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Non-reciprocal spin wave in a chiral antiferromagnet without the Dzyaloshinskii-Moriya interaction

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Non-reciprocal spin wave can facilitate the realization of spin wave logic devices. It has been demonstrated that the non-reciprocity can emerge when an external magnetic field is applied to chiral magnets whose spin structures depend crucially on an asymmetric interaction, that is, the Dzyaloshinskii-Moriya interaction (DMI). Here we demonstrate that the non-reciprocity can arise even without the DMI. We demonstrate this idea for the chiral antiferromagnet Ba₃NbFe₅Si₂O₁₄ whose DMI is very small and chiral spin structure arises mainly from the competition between symmetric exchange interactions. We show that when an external magnetic field is applied, asymmetric energy gap shift occurs and the spin wave becomes non-reciprocal from the competition between symmetric exchange interactions and the external magnetic field.

I. INTRODUCTION

In many physical systems, waves propagating in opposite directions share the same characteristics. In certain special systems, on the other hand, waves propagating in opposite directions may exhibit different characteristics. For instance, waves with wave vector ±k may have different frequencies. Such non-reciprocity may endow functionalities which are difficult to realize in reciprocal systems. In particular, it was suggested¹,² that spin wave non-reciprocity can facilitate the realization of spin wave logic devices, such as a spin current diode. For the spin wave non-reciprocity to emerge in chiral magnets, certain symmetries should be broken. In case of the chiral magnets depicted in Fig. 1, their spin Hamiltonians may be invariant under the time-reversal operation ⁹, which enforces the spin wave dispersion relation to be reciprocal, E(k) = E(−k). Thus the time-reversal symmetry should be broken by some means to induce the non-reciprocity.

Recently, the spin wave non-reciprocity in chiral magnets has been studied. The experimental results in chiral ferromagnets Cu₂OSeO₃⁴, MnSi⁵, FeGe and Co-Zn-Mn alloys⁶ indicate that the non-reciprocity arises in these noncentrosymmetric systems when an external magnetic field is applied. When the field direction is reversed, the sign of non-reciprocity is also reversed. In Cu₂OSeO₃, it was demonstrated⁴ that the sign of non-reciprocity depends not only on the field direction but also on the sign of crystal chirality. Non-reciprocal spin wave dispersion relation has been reported for chiral antiferromagnet α-Cu₂V₂O₇⁷ as well, for which, similar to chiral ferromagnets, the breakings of the time reversal and the spatial inversion symmetries are important. We remark that in these examples⁴⁻⁷, the very existence of the chiral magnetism relies crucially on the Dzyaloshinskii-Moriya interaction (DMI)⁸. Considering that the DMI itself requires some symmetries to be broken, it is natural in some sense to expect the spin wave dispersions to be non-reciprocal in these systems.

In this paper, we examine theoretically the spin wave dispersion in a chiral antiferromagnet Ba₃NbFe₅Si₂O₁₄ (BNFS) whose spin configuration forms the triangular-helical chiral magnetic order [Fig. 2(b)]. This system differs from the aforementioned chiral (anti)ferromagnets in that its chiral magnetic structure arises from the competition of symmetric exchange interactions⁹ instead of the DMI. We demonstrate that even without the DMI, the spin wave dispersion along the c-axis of BNFS becomes non-reciprocal and exhibits asymmetric energy shift when an external magnetic field is applied along the c-axis (parallel to k). In contrast, in the chiral antiferromagnet α-Cu₂V₂O₇ whose spin configuration relies crucially on the DMI, the asymmetric energy shift appears when an external magnetic field is applied and is perpendicular to k. We remark that in view of Ref.², which examines possible non-reciprocity based on symmetry considerations (or symmetry-operational equivalence), the non-reciprocal spin waves in BNFS (our work) and α-Cu₂V₂O₇ correspond to two distinct cases (Figs. 3(c) and 3(d) of Ref.², respectively), where the non-reciprocity is allowed by symmetries. In addition, we mention that there is a distinct difference between non-reciprocal spin wave propagation and non-reciprocal light propagation⁹,¹⁰; Light wave propagates with polarization, but spin wave has no polarization. Due to this difference, the physics of the spin wave non-reciprocity
Figure 2(a) shows that magnetic Fe angles within the same plane. But along the c axis, and the other is the triangular chirality ($\epsilon_T = 1$), which represents the winding direction of spin within each triangle. The neutron scattering study on BNFS reports $\epsilon_H = 1$ and $\epsilon_T = -1$.

