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Tuning a random field mechanism in a frustrated magnet

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We study the influence of spinless impurities on a frustrated magnet featuring a spin-density wave (stripe) phase by means of Monte Carlo simulations. We demonstrate that the interplay between the impurities and an order parameter that breaks a real-space symmetry triggers the emergence of a random-field mechanism which destroys the stripe-ordered phase. Importantly, the strength of the emerging random fields can be tuned by the repulsion between the impurity atoms; they vanish for perfect anticorrelations between neighboring impurities. This provides a novel way of controlling the phase diagram of a many-particle system. In addition, we also investigate the effects of the impurities on the character of the phase transitions between the stripe-ordered, ferromagnetic, and paramagnetic phases.

I. INTRODUCTION

Low-temperature phases of many-particle systems usually break one or several of the symmetries of the interactions spontaneously. This is well described by the concept of order parameters (OPs), quantities that vanish in the symmetric phase but are nonzero (and nonunique) in the symmetry-broken phase (see, e.g., Ref. 1). A simple example of an OP is the total magnetization which measures the degree to which the spin rotation symmetry is broken. In recent years, lots of attention has been attracted by phases that spontaneously break real-space symmetries in addition to spin, phase, or gauge symmetries, for example by rendering the x and y directions in a crystal inequivalent. Such phases include the charge-density wave or stripe phases in cuprate superconductors, the Ising-nematic phases in the iron pnictides, the Ising-nematic phases in the iron pnictides, and stripe phases as a function of temperature, two distinct symmetry-broken phases appear. For $g = |J_2|/J_1 < 1/2$, the system enters a ferromagnetic (FM) low-temperature phase that breaks the $Z_2$ Ising symmetry but none of the real-space symmetries. For $g > 1/2$, in contrast, the low-temperature phase displays a stripe-like spin order that breaks not only the Ising symmetry but also the $Z_2$ rotation symmetry of the square lattice. The Hamiltonian (1) is therefore particularly well suited for our study of impurity effects on different OPs as it allows us to contrast an OP that does not break any real-space symmetries (the ferromagnetic OP) with one that does (the stripe OP).

In this paper, we focus on the impact of random disorder on a phase that breaks a real space symmetry. To do so we turn our attention to a frustrated Ising model on a square lattice having ferromagnetic nearest-neighbor interactions and antiferromagnetic next-nearest-neighbor interactions. The disorder takes the form of spinless impurities or vacancies that dilute the magnetic lattice. The resulting Hamiltonian reads

$$ H = -J_1 \sum_{\langle ij \rangle} \rho_i \rho_j S_i S_j - J_2 \sum_{\langle ij \rangle} \rho_i \rho_j S_i S_j $$  \hspace{1cm} (1)

where the $S_i = \pm 1$ are classical Ising variables, while $J_1 > 0$ and $J_2 < 0$ are the nearest-neighbor and next-nearest-neighbor interactions, respectively. The $\rho_i$ are quenched random variables that take the values 0 (vacancy) with probability $p$ and 1 (site occupied by spin) with probability $1 - p$. We consider both uncorrelated randomness for which the $\rho_i$ are statistically independent and anticorrelated randomness for which repulsion between the impurities suppresses the simultaneous occupation of two nearest-neighbor sites by impurities.

In the absence of vacancies ($p = 0$), the phase diagram and the phase transitions of this system are well-understood (see Fig. 1 as well as Refs. 12–15 and references therein). At high temperatures, it features a conventional paramagnetic phase. Upon lowering the temperature, two distinct symmetry-broken phases appear. For $g = |J_2|/J_1 < 1/2$, the system enters a ferromagnetic (FM) low-temperature phase that breaks the $Z_2$ Ising symmetry but none of the real-space symmetries. For $g > 1/2$, in contrast, the low-temperature phase displays a stripe-like spin order that breaks not only the Ising symmetry but also the $Z_2$ rotation symmetry of the square lattice. The Hamiltonian (1) is therefore particularly well suited for our study of impurity effects on different OPs as it allows us to contrast an OP that does not break any real-space symmetries (the ferromagnetic OP) with one that does (the stripe OP).

