Cavity superconductor-polaritons

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Following the recent success of realizing exciton-polariton condensates in cavities, we examine the hybridization of cavity photons with the closest analog of excitons within a superconductor, states called Bardasis-Schrieffer modes. Though these modes do not typically couple linearly to light, one can engineer a coupling with an externally imposed supercurrent, leading to the formation of hybridized Bardasis-Schrieffer-polariton states, which we obtain both as poles of the bosonic Green's function and through the derivation of an effective Hamiltonian picture for the model. These new excitations have nontrivial overlap with both the original photon states and d-wave superconducting fluctuations. We conjecture that a phase-coherent density of these objects could produce a finite d-wave component of the superconducting order parameter – an $s \pm id$ superconducting state.

Strong light-matter interaction has been a field of continuing interest for many years[1], with exciton polaritons[2] in particular garnering much attention. Formed from strong coupling between microcavity photons and excitons within a semiconductor, exciton-polaritons and their condensation at high temperatures are by now a well-established experimental milestone[3–6]. These systems have recently seen application in the quantum simulation of solid state physics[7–10], acoustic black hole physics[11], and topological properties of quasicrystal states[12].

Similar cavity schemes have been proposed for superconducting systems in order to enhance the strength of superconductivity through various mechanisms [13–18]. Though there is a rough similarity between semiconducting and superconducting quasiparticle spectra, both featuring a gap, the BCS ground state is notably more complicated than that of a semiconductor, thus complicating the matter of superconductor-polariton formation.

The existence of internal exciton-like states of the superconducting order parameter was originally proposed by Bardasis and Schrieffer not long after the development of BCS theory[19]. Now named Bardasis-Schrieffer (BS) modes, these can be thought of as the excitation of Cooper-pairs into states with higher angular momentum than their ground state. More precisely, BS modes are gapped, undamped, in-gap fluctuations of the superconducting order parameter in a subdominant pairing channel with a $U(1)$ phase of $\pi/2$ relative to the ground state condensate. Typically d-wave fluctuations are considered about an s-wave ground state, as we will consider here. These modes have long been sought experimentally but are difficult to detect because they do not couple to electromagnetism at the level of linear response; their detection has only been recently reported through Raman spectroscopy in iron-based materials[20–22]. A variety of multiband effects are known to complicate their identification[23], which we will not consider for simplicity.

In our proposed model the BS mode can be hybridized with photons in a resonant cavity to form polaritons in analogy with the theory of exciton-polariton formation in semiconductors. Importantly we show how the lack of linear coupling to light, which would normally prohibit this hybridization, can be overcome by driving a supercurrent. This method has similarly been proposed for directly driving the Higgs mode with light[25] and has been recently implemented successfully in experiment[26]. A secondary implication of our work is that the BS mode could also be observed optically with a similar experimental protocol.

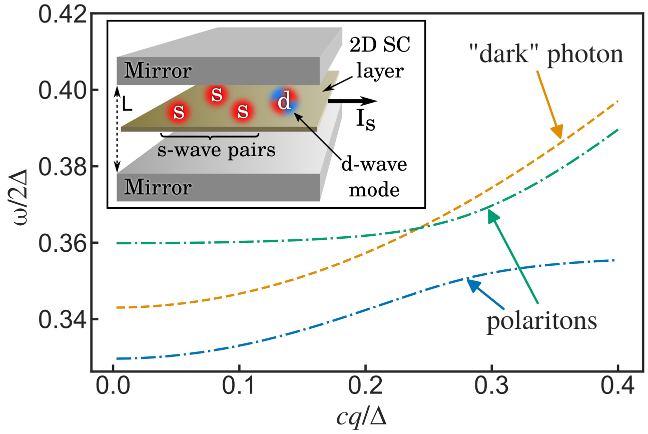

FIG. 1. (Color online) The dispersion of the Bardasis-Schrieffer-polariton modes (dot-dashed), calculated both numerically and with a simplified analytic method[24] – the two give visually identical results. An external supercurrent causes the BS mode and cavity photons to hybridize, and the polariton states have significant overlap with each. The "dark" photon mode (dashed) remains decoupled. The splitting of otherwise degenerate photon modes is a result of a supercurrent-induced self-energy contribution. Temperature and supercurrent angle are chosen to maximize hybridization (see Fig. 2). *Inset* — schematic of the system: a 2-dimensional superconductor with an applied supercurrent I_S at the center of a planar cavity.

