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Electron heating induced by microwave photo-excitation in the GaAs/AlGaAs 2D electron system

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We examined the influence of microwave radiation on both the amplitude of Shubnikov-de Haas (SdH) oscillations and the null field longitudinal magnetoresistance at liquid helium temperatures, in GaAs/AlGaAs Hall bar devices. Microwave radiation over the frequency range $25 \leq f \leq 50$ GHz with source power $0 \leq P \leq 4$ mW served to photo-excite the high mobility ($\simeq 10^7$ cm²/Vs) 2D electron system (2DES) as magnetoresistance traces were obtained as a function of the microwave power P and temperature T . Lineshape study of SdH oscillations has been carried out over the span $2.3 < \omega_c/\omega \leq 5.2$, where $\omega_c = eB/m^*$, $\omega = 2\pi f$, B is the magnetic field, m^* is the effective mass and f is the microwave frequency. Here, fits of the SdH lineshape served to determine the electron temperature (T_e) as a function of P and T .

Theory has proposed that, in the $\omega_c/\omega \geq 1$ regime, both the electron temperature and radiation energy absorption rate (S_p) exhibit relatively small response, while in the $\omega_c/\omega \leq 1$ regime, both T_e and S_p are enhanced and exhibit oscillatory behavior. We compare the experimental results with these theoretical predictions, and comment upon relative role of electron heating in the microwave photo-excited high mobility 2DES.

I. INTRODUCTION

Photo-excited transport has been a focus area in the study of transport at large filling factors in the high mobility 2D electron system (2DES) over the past decade.¹⁻⁷⁰ Of interest here are the microwave-induced zero-resistance^{1,2} states, which arise from the associated microwave induced magnetoresistance oscillations.¹⁻³ Such zero-resistance states are believed, in one interpretation, to represent a photo-induced “absence of backscattering” condition of the high-mobility 2DES.⁵⁴ Thus, under microwave photoexcitation, at low temperatures, the magnetoresistance in a high quality 2DES shows large, periodic-in- B^{-1} , magnetoresistance oscillations,¹⁻³ where the extrema are “1/4-cycle-shifted” with respect to cyclotron resonance and cyclotron resonance harmonics.^{1,5} At lower temperatures, moderate microwave intensity transforms the oscillatory minima into zero-resistance states. Interesting experimental features examined by experiment include the 1/4 cycle phase shift^{1,5,8}, the non-linear increase in the amplitude of the radiation-induced oscillations with the microwave power^{19,55}, observed correlations between the magnetoresistance oscillations and microwave reflection^{8,25} from the 2DES, polarization sensitivity,^{22,28,29,31} and magnetoresistive response under bichromatic excitation.^{10,42} Observed oscillatory phenomena in the photoexcited two-dimensional electron systems have been considered by the displacement model,^{43,46,48,49} the microwave driven electron orbital model,^{44,55} the inelastic model,⁵¹ and a memory effect theory.⁶⁹

A subject of experimental interest is the study of possible electron heating under photoexcitation, as the theory has predicted the possibility of variable, magnetic

field dependent, microwave-induced electron heating in the 2DES in the large-filling-factor, low-magnetic-field limit.^{50,52} Theory has examined the electron heating by microwave photo-excitation in a balance-equation scheme that takes into account photon-assisted electron transitions as well as radiation-induced change of the electron distribution for high-mobility two-dimensional systems. The results suggest that the electron temperature is a function of the magnetic field, the microwave intensity, and frequency, and it is determined by the balance between the energy absorption from the radiation field and the energy dissipation to the lattice through electron-phonon scattering.

This work indicates that microwave photoexcitation produces a small discernible increase in the electron temperature both at null magnetic field and at finite magnetic fields, in the examined range, in the GaAs/AlGaAs 2DES. The heating effect appears greater at null field in comparison to the examined magnetic field interval, in qualitative agreement with theory.⁵²

II. EXPERIMENT AND RESULTS

Lock-in-based electrical measurements were performed on photolithographically fabricated Hall bar from molecular beam epitaxy grown high mobility GaAs / AlGaAs heterojunctions. The Hall bar sample was mounted at the lower end of a long cylindrical waveguide sample holder which is inserted into a variable temperature insert (VTI), within the bore of the superconducting magnet. The sample temperatures was controlled and varied by pumping on and reducing the vapor pressure of the liquid helium within the VTI insert over the range $1.45 \leq T \leq 4.2$ K. The sample was immersed in liquid

helium for all the reported measurements. The Hall bar sample inside the VTI was pre-illuminated with red light to obtain a high mobility state. At 1.47 K, sample electron density (n_e) was $2.4 \times 10^{11} \text{ cm}^{-2}$ and mobility (μ_e) was $1.2 \times 10^7 \text{ cm}^2/\text{Vs}$.