The direct phase transition between the ferromagnetic and stripe phases as a function of $g$ is of first order. Extensive numerical simulations13,14 have also established that the transition from the stripe phase to the paramagnetic phase is first order for $g < g^* \approx 0.67$. The line of first order transition terminates at $g^*$ and gives rise to critical behavior in the Ashkin-Teller16,17 universality class. Finally, the transition from the ferromagnetic phase to the high temperature paramagnetic phase is known to lie in the Ising universality class.

To analyze how the site dilution influences the frustrated Ising model (1), we perform extensive Monte Carlo simulations. We also determine the exact ground states of small plaquettes to understand the disorder effects microscopically. Our results are illustrated by the phase diagram shown in Fig. 1 and can be summarized as fol-
focuses on the effect for the stripe-to-paramagnet transition. We are dedicated to a brief description of dilution on the temperature range we have been able to simulate. We have been able to identify such a phase beyond doubt in the temperature range. However, we have not been able to identify such a phase beyond doubt in the temperature range we have been able to simulate. The ferromagnetic low-temperature phase survives, albeit with a depressed critical temperature compared to the ferromagnetic phase. The fate of the various phase transitions under the influence of disorder is discussed in Sec. VI. We conclude with Sec. VII. Some technical details of our calculations are relegated to the Appendices.

In the rest of this paper, we discuss our simulations, explain the tunable random-field mechanism, and put our results into a broader perspective. The paper is organized in the following manner: Sec. II is dedicated to a description of the primary observables that we calculate via the Monte Carlo simulations. It also gives details of the system parameters. Sec. III focuses on the effect of impurities on the stripe phase. In Sec. IV, we describe the emergent random field mechanism that destabilizes the stripe phase. In this section, we also explain how the emergent random field mechanism can be tuned by introducing anticoalignations into the disorder distribution. Sec. V briefly describes the impact of dilution on the ferromagnetic phase. The fate of the various phase transitions under the influence of disorder is discussed in Sec. VI. We conclude with Sec. VII. Some technical details of our calculations are relegated to the Appendices.

II. MONTE CARLO SIMULATIONS

We employ standard single-spin flip Metropolis simulations of the Hamiltonian (1). We study square lattices of linear sizes between $L = 8$ to $80$, averaging the results over 500 to 1000 disorder configurations. Details of the simulation algorithm and parameter values can be found in Appendix A. The primary observables are the OPs for the ferromagnetic and stripe phases. The two-component stripe OP $\psi \equiv (\psi_x, \psi_y)$ is defined as

$$\psi_x = \frac{1}{L^2} \sum_i \rho_i S_i (-1)^{x_i}, \quad \psi_y = \frac{1}{L^2} \sum_i \rho_i S_i (-1)^{y_i},$$

where $(x_i, y_i)$ are the coordinates of site $i$ whereas the ferromagnetic OP, i.e., the magnetization, reads

$$m = \frac{1}{L^2} \sum_i \rho_i S_i.$$

We also analyze the corresponding susceptibilities $\chi_S = L^2 \left[ \langle \psi^2 \rangle - \langle |\psi| \rangle^2 \right] / T$ and $\chi_F = L^2 \left[ \langle m^2 \rangle - \langle |m| \rangle^2 \right] / T$ as well as the Binder cumulants

$$U_S = 2 \left( 1 - \frac{1}{2} \left[ \langle |\psi| \rangle^2 \right] \right), \quad U_F = 3 \left( 1 - \frac{1}{3} \left[ \langle m^2 \rangle \right] \right).$$

Here, $[\cdot \cdot \cdot]$ denotes the average over disorder realizations whereas $\langle \cdot \cdot \cdot \rangle$ indicates the usual thermodynamic (Monte Carlo) average. The Binder cumulants are normalized such that they take the limiting values $U_{F,S} \to 1$ deep in the corresponding ordered phases and $U_{F,S} \to 0$ deep in the disordered phase. The crossing of the Binder cumulant curves for different system sizes yields the location of the phase transition. The Binder cumulant also allows us to determine the order of the transition: For a continuous transition, it is a monotonic function of temperature. At a first-order transition, in contrast, the Binder cumulant shows a minimum that becomes more pronounced.