According to Ref. 13, physical phenomena in BNFS can be described through symmetric Heisenberg exchange interactions without invoking the DMI. We thus neglect the DMI and consider symmetric exchange interaction only. For the structure depicted in Fig. 2(a), the intra-layer spin exchange interaction $H_{\text{intra}}^{(l)}$ within the layer l, and the inter-layer spin exchange interaction $H_{\text{inter}}^{(l)-(l+1)}$ between the layer l and $l + 1$ are as follows:

$$H_{\text{intra}}^{(l)} = \sum_{\alpha \neq \alpha', \beta} \left[ J_1 S_{l, \alpha} \cdot S_{l, \alpha'} + J_2 S_{l, \alpha} \cdot S_{l, \beta} \right], \quad (1)$$

$$H_{\text{inter}}^{(l)-(l+1)} = \sum_{\alpha, \beta \neq \beta'} \left[ J_4 S_{l, \alpha} \cdot S_{l+1, \alpha} + J_5 S_{l, \alpha} \cdot S_{l+1, \beta} + J_3 S_{l, \alpha} \cdot S_{l+1, \beta} \right], \quad (2)$$

where $J_1, J_2, \cdots, J_5$ are exchange parameters [Fig. 2(a)], and spin operator $S_{l, \alpha}$ represents the magnetic moment at l, $\alpha$ site. $l$ represents plane index, and $\alpha, \beta$ represent a position vector within the ab plane. Strictly speaking, $\alpha$ denotes $R + a_j$, where $a_j$ is a lattice vector within each Fe$^{3+}$ triangle, $R$ is a reference point. Also $\beta$ denotes $R + R_k + a_j$, where $R_k$ ($k = 1, 2, 3$) is an inter-triangle position vector. When an external magnetic field $B$ is applied, there appears the Zeeman interaction,

$$H_{\text{Zeeman}}^{(l)} = J_0 \sum_{\alpha} S_{l, \alpha} \cdot B. \quad (3)$$

where $J_0 = 2\mu_B$, and $\mu_B$ is the Bohr magneton. We assume that $B$ is along the c axis, that is, $B = B \hat{z}$, where $\hat{z}$ denotes the c axis direction [Fig. 2(b)]. Then, the total spin Hamiltonian can be obtained by adding up $l$,

$$H_{\text{total}} = \sum_l \left( H_{\text{intra}}^{(l)} + H_{\text{inter}}^{(l)-(l+1)} + H_{\text{Zeeman}}^{(l)} \right). \quad (4)$$

To facilitate subsequent analysis, it is convenient to introduce local coordinate systems whose coordinate axes vary from atomic site to site and are aligned along the local equilibrium spin directions. At the site l, $\alpha$, the unit vectors for the local coordinate system are

$$\hat{X}'_{l, \alpha} = \hat{x} \cos \phi_{l, \alpha} \cos \theta_{l, \alpha} + \hat{y} \cos \phi_{l, \alpha} \sin \theta_{l, \alpha},$$

$$\hat{Y}'_{l, \alpha} = -\hat{x} \sin \phi_{l, \alpha} + \hat{y} \cos \phi_{l, \alpha},$$

$$\hat{Z}'_{l, \alpha} = -\hat{x} \sin \phi_{l, \alpha} \cos \theta_{l, \alpha} - \hat{y} \sin \phi_{l, \alpha} \sin \theta_{l, \alpha} + \hat{z} \cos \theta_{l, \alpha},$$

where $\phi_{l, \alpha}, \theta_{l, \alpha}$ are, respectively, azimuthal and polar angles of the equilibrium spin direction at l, $\alpha$. We assume that $\theta_{l, \alpha}$ is independent of l and $\alpha$, that is, $\theta_{l, \alpha} = \theta$. We also assume that $\phi_{l, R+R_k+a_j}$ follows the helical pattern, that is, $\phi_{l, R+R_k+a_j} = \epsilon_T (\tau l + \epsilon_T) 2\pi / 3)\| 11$, where $\tau$ is a helical period along c axis. Values of $\theta$ and $\tau$ will be determined below by minimizing the equilibrium energy.