The microwave radiation over the $25 \leq f \leq 50 \text{ GHz}$ band was generated with a commercially available microwave synthesizer and the specimen was illuminated with linearly polarized microwaves for the photo-excited transport measurements. The polarization of incident microwave was parallel to the long direction of the Hall bar device. The diagonal resistance (R_{xx}) is reported here at temperatures where microwave induced magnetoresistance oscillations and SdH oscillations are relatively strong, for a number of microwave source powers $0 \leq P \leq 4 \text{ mW}$. Here, we present the data at $f = 48.5 \text{ GHz}$, which are representative of the observations over the above mentioned frequency band.

Microwave radiation-induced magnetoresistance oscillations are observable for $B \leq 0.2 \text{ T}$ at $T = 1.47 \text{ K}$ in Fig. 1. It can be clearly seen in Fig. 1(a) that the amplitude of radiation-induced magnetoresistance oscillations grow nonlinearly with the microwave source power^{18,20}. Strong SdH oscillations are also observable under both the dark and microwave-irradiated conditions for $B \geq 0.2 \text{ T}$. In this study, we examine two observable characteristics in the R_{xx} vs. B traces with the parametric variation of P . These characteristics are (1) the R_{xx} at zero magnetic field is up-shifted to higher resistance values with the increment of the microwave source power as shown in inset of Fig.1 (the magnetic field dependence of R_{xx} or the line shape observed here will be a topic of study elsewhere), and 2) the amplitude of SdH oscillations decays with the increment of microwave source power at finite magnetic fields. We attribute these microwave-induced variations to a heating effect from the incident microwaves on the Hall bar device since the effect of increased microwave excitation on R_{xx} is analogous to the effect of increasing the sample temperature, in the absence of photoexcitation. Our aim here is to convince of electron heating for these two cases and extract the change of the electron temperature with photo-excitation.

We begin by considering the up-shifting of the R_{xx} traces with the incident microwave source power observable in the inset of Fig. 1(a). Since the null magnetic field R_{xx} increment appeared as a result of microwave photo-excitation, which could also plausibly produce electron heating, we examined the dependence of the null magnetic field R_{xx} on the temperature. Thus, the resistance, R_{xx} at $B = 0$ was measured as a function of the bath temperature, termed here the lattice temperature, T_L , under dark conditions, i.e., without microwave photo-excitation. Although we examined the temperature dependence of R_{xx} at $B = 0$ over a wide T -interval, the temperature interval of interest for comparing with the up-shift in R_{xx} observed under microwave excitation turned out to be only $1.47 \leq T \leq 1.92 \text{ K}$ and, therefore, Fig 2(a)