Our main results, presented in Fig. 1, demonstrate the hybrid Bardasis-Schrieffer-polariton dispersions calculated from a microscopic model of coupled fermions and cavity photons. We further show that these polariton states can be described to an excellent degree of approximation by an intuitive effective Hamiltonian picture of the coupled modes. Owing to the origin of the light-matter coupling, details of the hybridization can be straightforwardly controlled.

We consider a setup consisting of a two-dimensional electron system at the center of a perfectly reflecting parallel mirror QED cavity, as shown in the inset of Fig. 1. The 2D electron system is described by a single-band fermion action with a BCS interaction decomposed in angular momentum channels. With $\hbar = 1$ it is

$$S_\psi = \sum_{k,\sigma} \bar{\psi}_{k,\sigma} \left(-i\epsilon_n + \xi_k\right) \psi_{k,\sigma} - \frac{1}{\beta} \sum_q \sum_{\ell=s,d} g_\ell \bar{\varphi}^\ell_q \varphi^\ell_q, \quad (1)$$

with $\xi_k = k^2/2m^* - \mu$ the energy measured from the Fermi surface, σ labelling spin, k and q each representing momentum and Matsubara frequency, g_ℓ the interaction strength in the ℓ-channel, and the interaction written in terms of bilinears,

$$\varphi^\ell_q = \sum_k f_\ell(\phi_k) \psi_{-k+\frac{q}{2},\downarrow} \psi_{k+\frac{q}{2},\uparrow}. \quad (2)$$

Importantly, following Bardasis and Schrieffer[19] we assume the interaction is sizable in both s-wave and d-wave channels, but a stronger s-wave component, $g_s > g_d$, leads to a purely s-wave superconducting ground state. The form factors are taken to be $f_s(\phi_k) = 1$ and $f_d(\phi_k) = \sqrt{2}\cos(2\phi_k)$. This choice f_d breaks the model's full rotational symmetry by choosing an explicit reference axis from which the angle of $\mathbf{k}$, here called ϕ_k, is measured, which we expect to be chosen by the underlying crystal structure of the system — not explicitly present in our continuum model. The interaction can be decoupled in both angular momentum channels simultaneously with a Hubbard-Stratonovich transformation

$$S = \sum_k \bar{\Psi}_k \left(-i\epsilon_n \hat{\tau}_0 + \xi_k \hat{\tau}_3\right) \Psi_k + \frac{1}{\beta} \sum_{q,\ell} \frac{1}{g_\ell} |\Delta^\ell_q|^2 - \frac{1}{\beta} \sum_{k,q} \bar{\Psi}_{k+\frac{q}{2}} \sum_\ell f_\ell(\phi_k) \begin{pmatrix} 0 & \Delta^\ell_q \\ \bar{\Delta}^\ell_{-q} & 0 \end{pmatrix} \Psi_{k-\frac{q}{2}}, \quad (3)$$

where $\Psi_k = (\psi_{k,\uparrow}, \bar{\psi}_{-k,\downarrow})$ are Nambu spinors, $\hat{\tau}_i$ are the Pauli matrices in Nambu space with $\hat{\tau}_0$ the identity, and Δ^ℓ_q are the complex Hubbard-Stratonovich decoupling fields labeled by angular momentum channel.