FIG. 1. Phase diagram of $J_1$-$J_2$ Hamiltonian (1) for both uncorrelated and anti-correlated site dilution at an impurity concentration of $p = 1/8$ compared to the phase diagram of the undiluted system. For uncorrelated impurities, the emergent random field mechanism destroys the stripe-ordered phase. In contrast, this phase survives the introduction of anti-correlated disorder. The first-order phase boundaries of the clean model are depicted by dashed lines, the clean ferromagnet-to-paramagnet transition is marked by open squares; and the open triangles represent the Ashkin-Teller critical behavior for the stripe-to-paramagnet transition. The phase boundaries of the diluted system are marked by filled symbols.
and is caused by the ex-

ception of nearest-neighbor sites by impurities is forbid-

den). The frustration parameter is $g = 1$, dilution $p = 1/4$ and several system sizes. Data for un-
correlated vacancies are shown in panels (a) and (c) whereas panels (b) and (d) show results for anti-correlated vacancies.

with increasing system size\textsuperscript{25} and is caused by the ex-

istence of multiple peaks in the OP distribution. This non-
monotonic temperature dependence can serve as an indicator of a first-order transition.

III. STRIPE PHASE

We now turn to the central question of this manuscript, the fate of the stripe phase upon introducing spinless impurities. Figure 2 depicts the stripe OP and the associated susceptibility for dilution $p = 1/4$, contrasting the cases of uncorrelated impurities and perfectly anticorrelated impurities (where the simultaneous occupation of nearest-neighbor sites by impurities is forbid-
den). The frustration parameter is $g = |J_2|/J_1 = 1$ for which the undiluted system features a stripe-ordered low-
temperature phase. Figure 2(a) shows that the stripe order-parameter at low temperatures decreases with increasing system size for the case of uncorrelated impurities. In this case, the stripe susceptibility shown in Fig. 2(c) develops a pronounced secondary peak at low temperatures. As suggested in Ref. 20, these observations indicate the absence of long-range stripe order in the thermodynamic limit. In contrast, in the case of anti-
correlated disorder, the stripe order-parameter saturates at a size-independent nonzero value at low temperatures, as shown in Fig. 2(b). The corresponding stripe sus-
cceptibility, shown in Fig. 2(d), displays the conventional behavior associated with a continuous phase transition. These observations suggest that the stripe order survives in the case of anticorrelated impurities.

To provide further evidence, we compare the behavior of the stripe Binder cumulants $U_S(T)$ for uncor-
related and anticorrelated impurities. Figure 3 depicts the Binder cumulants for the same parameters used above, viz., $p = 1/4$ and $g = 1$. Focussing on Fig. 3(a), we see that for uncorrelated impurities, the Binder cumulant vs. temperature curves for different system sizes do not cross. With increasing size, the Binder cumulant shifts to smaller and smaller values, i.e., towards the disordered phase, confirming the absence of long-range stripe order for the case of uncorrelated impurities. The fate of the stripe phase can be further illustrated via the nematic OP $\eta = \psi^2_x - \psi^2_y$ which measures the local preference for vertical vs. horizontal stripes. The color plot in Fig. 3(c) shows the local nematic OP for each $2 \times 2$ plaquette, clearly demonstrating competing domains of horizontal and vertical stripes, (see Appendix C).