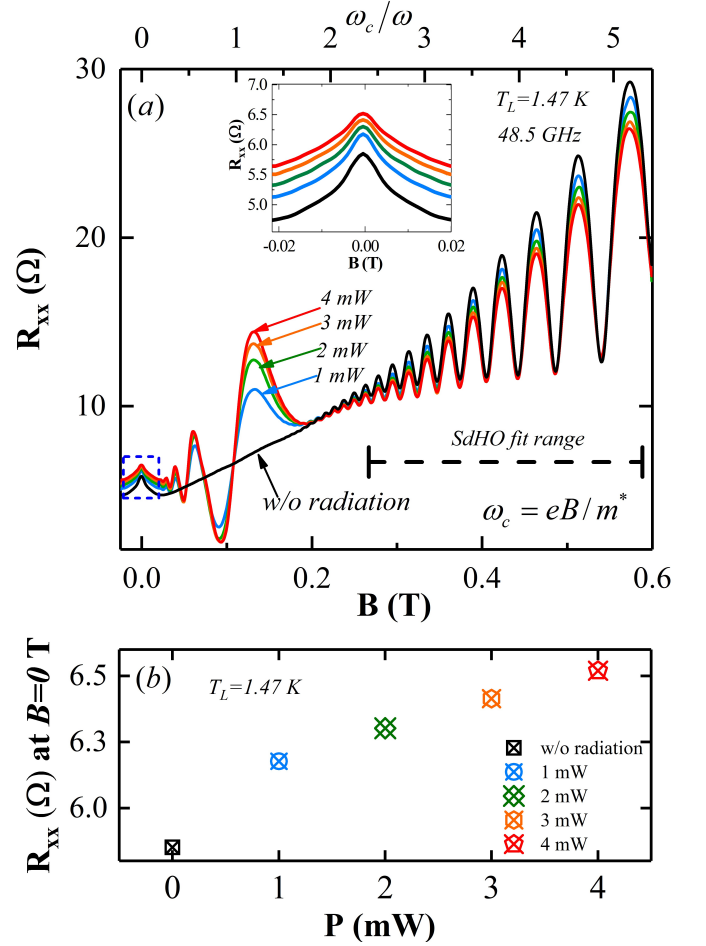


FIG. 1. (a) The diagonal resistance R_{xx} is shown vs. the magnetic field B , at 1.47 K for 48.5 GHz microwave excitation at the indicated microwave source-power. Here, R_{xx} exhibits radiation-induced magnetoresistance oscillations and Shubnikov-de Haas (SdH) oscillations. The observed SdH oscillations have been fit over the range of B that is indicated by the dashed line, which corresponds to $2.3 < \omega_c/\omega \leq 5.2$, where ω_c is the cyclotron frequency and $\omega = 2\pi f$, with $f = 48.5 \text{ GHz}$. The inset, which shows the R_{xx} over the range $-0.02 \leq B \leq 0.02 \text{ T}$, indicates that the R_{xx} is up-shifted with the increment of the microwave source-power. (b) The R_{xx} at null magnetic field extracted from the inset of plot (a) is plotted as a function of the microwave power.

shows the dark R_{xx} vs. B traces as Fig. 2(b) exhibits the dark R_{xx} at $B = 0$ vs the temperature T_L , for this temperature interval. The Fig. 2 (a) and 2(b) show that the zero-magnetic-field R_{xx} increases approximately linearly with T_L , as indicated by the red solid line in Fig. 2(b), which is a least squares linear fit. The linear fit gives the diagonal resistance as a function of the lattice temperature via the parametric equation: $R_{xx} = R_{xx}(T_L)$, i.e., $R_{xx}[\Omega] = 1.54[\Omega/\text{K}]T_L[\text{K}] + 3.58[\Omega]$. This equation for the dark diagonal resistance can be inverted to obtain $T = T(R_{xx})$, i.e., $T[\text{K}] = (R_{xx}[\Omega] - 3.58[\Omega])/1.54[\Omega/\text{K}]$. Such inversion is carried out so that the zero-magnetic-