Applying the Holstein-Primakoff transformation, the spin operators along local coordinate axes are written as $S_{l, \alpha}' = S - B_{l, \alpha}^1 b_{l, \alpha}'$, $S_{l, \alpha}' = \sqrt{S/2} (B_{l, \alpha}^1 + b_{l, \alpha})$, $S_{l, \alpha}'^\dagger = \sqrt{S/2} (B_{l, \alpha}^1 + b_{l, \alpha})^\dagger$, and

$$S_{l, \alpha}'^\dagger = \sqrt{S/2} (B_{l, \alpha}^1 + b_{l, \alpha})^\dagger.$$
The first order term is

$$\mathcal{H}_{\text{total}} = \mathcal{H}_{\text{total}}^{(0)} + \mathcal{H}_{\text{total}}^{(1)} + \mathcal{H}_{\text{total}}^{(2)} + \mathcal{O}(3\text{rd order terms in } b, b^\dagger),$$

(6)

where $\mathcal{H}_{\text{total}}^{(n)}$ denotes $n$-th order terms. First of all, $\mathcal{H}_{\text{total}}^{(0)}$ reads

$$\mathcal{H}_{\text{total}}^{(0)} = \sum_k \left[ S^2(J_1 + 2J_2) \left( -\frac{1}{2} \cos^2 \theta + \sin^2 \theta \right) + 2 \sum_{\nu=0}^2 S^2J_{3+\nu}\left( \cos(\tau + \epsilon T \varphi_\nu) \cos^2 \theta + \sin^2 \theta \right) \right],$$

(7)

where $\varphi_0 = 2\pi/3$, $\varphi_1 = 0$, $\varphi_2 = 4\pi/3$. Since $\mathcal{H}_{\text{total}}^{(0)}$ amounts to the equilibrium energy, we minimize it with respect to $\theta$ and $\tau$. From $\partial \mathcal{H}_{\text{total}}^{(0)}/\partial \theta = 0$ and $\partial \mathcal{H}_{\text{total}}^{(0)}/\partial \tau = 0$, one obtains

$$2 \sum_{\nu=0}^2 J_{3+\nu} \sin(\tau + \epsilon T \varphi_\nu) = 0,$$

(8)

$$\sin \theta = -\frac{J_0B_z/S}{2(J_1 + 2J_2) + 2 \sum_{\nu=0}^2 J_{3+\nu}\left( -\cos(\tau + \epsilon T \varphi_\nu) + 1 \right)},$$

(9)

$\tau$ in Eq. (8) agrees with the helical period along the $c$ axis reported in Ref.13. On the other hand, Eq. (9) indicates that $\theta = 0$ when $B_z = 0$ and thus equilibrium spins lie within the $ab$ plane. When $B_z$ is applied, however, the equilibrium spins deviate from the $ab$ plane [Fig. 2(b)]. Using the measured parameters in Table II13 that will be used henceforth, we predict $\theta = 2.2^\circ$ at $B_z = 6.8$ T. This value is similar to the value reported in Ref.16. Also, the first order term is

$$\mathcal{H}_{\text{total}}^{(1)} = i \cos \theta \sqrt{\frac{S^3}{2}} \sum_k \sin \theta \left( 3(J_1 + 2J_2) + 2 \sum_{\nu=0}^2 J_{3+\nu}\left( -\cos(\tau + \epsilon T \varphi_\nu) + 1 \right) \right) + \frac{J_0B_z}{S} \left( b_{k,j}^\dagger - b_{k,j} \right).$$

(10)

This square bracket becomes zero for the value of $\theta$ that minimizes $\mathcal{H}_{\text{total}}^{(0)}$.

The next order term is $\mathcal{H}_{\text{total}}^{(2)}$. In order to analyze $\mathcal{H}_{\text{total}}^{(2)}$, it is convenient to introduce the Fourier transformed bosonic operator $b_{k,j}$, which is related to $b_{l,R+a,j}$ as follows

$$b_{l,R+a,j} = \frac{1}{\sqrt{N}} \sum_k \exp[i(k \cdot (l a_z \hat{z} + R))] b_{k,j},$$

(11)

where $N$ is a number of layers, and $a_z$ is the inter-layer spacing. In terms of the Fourier transformed bosonic operators, one obtains

$$\mathcal{H}_{\text{total}}^{(2)} = \sum_{k,j,j^\prime} \frac{c_{k,j}^\dagger \left( \alpha_{k,j,j^\prime} \beta_{k,j,j^\prime}^\dagger \right) c_{k,j^\prime}}{\delta_{k,j,j^\prime}},$$