In contrast, for the case of anticorrelated impurities, the stripe Binder cumulants for different system sizes do cross as evidenced in Fig. 3(b). This indicates the ex-
istence of a phase transitions and thus the survival of the stripe-ordered low-temperature phase. Estimates of the transition temperature $T_c$ and the correlation length exponent $\nu$ can be obtained from finite-size scaling \textsuperscript{26,27} (for details, see Appendix B). Figure 3(d) shows the scaling collapse of the Binder cumulant in terms of the scaled variable $(T - T_c)L^{1/\nu}$, with $T_c = 1.1729(5)$ and $\nu = 1.26(3)$. The data collapse is very good: the un-
underlying least-square fit has a reduced $\chi^2 = 0.97$\textsuperscript{28}. Be-
cause our systems are only moderately large, the value of $\nu$ should be understood as an effective exponent rather than the true asymptotic exponent.
fields for the nematic OP

The fact that a local preference for vertical or horizontal stripes only appears if two impurities occupy two nearest-neighbor sites can be used to tune the strength of the emerging random field mechanism. If the probability for nearest-neighbor pairs of impurities is reduced, for example because of a repulsive interaction between the impurities, fewer random fields appear in the system. In the limit of perfectly anticorrelated impurities where such pairs are completely forbidden, the random-field mechanism is switched off. This explains why our simulations showed that the stripe-ordered phase survives for anticorrelated impurities.

V. FERROMAGNETIC PHASE

In contrast to the stripe OP, the total magnetization does not break a real-space symmetry. Therefore, spinless impurities do not create random fields coupling to the ferromagnetic order. Instead, they act as much more benign random-mass or random-$T_c$ disorder. Consequently, the ferromagnetic phase survives in the presence of impurities, be they uncorrelated or perfectly anticorrelated. However, the Curie temperature $T_c$ is reduced compared to the undiluted system, as is shown in the phase diagram in Fig. 1.

VI. PHASE TRANSITIONS

We now turn to the phase transitions between the paramagnetic, ferromagnetic, and stripe phases. As explained in the Introduction, the transitions of the undiluted system are well understood. There is a direct first-order phase transition between the ferromagnetic and stripe phases at low temperatures. The transition between the ferromagnetic and paramagnetic phases is continuous and belongs to the 2D Ising universality class. Extensive numerical simulations have also established that the transition from the stripe phase to the paramagnetic phase is of first order for $0.5 < g < g^* \approx 0.67$. The line of first-order transition terminates at $g^*$ and gives rise to critical behavior that belongs to the Ashkin-Teller universality class.

In the presence of anticorrelated disorder, the ferromagnetic and stripe phases both survive. According to Landau, phase transitions between two ordered phases that break different symmetries must be of first order. However, the Aizenman-Wehr theorem forbids first-order transitions in two-dimensional disordered systems. This implies that the ferromagnetic and stripe phases must be separated by an intermediate phase. This could simply be the paramagnetic phase extending all the way to zero temperature, or there could be a spin glass (SG) phase at low temperatures and $g$ close to 0.5. Unequivocally resolving the phases in this parameter region is beyond the scope of this paper. Interestingly, a similar glass phase was recently found via a Monte Carlo based anal-
and 34 by introducing quenched Wehr theorem to a continuous one, in agreement with the Aizenman-demonstrating that the first-order transition is rounded 0 purities on the phases of a frustrated Ising magnet. As discussed in Appendix B.

In contrast, in the diluted system with \( p \) different system sizes. (a) undiluted system, \( p = 0 \), \( g = 0.56 \); (b) anticorrelated impurities, \( p = 1/8 \), \( g = 0.56 \); (c) anticorrelated impurities, \( p = 1/8 \), \( g = 0.60 \); (d) minimum value \( U^* \) as a function of inverse system size.

VIII. ACKNOWLEDGEMENTS

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APPENDIX A: DETAILS OF MONTE-CARLO PROCEDURE

Even though cluster-flip methods such as the Swendsen-Wang\textsuperscript{37} and Wolff\textsuperscript{38} algorithms can be used to study random systems such as the disordered ferromagnetic Ising model\textsuperscript{39}, in the case of the disordered $J_1$-$J_2$ model, the efficiency of cluster algorithms is severely curtailed due to the presence of the frustrated interactions\textsuperscript{40}. Thus, we are restricted to the classical single-spin-flip Metropolis algorithm\textsuperscript{23} to perform our simulations. We study square lattices of linear size $L = 8$ to 80. Each Monte Carlo simulation consists of an equilibration period of $10^6$ Monte Carlo sweeps (a sweep corresponds to one attempted spin flip per lattice site), followed by a measurement period of another $10^6$ sweeps, with measurements taken after each sweep. To improve the equilibration performance, we adopt a cooling procedure. We start the simulations at high temperatures and lower the temperature in small steps, using the final state of the higher temperature simulation as the initial condition for the next lower temperature.