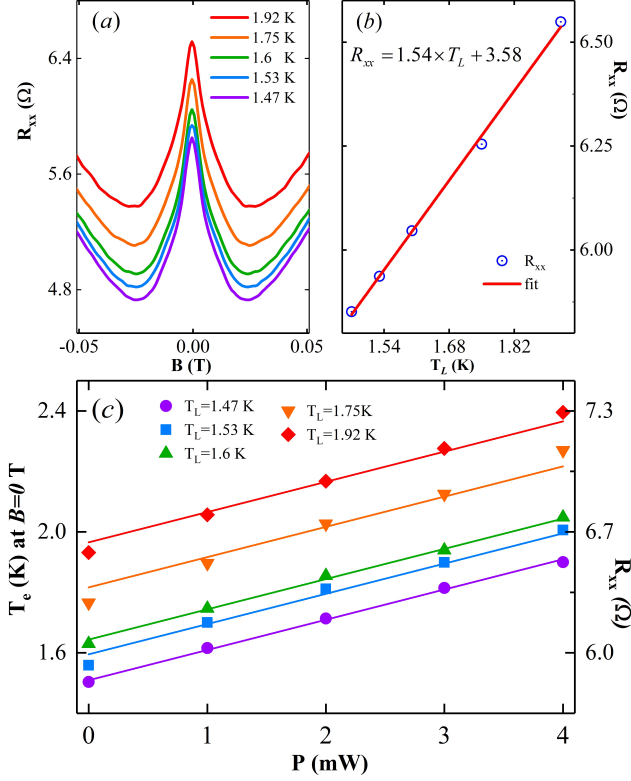


FIG. 2. (Color online) (a) R_{xx} is shown at various temperatures for $-0.05 \leq B \leq 0.05$ T in the absence of microwave excitation. (b) In this panel, the zero-field diagonal resistance $R_{xx}(B = 0T)$ is plotted vs. the bath or lattice temperature, T_L . This panel shows that $R_{xx}(B = 0T)$ increases linearly with T_L . Since, at constant lattice temperature, $R_{xx}(B = 0T)$ shows an upshift with the incident microwave power, P , see Fig. 1, the change in $R_{xx}(B = 0T)$ with P can serve to determine an electron temperature, T_e , which can differ from T_L under photoexcitation. (c) This panel shows the power dependence of zero field R_{xx} (right ordinate) for $0 \leq P \leq 4$ mW at different lattice temperatures. The enclosed solid symbols represent data points. The parametric conversion of R_{xx} to T_e is indicated on the left ordinate. In Fig. 2(b) and 2(c), the lines are a guide to the eye.

field R_{xx} can serve as a temperature gauge, even in the presence of microwave excitation. In the microwave irradiated condition, however, the diagonal resistance will serve as a gauge of the electron temperature, T_e , not the lattice temperature, T_L , since the electron system can potentially be decoupled from the lattice/bath in the presence of such drive.

Next, measurements as in Fig. 1(a), not shown here, of R_{xx} vs. B at various microwave powers, P , were carried out at each T_L . The right ordinate of Fig. 2(c) summarizes the zero-magnetic field results by plotting the null field R_{xx} vs. P (the abscissa) at a set of five T_L . The left-side ordinates in Fig. 2(c) shows the corresponding electron temperature scale obtained, as mentioned above, at

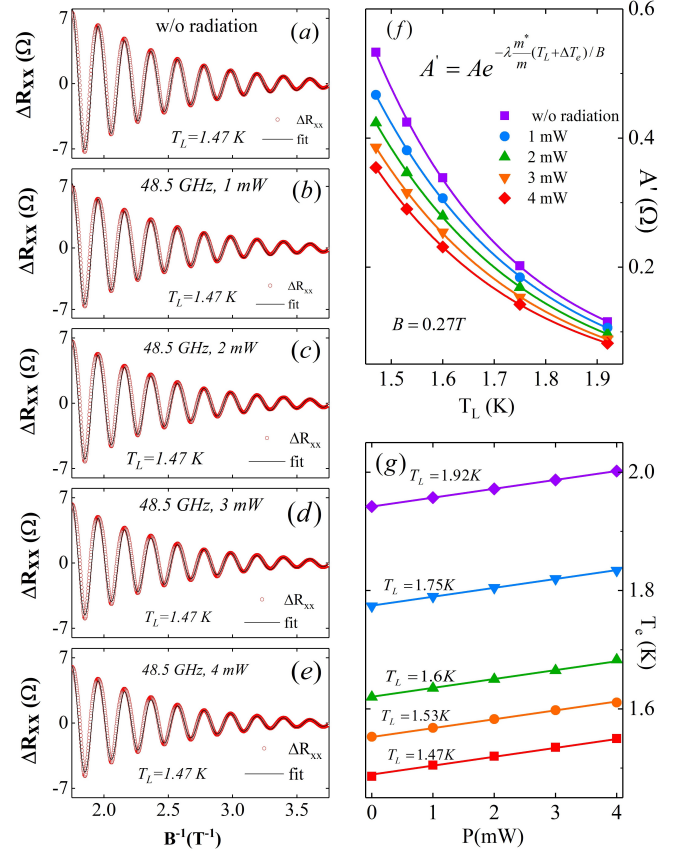


FIG. 3. (a)-(e) Shubnikov de Haas oscillations, which are periodic in B^{-1} , are observable in ΔR_{xx} that has been plotted vs B^{-1} over the span $1.7 \leq B^{-1} \leq 3.75$ T^{-1} at various microwave power levels. The red symbols represent the ΔR_{xx} data while the black lines represent numerical fits to $\Delta R_{xx} = -A e^{\alpha/B} \cos(2\pi F/B)$. The fits serve to extract the electron temperature, T_e , at each P and each T_L . (f) The SdH oscillation amplitude ($A' = A e^{-\lambda(T_L + \Delta T_e)(m^*/m_e)}$) extracted from ΔR_{xx} , see text, is decaying exponentially with the temperature. (g) The electron temperatures T_e extracted from fits of the SdH oscillations are shown as a function of the microwave source power at different lattice temperatures T_L . Solid symbols represent the extracted electron temperature and solid lines are linear fits.