The cavity is treated as perfectly reflecting boundaries at $z = 0, L$. The action for photons inside the empty cavity is (with $c = 1$)

$$S_{\rm cav} = -\frac{1}{2\beta} \sum_{q,n,\alpha} A_{\alpha,n,-q} \left[(i\Omega_m)^2 - \omega^2_{n,q}\right] A_{\alpha,n,q}. \quad (4)$$

Here α indexes the two cavity polarizations, n labels the quantized modes resulting from the confinement in z, and $\omega^2_{n,q} = \omega^2_{n,0} + q^2$, with $\omega_{n,0} = n\pi/L$, is the dispersion of photons inside the cavity. We consider just the $n = 1$ mode and drop the index; all other modes are higher in energy and far from the resonance we tune to later. The vector potential is written in terms of polarizations as $\mathbf{A}_q(z) = \sum_\alpha \boldsymbol{\epsilon}_{\alpha,\mathbf{q}}(z) A_{\alpha,q}$, with $\boldsymbol{\epsilon}_{\alpha,\mathbf{q}}(z)$ the polarization vectors inside the cavity[24]. The electron system is located in the middle of the cavity, so only $z = L/2$ must be considered. Minimal coupling between the cavity photon and the electron system generates a paramagnetic term proportional to $e\mathbf{v_k} \cdot \mathbf{A}_q$, with the electron velocity operator $\mathbf{v_k} = \mathbf{k}/m^*$, and a diamagnetic term proportional to $e^2 A^2_q$. We drop the diamagnetic term since it is unimportant both in the weak-field regime[27] and for the cavity photon self-energy in the presence of disorder, which is ubiquitous in 2D[28,29].

Note that our cavity geometry is chosen for calculation simplicity, but in real microwave cavities the transverse nature of the of photon amplitude envelope is more complicated. The effect of this is to increase the strength of the paramagnetic coupling, which we include via a phenomenological enhancement in the light-matter coupling term[18,27,30].

We now consider externally driving a homogeneous supercurrent through the system. A supercurrent can be understood as the superconducting condensate moving at constant uniform velocity with respect to the lab frame, with Bogoliubov quasiparticles being defined in the comoving frame, i.e. the supercurrent can be included via a simple Galilean transformation. Calling the condensate superfluid velocity $\mathbf{v}_S$, we have $\mathbf{v_k} \to \mathbf{v_k} + \mathbf{v}_S$. The angle of $\mathbf{v}_S$ with respect to the axis defined by $f_d(\phi_k)$, as depicted in the inset in Fig. 2, is denoted θ_S. This modifies the quasiparticle dispersion in the lab frame

$$\xi_k \to \xi_k + \mathbf{k} \cdot \mathbf{v}_S + \frac{1}{2} m v^2_S \equiv \xi^S_k + \mathbf{k} \cdot \mathbf{v}_S. \quad (5)$$

The term linear in $\mathbf{k}$ is a Doppler shift in the energy while the v^2_S term can be absorbed into a (negligible) redefinition of the chemical potential. The velocity shift also affects the paramagnetic coupling,

$$S_{\psi-A} \to \frac{X}{\beta} \sum_{k,q} \bar{\Psi}_{k+\frac{q}{2}} \underbrace{\left(-e\mathbf{v_k}\hat{\tau}_0 - e\mathbf{v}_S\hat{\tau}_3\right) \cdot \mathbf{A}_q}_{\equiv \hat{\chi}_{k,q}[A]} \Psi_{k-\frac{q}{2}}. \quad (6)$$

Here X denotes the phenomenological coupling enhancement described above[18,27,30], which we absorb into a redefinition of the charge. Crucially the Nambu structure for the paramagnetic and supercurrent-induced terms are different, since particle and hole velocities are shifted oppositely, ultimately allowing the coupling of the BS mode to light. The supercurrent can equivalently be included as a uniform phase winding of Δ^s which, upon appropriate gauge transformation, reproduces these results while maintaining explicit gauge invariance.