(12)

where $\alpha_{k,j} = \left( b_{k,j} b_{k,j}^\dagger \right)^T$ and $\alpha_{k,j,j^\prime}$, $\beta_{k,j,j^\prime}$ are given in Appendix. With these values, it is straightforward to verify that Eq. (12) is hermitian. The equation of motion approach17,18 is a commonly used technique to obtain eigenvalues of the bosonic quadratic Hamiltonian [Eq. (12)]. To utilize this approach, we transform Eq. (12) into a standard form of boson quadratic Hamiltonian in Ref.18 by extending the boson basis $c_k$ into $d_{k,j} = \left( b_{k,j} b_{k,j}^\dagger b_{k,j}^\dagger b_{k,j}^\dagger \right)^T$. Then, Eq. (12) can be rewritten as follows:

$$\mathcal{H}_{\text{total}}^{(2)} = \frac{1}{2} \sum_k \sum_{j,j^\prime} d_{k,j} \left( A_{k,j,j^\prime}^T B_{k,j,j^\prime} A_{k,j,j^\prime} B_{k,j,j^\prime}^T \right) d_{k,j^\prime},$$

(13)

which is in the standard form of the boson quadratic Hamiltonian. Here, $6 \times 6$ matrices $A_k$, $B_k$ are

$$A_k = \left( \alpha_k^T 0 0 \delta_k \right), \quad B_k = \left( 0 \beta_k^T 0 \right),$$

(14)

where $\alpha_k$ and $\beta_k$ are $3 \times 3$ matrices with $\alpha_{k,j,j^\prime}$ and $\beta_{k,j,j^\prime}$ as their matrix elements, respectively. Then, one obtains the following associated matrix18 $M_k$

$$M_k = \begin{pmatrix} A_k & -B_k^\dagger \\ B_k & -A_k^\dagger \end{pmatrix} = \begin{pmatrix} \alpha_k & 0 & 0 \\ 0 & \alpha_k^T & 0 \\ 0 & 0 & \alpha_k^T \end{pmatrix}.$$

(15)

The eigenvalues of this $12 \times 12$ matrix $M_k$ consists of $E(k)$, $E(-k)$, $-E(k)$, and $-E(-k)$ for three spin wave branches.

III. RESULT AND DISCUSSION

We investigate the spin wave dispersion of BNFS by numerical calculation. The spin wave dispersion as a function of $L = k a_z/2\pi$, and $B = B_z \hat{z}$ is shown in Fig. 3(a). Here we assume that $k = (0,0,k)$. The different colors indicate different values of $B_z$. For $B = 0$ (black solid lines), there are three branches of spin wave excitations. Each of them becomes gapless at $L = 0$ ($c$-mode), $L = +\tau/2\pi$ ($w_1$-mode), and $L = -\tau/2\pi$ ($w_2$-mode), where $\tau/2\pi \approx 0.14$. Note that for $B = 0$, $E_{w_1}(k) = E_{w_2}(-k)$ and $E_{w_1}(k) = E_{w_2}(-k)$. Thus the dispersion relations are symmetric. As $B_z$ increases from 0, $E_{c}(k)$ remains essentially unchanged, but $E_{w_1}(k)$ and
$E_{w_2}(\mathbf{k})$ are progressively modified. For both $E_{w_1}(\mathbf{k})$ and $E_{w_2}(\mathbf{k})$, gapless points disappear and are replaced by quadratic dispersions. Note that the resulting energy gap is significantly bigger for the $w_1$-mode than for the $w_2$-mode. Thus the relation $E_{w_1}(\mathbf{k}) = E_{w_2}(-\mathbf{k})$ becomes broken and the dispersions for the $w_1$- and $w_2$-modes become asymmetric, acquiring the non-reciprocity. In addition, we remark that the sign of the non-reciprocity [Fig. 3(a)] can be reversed when the sign of the magnetic chirality $\epsilon_{TFH}$ is reversed, although the magnetic chirality reversal is difficult to realize in experiments because this reversal requires energy costs.