each lattice temperature. The results of Fig. 2(c) suggest that $\Delta T_e / \Delta P \approx 0.1$ K/mW of source microwave power at null magnetic field. In Fig. 2(c) the lines shown are simply guides to the eye. Since the ordinate scale in Fig. 1(b) is much expanded compared to the scale on the right ordinate of Fig. 2(c), the non-linearity observable in Fig. 1(b) is not so evident in Fig. 2(c).

In the second part of our study, we examined the influence of microwave-excitation on the SdH oscillations. For this purpose, we examined the SdH lineshape over the span $2.3 < \omega_c/\omega \leq 5.2$, which is indicated in Fig. 1(a). In order to facilitate lineshape fits, a monotonic background R_{xx} term was subtracted from the raw magnetoresistance data to obtain the oscillatory ΔR_{xx} term.

This term was then plotted vs. the inverse magnetic field for different microwave source powers as shown in Fig.3(a)-3(e) for $T_L = 1.47K$. Here, the red-color open-circles represent data.

As mentioned previously, the amplitude of the SdH oscillations decays with the increment of microwave source power. To extract the amplitude of the SdH oscillations, a standard nonlinear least square fit (NLSF) was performed on ΔR_{xx} data with an exponentially damped sinusoidal function, i.e., $\Delta R_{xx} = -A e^{\alpha/B} \text{Cos}(2\pi F/B)$, where A is the amplitude, F is the SdH frequency.^{15,71-73} Since, the parameter F is insensitive to the incident radiation at a constant lattice temperature, the F was fixed to a constant value. As an example, the fit of the ΔR_{xx} data at $1.47 K$ for various P spanning $0 \leq P \leq 4 mW$ are shown in Fig.3(a)-3(e) as solid black lines. These panels propose good agreements between data and fit. For the sake of illustration, the temperature dependence of the SdH oscillation amplitude $A' = Ae^{\alpha/B}$ are exhibited vs. the microwave power in Fig.3(f). It can be clearly seen that A' decays exponentially with increasing lattice temperature as well as increasing microwave source power.

Next, we extracted the electron temperature, T_e , at finite magnetic fields from the SdH oscillation amplitude. To extract this T_e from the SdH amplitude in these data, the damping constant, α , in the fitting model, $\Delta R_{xx} = -A e^{\alpha/B} \text{Cos}(2\pi F/B)$, became $\alpha = -\lambda(T_L + T_D + \Delta T_e)(m^*/m_e)$, here $\lambda = 2\pi^2 k_B m_e / e$, m^*/m_e is effective electron mass ratio, T_L is the lattice temperature, ΔT_e is the electron temperature increment with respect to the lattice, and T_D is the Dingle temperature. In practice, it turned out that T_D is very small compared to T_L . Thus, $T_L + T_D \approx T_L$. And, the fit function became $\Delta R_{xx} = A' \text{Cos}(2\pi F/B) = -A e^{-\lambda(T_L + \Delta T_e)(m^*/m_e)/B} \text{Cos}(2\pi F/B)$.

In fig. 4, we compare the microwave power variation of electron temperature, $T_e = T_L + \Delta T_e$, at zero-magnetic field extracted from the power variation of the zero-field R_{xx} , with the power-variation of the electron temperature at finite magnetic fields extracted from the SdH oscillations. Fig.4(a) - 4(e) show that the electron temperatures increased with the incident microwave source powers, both in the absence of a magnetic field and also at small finite magnetic fields. However, the increase in the electron temperature at zero magnetic field under microwave excitation is $\approx 6x$ higher than the extracted electron temperature increase over the SdH oscillation region. Fig. 4(a) - 4(e) indicate that $\Delta T_e / \Delta P \approx 0.1 K/mW$ at null magnetic field and $\Delta T_e / \Delta P \approx 0.015 K/mW$ at finite magnetic fields.