We make the mean-field approximation on the s-wave gap function

$$S = S_{\Delta,s} + S_{\Delta,d} + S_{\rm cav} - \sum_k \bar{\Psi}_k \hat{G}^{-1}_k \Psi_k + \frac{1}{\beta} \sum_{k,q} \bar{\Psi}_{k+\frac{q}{2}} \left(\hat{\chi}_{k,q}[A] - \hat{\Delta}^d_{k,q}\right) \Psi_{k-\frac{q}{2}}, \quad (7)$$

with $S_{\Delta,s} = \beta|\Delta|^2/g_s$ describing the static, homogeneous s-wave component Δ, $S_{\Delta,d} = \beta^{-1}\sum_q |\Delta^d_q|^2/g_d$ describing the d-wave fluctuations, $\hat{G}_k^{-1} = (i\epsilon_n - \mathbf{k}\cdot\mathbf{v}_S)\hat{\tau}_0 - \xi^S_k\hat{\tau}_3 + \Delta\hat{\tau}_1$ the inverse Nambu Green's function, and

$$\hat{\Delta}^d_{k,q} = f_d(\phi_k)\begin{pmatrix} 0 & \Delta^d_q \\ \bar{\Delta}^d_{-q} & 0 \end{pmatrix}. \tag{8}$$

The mean field value of Δ is obtained as the saddle point solution in the absence of $\mathbf{A}$ and Δ^d but in the presence of the supercurrent, in keeping with the approximation that Δ is unaffected by d-wave fluctuations and photons.

We now integrate out the fermions and expand to second order in $\hat{\Delta}^d$ and $\hat{\chi}$, giving

$$S_{\text{eff}} = S_d + S_A + S_{d-A}. \tag{9}$$

These three terms are defined as the parts of the action describing free d-wave fluctuations, cavity photons in the presence of the superconducting system, and the supercurrent-generated coupling between them, respectively.

Since the d-wave fluctuations have much greater kinetic mass than photons, we approximate them with a flat dispersion: their energy in the limit $\mathbf{q}\to 0$. Additionally, we drop all terms which vanish in the quasiclassical ξ-approximation. Writing Δ^d in terms of its real and imaginary components, S_d decouples into an action for each. The real mode is within the Bogoliubov quasiparticle continuum, and is therefore overdamped[19,31]. It also remains decoupled from photons despite the supercurrent so we do not consider it further. The imaginary mode is the in-gap Bardasis-Schrieffer mode. Naming this mode d_q, the BS mode action is

$$S_d = \frac{1}{\beta}\sum_q d_{-q}\left[\frac{1}{g_d} + \sum_{\mathbf{k}} f_d(\phi_k)^2 \frac{2\lambda_k\,\delta n_{\mathbf{k}}}{(i\Omega_m)^2 - (2\lambda_k)^2}\right] d_q, \tag{10}$$

where $\bar{d}_q = d_{-q}$, $\lambda_k = \sqrt{\left(\xi^S_k\right)^2 + \Delta^2}$ is the quasiparticle energy in the comoving frame, $\delta n_{\mathbf{k}} = n_F(E^-_{\mathbf{k}}) - n_F(E^+_{\mathbf{k}})$, where n_F is the Fermi function, and $E^{\pm}_{\mathbf{k}} = \pm\lambda_k + \mathbf{k}\cdot\mathbf{v}_S$ is the Doppler-shifted energy.

The photon sector of the action consists of the empty cavity action S_{cav} plus a self-energy term due to the superconductor,

$$S_A = -\frac{1}{2\beta}\sum_{q,\alpha,\beta} A_{\alpha,-q}\left[\left((i\Omega_m)^2 - \omega_q^2\right)\delta_{\alpha\beta} - \Pi_{\alpha\beta,q}\right] A_{\beta,q}. \tag{11}$$

The matrix $\Pi_{\alpha\beta,q}$ is the electromagnetic linear response function of the superconducting system written in the cavity polarization basis[24].

