant. Such terms are automatically invariant under the full group $\mathcal{G}$. For a detailed calculation of these covariants derivatives and their coupling to quasi-particles see Ref. [18].

For the broken generators considered in our set up, we have the commutation relation $[H, C] = iD$ which gives rise to an IHC. Using (6) we find

$$\nabla_0\phi = \Lambda + \partial_t\phi + ... \qquad (7)$$

and by setting this result to zero, we may eliminate Λ (to lowest order in derivatives) in favor of the *dilaton*. Another IHC arises as a consequence of $[P_i, C] = -iK_i$ via the covariant derivative

$$\vec{\nabla}\cdot\vec{\eta} = 3\Lambda + \vec{\partial}\cdot\vec{\eta} + ... \qquad (8)$$

Setting these covariant derivatives to zero (i.e. imposing the IHC) is consistent with the symmetries. Nature may or may not choose to impose any or all of these constraints. The final possibility is that there are no Goldstone modes at all due to DIHCs. To understand how this scenario can arise let us first discuss how the DIHC is generated in the canonical Fermi liquid.

HOW DO DYNAMICAL INVERSE HIGGS CONSTRAINTS ARISE?

We begin by studying the Fermi liquid away from unitarity. The framon both acts as a gauge field (A_μ) and shows up in the vielbein in Eq. (6) and its coupling to the quasi-particles is given by replacing the normal derivatives in Eq.(1) with the covariant derivatives [18],[39]

$$\begin{aligned}\nabla_t\psi &= \partial_t\psi + \vec{\eta}\cdot\vec{\partial}\psi + \frac{i}{2}M\vec{\eta}^2\psi \\ \nabla^i\psi &= \partial^i\psi - iM\eta^i\psi.\end{aligned} \qquad (9)$$

For example the quadratic term in the quasi-particle action is now given by,

$$S_0 = \int \frac{d^dpdt}{(2\pi)^d}\ \psi^\dagger_{\mathbf{p}}(t)[(i\partial_0 - \vec{\eta}\cdot\mathbf{p} - \tilde{\varepsilon}(\vec{p}+m\vec{\eta}) + \mu_{\rm F} + \mathcal{O}(\eta^2)]\ \psi_{\vec{\mathbf{p}}}(t). \qquad (10)$$

Similarly we can write down a boost invariant quartic term by using the covariant derivatives given in Eq.(9)

[18]. The kinetic term for the framon can also be constructed from the following covariant derivatives

$$\nabla_j \eta^i \;=\; \partial_j \eta^i \tag{11}$$

$$\nabla_0 \eta^i \;=\; -(\dot{\eta}^i + \vec{\eta}\cdot\vec{\partial}\eta^i). \tag{12}$$

Note that there are no linear terms in η in the covariant derivatives and hence there is no mass term for η i.e. the framon is not gapped.

Nature may choose to realize the broken boost symmetry in one of two ways, either with a framon or without it. We may generate an invariant theory, without the framon by treating it as a compensator field and imposing a non-trivial constraint, or we may keep the framon in the spectrum as a propagating degree of freedom, though as we shall see below, this latter choice is not available in two spatial dimensions.

Given that the framon transforms non-linearly under boosts, we may generate an invariant action by simply noting that, under a boost transformation, the action must satisfy

$$\delta S = \frac{\delta S}{\delta\psi}\delta\psi + \frac{\delta S}{\delta\eta}\delta\eta = 0, \tag{13}$$

and by imposing the operator constraint $\frac{\delta S}{\delta\eta} = 0$, and dropping the pieces in the action that involve only η we are ensured to have an invariant action for the quasi-particles. The constraints that are generated are strong in the technical sense, and relate operators that are quadratic and quartic in the fields. For these constraints to be satisfied for arbitrary quasi-particle states necessitates very restrictive dynamics. Indeed, as was shown in [18] that all of the restrictions on low energy scattering processes generated by the Fermi surface are *necesesitated* by these "dynamical inverse Higgs" (DIHC) constraints. That is, one can derive the fact that the only relevant processes in a Fermi liquid are the forward scattering and BCS channel, purely from symmetry considerations without going through the usual kinematic analysis (see e.g. [14]).