III. DISCUSSION

Theory⁵² suggests that stationary microwave photoexcitation can heat highly mobile electrons in the 2D system. The absorbed energy from the radiation field is transferred to the lattice by electron-phonon scatter-

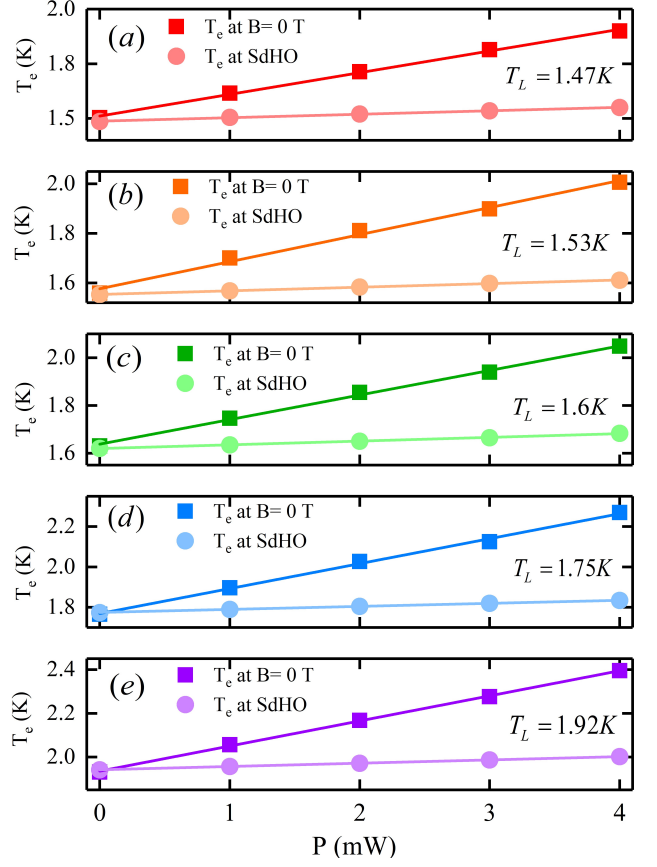


FIG. 4. (a)-(e): The extracted electron temperature T_e is plotted vs. the microwave power, P , in the vicinity of $B = 0T$ (square symbols) and in the regime of SdH oscillations, i.e., $2.3 < \omega_c/\omega \leq 5.2$, (disk symbols) for various bath/lattice temperatures, T_L . The figure shows electron heating under microwave excitation in both regimes. However, the heating appears more pronounced in the vicinity of $B = 0T$, in agreement with theoretical prediction.

ing through bulk longitudinal acoustic (LA), transverse acoustic (TA), and longitudinal optical (LO) phonons. The electronic energy absorption rate is strongly magnetic field dependent and also shows oscillations with periodicity in the inverse magnetic field reflecting the periodicity of both microwave induced magneto-resistance oscillations and Shubnikov-de Haas oscillations.⁵² It turns out that the electron temperature, T_e , reflects the features observed in the energy absorption rate.⁵² Simulations for typical experimental parameters suggest that at lower magnetic fields, i.e., $\omega_c/\omega \leq 1.4$, energy absorption occurs via inter Landau level transitions, leading to a significantly enhanced T_e , with $T_e \approx 10K$ for electric fields at the specimen of the order of $3.5V/cm$ at $f = 50 GHz$ for a specimen mobility $\mu = 2.5 \times 10^7 cm^2/Vs$.⁵² As the magnetic field increases, the inter Landau level transitions weaken and the absorbed energy decreases rapidly, leading to an electron temperature that is only slightly higher than the lattice temperature at $\omega_c/\omega \approx 3$.⁵² It is

worth pointing out that analytic expressions for the absolute absorption coefficient as a function of the magnetic field, in this B -field range of interest where the absorption changes rapidly with the magnetic field, do not occur in the literature, to our knowledge.