To understand this result, it is useful to consider the nature of spin wave “vibrations”. Figure 3(b) shows schematically the spin vibration patterns for the $c$-mode excitation (left), and the $w_1$-$w_2$-mode excitation within a Fe$^{1-3}$ triangle. In the $c$-mode, all spins vibrate without altering their net in-plane component, hence $\sum_{j=1}^{3} \delta S_j^{z,R+a} = 0$ within the triangle. Here, $\parallel$ denotes in-plane components. For this mode, the system has the rotation symmetry around the $c$ axis regardless of whether $B_z$ is applied. Thus $E_c(\mathbf{k} = \mathbf{0})$ for arbitrary $B_z$, since this particular mode amounts to the Goldstone mode for the symmetry. In case of $w_1$-$w_2$-modes, on the other hand, the spins vibrate without altering their net $c$ component, hence $\sum_{j=1}^{3} \delta S_j^{z,R+a} = 0$ within the triangle. Here, $\perp$ denotes out-of-plane components.

The blue plane in Fig. 3(b), which is defined by connecting the end points of the vibrating spins, shows the out-of-plane vibration clearly. In the $w_1$- and $w_2$-modes, the normal vector to the blue plane precess around the $c$ axis anticlockwise and clockwise, respectively. If $\mathbf{B} = \mathbf{0}$, the anticlockwise and clockwise precessions share the same vibration frequencies, resulting in $E_{w_1}(\mathbf{k}) = E_{w_2}(-\mathbf{k})$ [black solid line in Fig. 3(a)]. For $B_z \neq 0$, on the other hand, the field itself tends to induce the precession of the normal vector in one particular direction, thus introducing the difference between the anticlockwise and clockwise precession frequencies. This explains the non-reciprocity, $E_{w_1}(\mathbf{k}) \neq E_{w_2}(-\mathbf{k})$. Figure 3(d) shows that the difference $E_{w_1}(\mathbf{k} = \tau/a_2 \hat{z}) - E_{w_2}(-\mathbf{k} = -\tau/a_2 \hat{z})$ between the energy gaps of the $w_1$- and $w_2$-modes increases with increasing $B_z$. For $B_z = 6.8$ T, the energy gaps for the $w_1$- and $w_2$ modes are 0.36 meV, and 0.07 meV, respectively. Then, one obtains the gap size difference of 0.29 meV.

In some respect, our result is similar to Ref.\textsuperscript{7} that reports the non-reciprocal spin waves in a chiral antiferromagnet $\alpha$-Cu$_2$V$_3$O$_7$ with B. However, there are distinctions between BNFS and $\alpha$-Cu$_2$V$_3$O$_7$ systems. In BNFS, the chiral antiferromagnetic order arises from competition between symmetric exchange interactions whereas in $\alpha$-Cu$_2$V$_3$O$_7$, it arises from the DMI. Another important difference is the spin wave propagation direction. In BNFS, spin waves propagating parallel to the external magnetic field are non-reciprocal where in $\alpha$-Cu$_2$V$_2$O$_7$, spin waves propagating perpendicular to the external magnetic field are non-reciprocal.

Let us investigate the structure of Eq. (15) more closely to better understand reciprocal (non-reciprocal) spin wave without (with) $B_z$. First of all, we remark that eigenvalues of $M_{\mathbf{k}}$ are real even though $M_{\mathbf{k}}$ is not hermitian. Given this information, we can understand the reciprocity (non-reciprocity) as follows. The characteristic equation $\det [M_\mathbf{k} - x \mathbf{I}] = 0$ is rewritten as

$$\begin{vmatrix} \delta_k - x \mathbf{I} & -\beta^\dagger_k \\ -\alpha_k^{\dagger} - x \mathbf{I} & \beta_k \end{vmatrix} \times \begin{vmatrix} \delta_k - x \mathbf{I} & -\beta^\dagger_k \\ -\alpha_k^{\dagger} - x \mathbf{I} & \beta_k \end{vmatrix} = 0, \quad (16)$$

where $\mathbf{I}$ is $3 \times 3$ identity matrix, $x$ is a real eigenvalue of $M_{\mathbf{k}}$.
energy gap opening

19

such small DMI can generate observable effects such as
dorders of magnitude smaller than

J

DMI may also exist in BNFS since it is noncentrosymmetry
non-reciprocity from symmetric exchange interactions
(energy gap difference between the

w

not significantly affect the degree of the non-reciprocity
dispersion in this case can becomes non-reciprocal. The ro-
eration that links the two spin waves. Hence the spin wave
parallel to each other, there does not exist any symmetry op-
direction links the two spin waves
is parallel to