We investigate frustration parameters $g = |J_2|/J_1$ between 0.1 and 1.0. To study the influence of disorder, a total number of $N_{\text{imp}} = pL^2$ spinless impurity sites are introduced into the lattice. These impurities are either completely uncorrelated or they are perfectly anticorrelated such that the simultaneous occupation of nearest-neighbor sites by impurities is forbidden. We simulate dilutions of $p = 1/8$ and $1/4$. We expect, however, that the qualitative results hold for all values of $p$ that are sufficiently small such that lattice percolation effects do not play a role. All observables are averaged over a large number of impurity configurations. Specifically, we use 1000 configurations for the smaller system sizes, $L = 8$ to 32, and 500 configurations for the larger sizes. Using comparatively short Monte Carlo runs for a large number of disorder configurations improves the overall numerical efficiency (see Ref. 41 and references therein).

APPENDIX B: FINITE-SIZE SCALING ANALYSIS

In this section we describe the methodology adopted to extract the critical temperature $T_c$ and the critical exponents from the Monte Carlo data of the site-diluted $J_1$-$J_2$ model. The analysis is based on finite-size scaling\textsuperscript{26,27} of the stripe and ferromagnetic Binder cumulants $U_S$ and $U_F$ as well as the corresponding susceptibilities $\chi_S$ and $\chi_F$.

![Scaling plots](image)

**FIG. 6.** Scaling collapse of the ferromagnetic binder cumulant $U_F$ [panels (a) and (b)] and the scaled ferromagnetic susceptibility $\chi_F L^{-7/4}$ [panels (c) and (d)] for uncorrelated impurities at $p = 1/8$ and frustration parameters $g = 0$ and 0.3.

**II: Ferromagnetic transition**

We start by analyzing the ferromagnetic Binder cumulant $U_F$ defined as

$$U_F = \frac{3}{2} \left( 1 - \frac{1}{3} \frac{\langle m^4 \rangle}{\langle m^2 \rangle^2} \right).$$

(5)

According to finite-size scaling, the Binder cumulant values for different system sizes $L$ and temperatures $T$ should collapse onto a single master curve when plotted as a function of the scaling variable $x = (T - T_c) L^{1/\nu}$ where $\nu$ is the correlation length critical exponent. Moreover, as the Binder cumulant is a dimensionless quantity, its value right at $T_c$ should be size-independent, implying a Taylor expansion

$$U_{F,S}(T,L) = f(x) = a_0 + a_1 x + a_2 x^2 + \ldots$$

(6)

sufficiently close to the critical point. Figures 6(a) and (b) show examples of such scaling plots for uncorrelated impurities at concentration $p = 1/8$ and frustration parameters $g = 0$ and 0.3, respectively. The values of $T_c$ and $\nu$ are extracted from fits of the $U_F$ data to the expansion (6) truncated after the quadratic term. The quality of the fit can be estimated from the reduced sum of squared errors (per degree of freedom) $\bar{\chi}^2$ defined as

$$\bar{\chi}^2 = \frac{1}{N - M} \sum_{i=1}^{N} \frac{[U_{F,i} - f(x_i)]^2}{\sigma_i^2},$$

(7)

Here, $N$ is the number of data points, $M$ is the number of fit-parameters, and $\sigma_i^2$ is the (Monte Carlo) variance of the value $U_{F,i}$. The fits are considered of good quality.
and references therein). Re- 

demonstrate, however, that the 

ducing values are summarized in Table 

The resulting values are summarized in Table II. They

TABLE I. Critical temperatures \(T_c\), effective correlation length exponents \(\nu\), and reduced error sums \(\bar{\chi}^2\) obtained from the scaling analysis of the ferromagnetic Binder cumulant \(U_F\). Results are shown for various values of the frustration parameter \(g\) and dilution \(p = 1/8\) for both uncorrelated and anti-