The experimental results reported here are in qualitative agreement with these theoretical expectations. The observed increase of the T_e is much greater at null magnetic field than in the regime of Shubnikov de Haas oscillations. Indeed, the rate of increase of the T_e with P at null field is $\approx 6\times$ greater than at finite magnetic fields. On the other hand, the maximum observed increase here in the electron temperature is $\approx 0.4K$ at null magnetic field with a bath temperature of $\approx 1.47K$ and a source power of $4mW$, see Fig. 4, while theory suggests that the increase can be as large as $10K - 20K$ at $T_L \approx 1K$. We attribute this difference to two factors: (a) our specimen mobility is smaller in comparison to the value used in the theoretical calculation. A larger mobility provides for a longer elastic mean free path and phase coherence length. It appears plausible that over these longer length scales/times, more energy can be absorbed from the radiation field, leading to enhanced heating. That is, a lower mobility in our specimens in comparison to the theoretical calculation might lead to a reduced heating effect in experiment, especially at null magnetic field. (b) In our setup, there is significant microwave attenuation between the source and the specimen. Thus, the power level at the specimen can be reduced by $\approx 10 - 16dB$ or more compared to the power level at the source, which is the power level specified in experiment. This implies that the electric fields at the specimen are much lower, perhaps 10-40 times lower, than the values utilized for the theoretical simulation. As a consequence, the presented curves in the theory⁵² overestimate the heating effect. Thus, the results generally support the conclusion that there is electronic heating, although it is not as large as expected from theory.

IV. SUMMARY AND CONCLUSION

This study indicates that microwave photoexcitation produces a small discernable increase in the electron temperature both at null magnetic field and at finite magnetic fields in the GaAs/AlGaAs 2D electron system. The heating effect appears greater at null field in comparison to the examined finite field interval, in line with theoretical predictions. However, the increase in the electron temperature in the zero-field limit is smaller than theoretical predictions mostly because theory assumes no microwave attenuation between source and sample while, in our experiment, the attenuation appears substantial.⁵²

V. ACKNOWLEDGEMENTS

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VI. APPENDIX

At $f = 50GHz$, the microwave wavelength is $6mm$ while the sample width is $0.2mm$. Thus, the sample size is relatively small compared to the wavelength. The sample then is like a short antenna (size small compared to wavelength) and it becomes a poor antenna both for transmission and reception. This feature could also be influential in the observed reduced heating in our experiments. If the thickness of the 2DES is estimated to be $10 - 20nm$ when the skin depth at these frequencies is in the micro-meter range, i.e., sample thickness is small compared to skin depth, then the microwave intensity is uniform through the thickness of the specimen and the microwave radiation passes through the specimen. According to microwave absorption theory, to efficiently heat a material, it helps to have an absorption mechanism at the frequency of interest. In the GaAs/AlGaAs 2DES, at low magnetic fields, i.e., $B < B_f = 2\pi f m^*/e$, the microwave frequency can span an integer number of Landau levels and, therefore, there is potentially a mechanism available. While at $B > B_f$, this mechanism is missing. This provides one avenue for qualitatively understanding the observed reduced heating in the regime of SdH oscillations.

For these experiments, an approximately 6-foot-long cylindrical wave guide with $11mm$ inner diameter, was used to transmit the microwave radiation to the sample. The inner cross section of the wave guide is $A = \pi r^2 = 0.95cm^2$. Assume that attenuation due to the wave guide is $10dBm$ and the sample area is $\approx 1mm^2$. Then, for a source power of $1mW$, the power incident on the sample is $0.1(0.01/0.95) \approx 1\mu W$. It is understood that the absorption coefficient in the low magnetic field regime is dependent on the microwave frequency, the magnetic field, and sample quality. To our knowledge, an analytic expression has thus far not been presented for this regime for the absolute absorption coefficient as a function of the magnetic field. For the sake of estimation, we utilize an absorption coefficient of 0.1 at $B = 0$ to suggest an absorbed power of $\approx 0.1\mu W$ at $B = 0$. If, as suggested by the theory,⁷⁴ the absorption coefficient is reduced by a factor of 20 to $B = 0.4T$ at $50GHz$, then the absorbed power would be $\approx 0.005\mu W$ at $B = 0.4T$.

To estimate the expected reduction in heating due to the expected change in the σ_{xx} with the magnetic field, we utilize the expression $P = \sigma_{xx}(B)E^2$, where P is the Joule heating power, and E is the microwave electric field. For simplicity, we consider the case where E

is constant. Then, the change in P with the magnetic field would reflect the change in σ_{xx} . A simple calculation indicates that σ_{xx} changes by nearly five orders-of-magnitude between $B = 0$ and $B = 0.5T$, while the

experimental results indicate that the change $\Delta T_e/\Delta P$ differs at most by a factor of six, $\Delta T_e/\Delta P = 0.1K/mW$ at $B = 0T$ and $\Delta T_e/\Delta P = 0.015K/mW$ at around $B = 0.4T$, between the null field and finite field cases.

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