B

cause the combined symmetry operation of the time-reversal
and the second (for

B

(17)

Then comparing the first and second determinants, and
recalling that

z

M

k,

M

k

and the first (second) determinant in the left hand
side is for

−k

(+k). When

B

= 0, matrix elements for

k

and

k

are related (see Appendix) as follows:

\[
\begin{pmatrix}
\delta_k - \beta_k^* \\
\beta_k - \alpha_k
\end{pmatrix}^* = \begin{pmatrix}
\delta_k - \beta_k^* \\
\beta_k - \alpha_k
\end{pmatrix}.
\]

(17)

Therefore, the low energy excitation spin wave spectrum
may become non-reciprocal, \(E(+k) \neq E(-k)\).

So far our analysis of BNFS has focused on the
non-reciprocity from symmetric exchange interactions
[Eq. (1) and (2)] and neglected the DMI. To be strict, the
DMI may also exist in BNFS since it is noncentrosymmetric.
According to\textsuperscript{19}, the energy scale of the DMI is three
orders of magnitude smaller than

J

1, and about two orders of magnitude smaller than

J

2, \(J_3, J_4, J_5\). Although such small DMI can generate observable effects such as
energy gap opening\textsuperscript{19} to the

w

1- and

w

2-modes, it can not significantly affect the degree of the non-reciprocity
(energy gap difference between the

w

1- and

w

2-modes)
simply because the DMI energy scale is much smaller than those of
\(J\).

Our examination of the non-reciprocal spin waves in
BNFS focused on the case when

k

and

B

are parallel to the
c-axis [Fig. 2(b)]. Figure 4 shows possible other con-
figurations of

k

and

B

, which are not examined in this paper.
For the three cases with

k

and

B

perpendicular to each other [depicted in Figs. 4(a), 4(b), and 4(c)], one can show by using simple symmetry argument\textsuperscript{2} that the non-
reciprocity is not possible. First, for the case in Fig. 4(a),
the spin waves propagating along the

(+a)-axis and

(−a)-axis

B

symmetry argument applies to the spin waves propagating along

(+b)-axis and

(−b)-axis. However this argument
does not apply to the spin waves propagating along the
direction which deviates from the
\(\pm\alpha\) or \(\pm\beta\) directions,
even though the propagation direction lies within the

ab

plane. For example, when

k

is parallel to

(\(a + 2b\))/\(\sqrt{3}\),
the spin wave dispersion [Fig. 5(a)] shows much weaker
non-reciprocity than Fig. 3(a). Here an external mag-
netic field is along the
c-axis. In this case, each spin wave
branch is asymmetric. In order to show asymmetric spin
wave clearly, we provide the energy difference between

+L

x

and

−L

x

as function of

L

x

[Fig. 5(b)]. Here the non-
reciprocal effect increases with increasing spin wave
propagation vector. However, this effect is strongly sup-
pressed in the vicinity of

L

x

= 0. Thus, we expect it to be
weak at least for the long wave length spin wave. Second,
when

B

is applied in the

ab

plane [Figs. 4(b) and 4(c)],
the actual calculation of spin wave excitations become
complicated since the ground state spin configuration is
not known for this case. But the symmetry analysis may
be still possible if a net magnetization in BNFS is par-
allel to

B.

Then, spin waves in Figs. 4(b) and 4(c) are
reciprocal, since spin waves with

±k

can be connected by a
rotation around the

B

direction. We remark that this
symmetry argument becomes exact when

B

is parallel to

±\alpha-axis or \(±\beta\)-axis. Finally, we consider the case in
Fig. 4(d), where

k

and

B

are parallel to each other, and these
are within the

ab

plane. In this case, two spin waves with

k || B

and

−k || B

cannot be connected by any opera-
tions, that is, this spin wave can become non-reciprocal. Therefore, the spin wave in BNFS may be non-reciprocal (reciprocal) when \( k \) and \( B \) are parallel (perpendicular) to each other.

IV. SUMMARY

In summary, we have shown theoretically that the spin wave in a chiral antiferromagnet BNFS becomes non-reciprocal when an external magnetic field is applied along the \( c \)-axis. Unlike other chiral ferromagnets or chiral antiferromagnets, where DMI is crucial for the non-reciprocity, the non-reciprocity in BNFS, which has very small DMI, can arise purely from the competition between symmetric exchange interactions and an external magnetic field. Thus our work demonstrates that the DMI is not crucial for non-reciprocal spin waves. Our work also widens material choice for non-reciprocal spin waves.