Now we will show that in two dimensions one is actually forced into realizing the broken spacetime symmetries via a DIHC as consequence of the renormalization group scaling of the η field. To see this note that by the standard power counting arguments, close to the Fermi surface [14, 15], the energy $(\varepsilon(p)-\mu)$ and the component of quasi-particle momentum normal to Fermi surface (l) scale as λ, where λ is the rescaling parameter of the RG transformation and $\lambda \to 0$ as we approach the Fermi surface. Since the covariant derivatives in Eq.(9) for the quasi-particles must transform homogeneously under a RG transformation, so the symmetry implies that the scaling of η is the same as l i.e $\eta \sim \lambda$. Any other choice would lead to a non-invariant action.

We can now deduce the scaling of η momentum by using the canonical commutation relation,

$$[\eta^i(x), \dot{\eta}^j(x')] = \delta^d(x-x')\delta^{ij}. \tag{14}$$

Since the framon has a standard bosonic dispersion relation, $E \sim k$, so if we assume that framon momentum $k_\eta \sim \lambda^n$, then from Eq.(14) we get,

$$n = \frac{2}{d-1}. \tag{15}$$

So in $d = 2$, $k_\eta \sim \lambda^2$ which is sub-leading to the quasi-particle momentum and so the $\eta(x)$ field must be multipole expanded [31] for the power counting to be consistent. This implies that the η field has no dynamics (the kinetic contribution to the action is ignored) and at leading order acts as an auxiliary field which can be removed from the theory by using leading order equations of motion. i.e. η must operate as a compensating field. Varying the quasi-particle action (including quadratic and quartic terms) with respect to η generates the following operator constraint [18],

$$O_i^B = \Big(\int \frac{d^dp}{(2\pi)^d}\,\psi_p^\dagger(p_i - m\frac{\partial\varepsilon_p}{\partial p_i})\psi_p - \frac{m}{2}\int\prod_{a=1}^{4}\frac{d^dp_a}{(2\pi)^d}$$
$$\delta^{(d)}(p_1+p_2-p_3-p_4)\Big(\sum_i \frac{\partial g(p_a)}{\partial p_{i,a}}\Big)\psi_{p_4}^\dagger\psi_{p_3}^\dagger\psi_{p_2}\psi_{p_1}\Big) = 0. \tag{16}$$

Taking one-particle matrix element of this operator results in a non-trivial relation between the two parameters (m^*, g_1) of the theory, which is nothing but the Landau relation in Eq.(4).

In $d = 3$ we are no longer forced to perform the multipole expansion, since the framon momentum scales in the same way as the quasi-particles' , $k \sim \lambda$. Thus in a canonical FL (i.e. not at unitarity) *a priori* there is no reason why nature could not choose to realize the broken boost invariance via the framon. However, if this were indeed the case then coupling of η to the quasi-particles would give rise to marginal interactions and result in Marginal Fermi liquid behavior.

The alternative realization of the theory is trivial to find via a DIHC, since the same line of reasoning holds as in the two dimensional case, except now ignoring the kinetic part of the framon field is simply a mathematical trick to find an invariant action.

GOING TO UNITARY LIMIT

Let us now consider the Fermi liquid at unitarity in d dimensions and ask whether or not the symmetries can be realized in a way consistent with Fermi liquid theory. Again we will consider the two logical possibilities of either having Goldstones in the spectrum or eliminating them using DIHCs'.

Since $[H,C] \sim D$ and $[P,C] \sim K$, we may choose to impose these two IHCs and eliminate two of the three Goldstones' by setting the covariant derivatives Eq.(7) and Eq.(8) to zero. In so doing we generate the relation

$$\partial \cdot \eta = \dot{\phi}. \tag{17}$$

In the case of two dimensions, this condition implies that not only is the framon non-dynamical but so is the dilaton. Thus we will generate two DIHCs. While the constraint from boost invariance will be identical to Eq.(16), varying the quasi-particle action with to respect to the dilaton leads to the vanishing of the operator