Within the approximations discussed above the coupling between photons and the BS mode arises *entirely* through the supercurrent-induced term,

$$S_{d-A} = -\frac{ie\Delta}{\beta}\sum_{\mathbf{k},q,\alpha} f_d(\phi_k)\frac{i\Omega_m\,\delta n_{\mathbf{k}}}{(i\Omega_m)^2 - (2\lambda_k)^2}\frac{\mathbf{v}_S\cdot\boldsymbol{\epsilon}_{\alpha,q}}{\lambda_k} \times \left(A_{\alpha,q}\,d_{-q} - A_{\alpha,-q}\,d_q\right), \tag{12}$$

consistent with the known result that the BS mode does not normally couple linearly to light. As a consequence, the BS mode only couples to the component of the vector potential parallel to the supercurrent.

The action is then straightforwardly written in terms of a hybrid inverse Green's function

$$S_{\text{eff}} = \frac{1}{2\beta}\sum_q (d_{-q}, A_{\alpha,-q})\begin{pmatrix} D^{-1}_{\text{BS},q} & g_{\alpha,q}\delta_{\alpha\beta} \\ g^*_{\alpha,q}\delta_{\alpha\beta} & D^{-1}_{\alpha\beta,q}\end{pmatrix}\begin{pmatrix} d_q \\ A_{\beta,q}\end{pmatrix}, \tag{13}$$

with sums over repeated indices and with $D^{-1}_{\text{BS},q}$, $D^{-1}_{\alpha\beta,q}$, and $g_{\alpha,q}$ defined implicitly through Eqs. (10)–(12). A more intuitive description can be obtained by first making a harmonic approximation to the BS action: continue D^{-1}_{BS} to complex frequency, expand around the saddle point solution Ω_{BS}, then restrict back to imaginary frequency. In our clean model Ω_{BS} is purely real, so the BS mode is undamped. We then expand in terms of BS and photon mode operators, $d_q = (b_q + \bar{b}_{-q})/\sqrt{2K\Omega_{\text{BS}}}$ and $A_{\alpha,q} = (a_{\alpha,q} + \bar{a}_{\alpha,-q})/\sqrt{2\omega_q}$, where K is a constant coming from the harmonic expansion. We make the standard approximation of dropping the counter-rotating terms $(aa, \bar{a}\bar{a})$ – an approximation we verify post-hoc – and perform a change of basis from photon polarizations to components parallel and perpendicular to the supercurrent. Inside the coupling and photon terms, we analytically continue to real frequency $i\Omega_m \to \Omega + i0$, then expand around relevant frequencies. The imaginary parts exactly vanish, and the action becomes

$$S_{\text{eff}} \approx \frac{1}{\beta}\sum_q \left(\bar{b}_q, \bar{a}^{\parallel}_q, \bar{a}^{\perp}_q\right)\left(-i\Omega_m\mathbb{1} + \check{H}^{\text{eff}}_{\mathbf{q}}\right)\begin{pmatrix} b_q \\ a^{\parallel}_q \\ a^{\perp}_q\end{pmatrix}, \tag{14}$$

now written in terms of an effective Hamiltonian[24]

$$\check{H}^{\text{eff}}_q = \begin{pmatrix} \Omega_{\text{BS}} & g_q & 0 \\ g_q & \omega_q + \Pi^S_q & 0 \\ 0 & 0 & \omega_q \end{pmatrix}, \tag{15}$$

where $q = |\mathbf{q}|$, Π^S_q is a self-energy shift in the photon mode polarized parallel to the supercurrent, coming from a supercurrent-dependent term in $\Pi_{\alpha\beta,q}$, and

$$g_q = ev_S\Delta\sqrt{\frac{2\,\Omega_{\text{BS}}}{L\,K\omega_q}}\sum_{\mathbf{k}}\frac{f_d(\phi_k)}{\lambda_k}\frac{\delta n_{\mathbf{k}}}{\Omega^2_{\text{BS}} - (2\lambda_k)^2}. \tag{16}$$