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Appendix: Components of Eq. (12)

\( \alpha_k, \beta_k \) are presented as below:

\[
\alpha_k = \left[ -\frac{1}{2} J_0 B \sin \theta + \frac{1}{2} (J_1 + J_2) S (1 - 3 \sin^2 \theta) \right. \\
- \sum_{\nu=0}^{2} J_{4+\nu} S \left\{ \cos(\tau + \epsilon_T \varphi_{\nu}) \cos^2 \theta + \sin^2 \theta \right\} I \\
+ J_1 S \left[ \frac{1}{8} (1 - 3 \sin^2 \theta) \Sigma_{3x} - \frac{\sqrt{3}}{4} \epsilon_H \epsilon_T \sin \theta \Sigma_{3y} \right] \\
+ J_2 S \left[ \frac{1}{8} (1 - 3 \sin^2 \theta) \mathcal{A}_{3k}^+ + \frac{\sqrt{3}}{4} \epsilon_H \epsilon_T \sin \theta \mathcal{A}_{3k}^- \right] \\
+ \frac{S}{4} (1 + \sin^2 \theta) \mathcal{B}_{3k}^+ + \frac{S}{4} \cos^2 \theta \mathcal{C}_{3k} - \frac{S}{2} \epsilon_H \sin \theta \mathcal{B}_{3k}, \tag{A.1}
\]

\[
\beta_k = -\frac{3}{8} S (1 - \sin^2 \theta) (J_1 \Sigma_{3x} + J_2 \mathcal{A}_{3k}^+) \\
+ \frac{1}{4} S (1 - \sin^2 \theta) (\mathcal{B}_{3k}^+ - \mathcal{C}_{3k}) S, \tag{A.2}
\]

where

\[
\Sigma_{3x} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad \Sigma_{3y} = \begin{pmatrix} 0 & -i & i \\ i & 0 & -i \\ -i & i & 0 \end{pmatrix}, \tag{A.3}
\]

\[
(A_{3k}^+)_l m = \hat{e}_l \times \hat{e}_m \left( e^{i k R_l} + e^{-i k R_m} \right), \tag{A.4}
\]

\[
(A_{3k}^-)_l m = \hat{e}_l \times \hat{e}_m (\hat{e}_n \cdot e^{i k R_l} + e^{-i k R_m}), \tag{A.5}
\]

\[
(B_{3k}^+)_l m = 2 \delta_{lm} J'_3 \cos k_z a_z + \hat{e}_l \times \hat{e}_m \left( J'_+ \cos k_z a_z \right. \\
+ i (\hat{e}_l \times \hat{e}_m) \cdot \hat{e}_n J'_- \sin k_z a_z, \tag{A.6}
\]

\[
(B_{3k}^-)_l m = 2 \delta_{lm} J''_3 \sin k_z a_z + \hat{e}_l \times \hat{e}_m \left( J''_+ \sin k_z a_z \right. \\
- i (\hat{e}_l \times \hat{e}_m) \cdot \hat{e}_n J''_- \cos k_z a_z, \tag{A.7}
\]

\[
(C_{3k})_l m = 2 \delta_{lm} J_4 \cos k_z a_z + \hat{e}_l \times \hat{e}_m \left( J_+ \cos k_z a_z \right. \\
+ i (\hat{e}_l \times \hat{e}_m) \cdot \hat{e}_n J_- \sin k_z a_z. \tag{A.8}
\]

Here, \( n \neq l, m \), and \( l, m = 1, 2, 3 \). \( \hat{e}_l \) is a unit vector. 
\( J_\pm = J_5 \pm i J_6, J'_\pm = J_{4+\nu} \cos (\tau + \epsilon_T \varphi_{\nu}), \) and \( J''_\pm = J_{4+\nu} \sin (\tau + \epsilon_T \varphi_{\nu}). \) Since we assume that \( k = (0, 0, k) \), Eqs. (A.4), (A.5) can be written as \( A_{3k}^+ = 2 \Sigma_{3x}, A_{3k}^- = 2 \Sigma_{3y}. \)

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8 T. Moriya, Phys. Rev. 120, 91 (1960).