$$\begin{aligned}\mathcal{O}_\phi &= \sum_{\vec{\mathbf{k}}} \int d^d p dt \;\; \psi^\dagger_{\vec{\mathbf{p}}}(t) \left[2\varepsilon(p) - p^i \frac{\partial \varepsilon}{\partial p_i}\right] \psi_{\vec{\mathbf{p}}}(t) \\ &+ \frac{1}{2} \prod_{a=1}^{4} \int d^d p_a dt \;\; \left[(2-d) g(\vec{\mathbf{p}}_i, \mu) - \vec{\mathbf{p}}_i \cdot \frac{\partial g(\vec{\mathbf{p}}_i, \mu)}{\partial \vec{\mathbf{p}}_i} - \beta(g)\right] \\ &\times \;\; \psi^\dagger_{\vec{\mathbf{p}}_1}(t) \psi_{\vec{\mathbf{p}}_2}(t) \psi^\dagger_{\vec{\mathbf{p}}_3}(t) \psi_{\vec{\mathbf{p}}_4}(t) = 0 \end{aligned} \tag{18}$$

where $\beta(g) = \mu \frac{\partial g}{\partial \mu}$ is the beta function. A detailed derivation of this result can be found in [18]. A short path to the derivation follows by calculating the Noether charges from the action in Eq.(1) for the Schrodinger group and imposing the commutation relation $[D,H] = 2iH$.

Let us now see if a Fermi liquid description is consistent with these constraints. Given our assumption of rotational invariance and the notion of a well defined Fermi surface, the marginal coupling is only a function of the angles which are scale invariant. Thus the second term in the second line of Eq.(18) vanishes ($\vec{\mathbf{p}} \cdot \frac{\partial g(\vec{\mathbf{p}}, \mu)}{\partial \vec{\mathbf{p}}} = 0$) , as such if we take the one particle matrix element we see that the quadratic and quartic terms must vanish separately since the quadratic term will depend upon the amplitude of the incoming external momentum whereas the quartic term will be independent of it due to rotational invariance. This gives the constraint

$$(2-d) g(\vec{\mathbf{p}}, \mu) = \beta(g) \tag{19}$$

In three dimensions we see that the coupling has power law running which is inconsistent with Fermi liquid theory, and in two dimensions the theory is free. Thus we conclude that, if the symmetries are realized without the existence of Goldstones then fermions at unitarity do not behave like Fermi liquid. In two dimensions the Goldstone can not be dynamical and as such we have ruled out any other way to realize the symmetry. In three dimensions, where the Goldstone can be dynamical, they lead to overdamping of quasi-particles, and thus we may conclude that *fermions at unitarity can not be be described as a Fermi liquid.* However, our method does not shed light on the question of whether the breakdown is due to pseudogaps.

We can also consider how these symmetry constraints can be utilized if we assume that the microscopic theory is defined via the action Eq.(1) independent of any Fermi liquid description. For the one-particle matrix elements of Eq.(18), it is still true that the quadratic and quartic terms must vanish separately, even if $\vec{\mathbf{p}}_i \cdot \frac{\partial g(\vec{\mathbf{p}}_i, \mu)}{\partial \vec{\mathbf{p}}_i} \neq 0$, since the quartic term is independent of the external momenta while the quadratic is not. In this case we have the constraints

$$\begin{aligned} \varepsilon &= \frac{p^2}{2m^\star} \\ 0 &= (2-d) g(\vec{\mathbf{p}}_i, \mu) - \vec{\mathbf{p}}_i \cdot \frac{\partial g(\vec{\mathbf{p}}_i, \mu)}{\partial \vec{\mathbf{p}}_i} - \beta(g). \end{aligned} \tag{20}$$

For S-wave scattering ($g(p)$=constant), $m = m^\star$ but for higher angular momentum channels, Eq.(20) gives us the beta function to all orders.

CONCLUSIONS

In this letter we have addressed the open question of whether or not three dimensional Fermi gases at unitarity behave like Fermi liquids. We have shown that for temperatures in the range $T_F > T > T_C$ the system will not manifest Fermi liquid behavior. Our arguments are based solely on the symmetry breaking pattern. In particular, we use the fact that while the conformal and boost symmetries are spontaneously broken, these symmetries must still be realized, albeit non-linearly, in the low energy theory. This this can happen either via the existence of the appropriate Goldstone modes, or through a Dynamical Inverse Higgs Constraint, whereby a nontrivial operator constraint must be obeyed to manifest the symmetry. We have shown that in three dimensions, neither of these possibilities is consistent with Fermi liquid behavior. We have also derived an exact result for the beta function for the coupling function in the unitary limit.

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[37] In a metal of course this is broken and the arguments here change slightly [18].
[38] Away from unitarity.
[39] These covariant derivatives are calculated for a normal fermi liquid i.e dilatations and special conformal transformations are unbroken.