We keep only to lowest order in $\mathbf{q}$. Only one photon mode hybridizes with the BS mode in the Hamiltonian approximation. This photon mode and the BS mode can be made resonant by tuning parameters of the system,

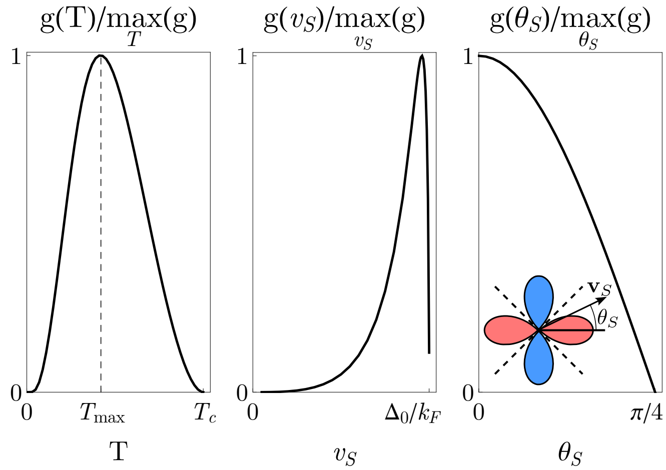

FIG. 2. (Color online) The hybridization matrix element g in the effective Hamiltonian as a function of temperature, superfluid velocity, and θ_S, the angle between the direction of the supercurrent and the axis defined implicitly by the d-wave form factor $f_d(\phi_k)$, all scaled by their respective maxima. (Left) $g(T)$ is maximized for a temperature $T_{\max} \approx 0.42T_c$. (Center) $g(v_S)$ is sharply peaked for large superfluid velocity around $v_S \approx 0.96\Delta(v_S = 0)/k_F$. (Note, $\Delta_0 \equiv \Delta(v_S = 0)$.) (Right) $g(\theta_S)$ is maximal for $\theta_S = m\pi/2$, $m \in \mathbb{Z}$, and vanishes when the supercurrent runs along a node of f_d, $\theta_S = (2m+1)\pi/4$. *Inset* — the orientation of the supercurrent with respect to the d-wave form factor. The color of the lobes gives the relative sign of f_d for different angles, and the dashed lines are the nodes where $f_d = 0$. The plots use $T = T_{\max}$, $v_S = 0.9\Delta(v_S = 0)/k_F$, and $\theta_S = 0$ where applicable, and fixed detuning $\omega_0 = 0.96\Omega_{\rm BS}$.

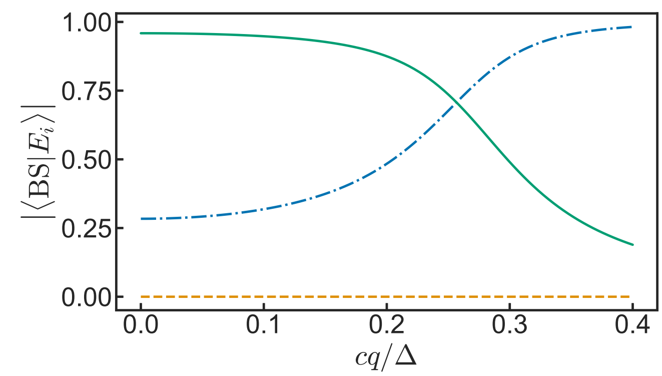

FIG. 3. (Color online) The Bardasis-Schrieffer component of the eigenvectors of the effective Hamiltonian, Eq. (15)[24]. The upper (solid) and lower (dot-dashed) polaritons have significant photon and Bardasis-Schrieffer character, indicating strong hybridization between the systems. One can also clearly see the "dark" photon mode (dashed) which does not hybridize with the superconductor's collective mode.

most straightforwardly the cavity size L, allowing them to strongly hybridize.

For numerical calculations we use material parameters motivated by iron-based superconductors[32–35], where BS modes have been experimentally detected. We set the Fermi energy $\epsilon_F = 100$ meV, the effective mass $m^* = 4m_e$[36], where m_e is the electron mass, and critical temperature $T_c = 35$ K. We put $1/g_d - 1/g_s = 0.1\nu$, where $\nu = m^*/2\pi$ is the density of states, and tune the size of the cavity L so that $\omega_0 = \pi/L = 0.96\,\Omega_{\rm BS}(\theta_S = 0)$, putting photons and the BS mode very near resonance. Finally, we set the phenomenological coupling enhancement to $X = 10$, although enhancements of $X = 10^2$ or greater have been predicted in similar cavity systems[18,27,30].

First consider the dependence of coupling strength g_q on temperature, superfluid velocity v_S, and supercurrent angle θ_S, as shown in Fig. 2. The coupling is mediated by thermally excited quasiparticles and so vanishes for $T \to 0$. It also vanishes for $T \to T_c$ since $\Delta \to 0$. The result is a unique maximum of $g(T)$ at an intermediate temperature, $T_{\max} \approx 0.42T_c$, which we use for all other computations. Similarly, g vanishes for small v_S — this can be verified by expansion of $\delta n_{\mathbf{k}}$ — and also as v_S approaches a value corresponding to the critical current, where the superconducting state vanishes. We set $v_S = 0.9\Delta(v_S = 0)/k_F$ in our calculations, near the value giving the maximum coupling but not too near the critical value[37]. Dependence on the supercurrent angle θ_S comes through the d-wave form factor. The coupling is strongest when the supercurrent is along an antinode of the form factor – $\theta_S = m\pi/2$, $m \in \mathbb{Z}$ – and vanishes when the supercurrent is along a node – $\theta_S = (2m+1)\pi/4$. We use $\theta_S = 0$ for all other calculations.

To obtain the polariton modes we both directly solve for the poles of the hybridized Green's function (13) and calculate the eigenvalues of the effective Hamiltonian (15), which can be diagonalized analytically[24]. The results of both approaches are in excellent agreement; the dispersions are plotted for both methods in Fig. 1. One of the photon modes can be made to strongly hybridize with the BS mode, while the other "dark" photon remains distinct. This is made especially clear by examining the BS component of the eigenvectors of the effective Hamiltonian, as shown in Fig. 3. Because the strength of the hybridization is controlled exclusively by g, any of the parameters on which it depends, namely T, v_S, or θ_S, can be used to directly control the strength of the effect.

In this work we have shown that driving a supercurrent through a superconductor in a planar microcavity leads to hybridization of cavity photons with a collective mode of the superconductor. In particular two polariton bands form which have significantly mixed character. This provides a means for observation and control of the Bardasis-Schrieffer mode, and, as for exciton-polaritons, these dispersions could in principle be measured with k-space imaging of the photonic component of the polariton states[38]. The nature of the construction allows for tuning of the hybridization strength, and therefore the polariton states, *in situ* through the externally applied supercurrent.

We speculate that the condensation observed in exciton-polariton systems[4–6] suggests proper driving of these superconductor-polariton modes could lead to their condensation and the formation of a non-equilibrium $s \pm id$ superconducting state. There is reason to suspect that condensation is a reasonable prospect; interactions giving thermalization arise at quartic order in perturbation theory, and the polariton lifetime is set by the cavity photon lifetime — the BS mode is in-gap and therefore undamped in this clean model. For a high enough Q-factor it is in principle possible for polaritons to thermalize before decaying, allowing for a transient quasi-thermal ensemble. More work must be done, however, before definitive statements can be made about a condensed state, especially regarding spontaneous coherence of the condensate. Finally, we note that finite polariton density with coherence imposed externally, e.g. from a coherent drive, would produce a non-equilibrium state with $s \pm id$ character, which one would expect to be distinct in nature from a thermodynamic $s \pm id$ state.

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