When \(\bar{\chi}^2 \lesssim 1\). Results of this analysis for both uncorrelated and ant

How do our results for the correlation length exponent \(\nu\) compare to theoretical predictions? The ferromag

Further evidence is provided by the ferromagnetic suscepti

The stripe-ordered to paramagnetic transition can be nalyzed along the same lines as the ferromagnetic tran

BII: Stripe transition

The stripe-ordered to paramagnetic transition can be analyzed along the same lines as the ferromagnetic tran

agree well with those from the analysis of the Binder cumulant. (For the effective exponent \(\nu\), the deviations are within one standard deviation: for \(T_c\) they are within two standard deviations.)

The results are summarized in Table III. In the undiluted, clean system, the stripe to paramagnetic transition is either of first-

FIG. 7. Scaling plots of the stripe cumulant \(U_S\) [panels (a) and (b)] and the stripe susceptibility \(\chi_S\) [panels (c) and (d)] for anticorrelated impurities of concentration of \(p = 1/8\) and frustration parameters \(g = 0.75\) and \(g = 1\).
order (for $g < g^* \approx 0.67$) or belongs to the Ashkin-Teller universality class (for $g > g^*$)\textsuperscript{12-15}. We have shown in the main text that the first-order transition is rounded to a continuous one in the presence of anticoherent impurities, as is expected from the Aizenman-Wehr theorem\textsuperscript{21}. Our results in Table III show that the critical exponent $\nu$ of the diluted system is close to the clean Ising value of unity for all studied values of $g$. In particular, $\nu$ does not vary systematically with $g$ as would be expected for the clean Ashkin-Teller universality class. The effects of disorder on the Ashkin-Teller universality class were studied by Murthy\textsuperscript{44} and Cardy\textsuperscript{45} via a renormalization group analysis that predicted clean Ising critical behavior with universal logarithmic corrections just as in the disordered Ising model. This was recently confirmed by large-scale simulations\textsuperscript{41}. As in the case of the ferromagnetic transition above, the system sizes in our present work are too small to extract logarithmic corrections. However, the effective $\nu$ values in Table III are close to the clean two-dimensional Ising value of unity. We conclude that our results are consistent with the critical behavior of the stripe transition belonging to the disordered Ising universality class.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
$g$ & $T_c$ & $\nu$ & $\bar{\chi}^2$ & $T_c$ & $\nu$ & $\bar{\chi}^2$ \\
\hline
0.60 & 0.707669(9) & 0.93(2) & 0.92 & & & \\
0.70 & 0.9872(1) & 0.99(3) & 1.10 & 0.9838(1) & 1.04(2) & 1.57 \\
0.75 & 1.1020(1) & 1.00(2) & 1.09 & 1.1029(1) & 1.04(2) & 1.34 \\
1 & 1.6361(1) & 1.05(2) & 1.09 & 1.6362(1) & 1.07(1) & 1.01 \\
\hline
\end{tabular}
\caption{Critical temperatures $T_c$, effective correlation length exponents $\nu$, and reduced error sums $\bar{\chi}^2$ obtained from the scaling analysis of the stripe Binder cumulant $U_S$ and the stripe susceptibility $\chi_S$. Results are shown for various values of the frustration parameter $g$ and dilution $p = 1/8$ for perfectly anticoherent impurities.}
\end{table}

\section*{APPENDIX C: DOMAINS}

FIG. 8. Local nematic order parameter $\eta_i$ for each $2 \times 2$ plaquette of a single system of $100 \times 100$ sites for $T = 0.55$, $g = 1$, and uncorrelated impurities of concentration $p = 1/8$ (left panel) and $p = 1/4$ (right panel).

As discussed in the main text, spinless impurities in the $J_1$-$J_2$ Hamiltonian create random fields for the nematic order parameter $\eta = \psi_x^2 - \psi_y^2$ which measures the local preference for vertical vs. horizontal stripes. These random fields destroy the long-range stripe order via domain formation. In order to image these domains, we define a local version of the nematic order parameter via $\eta_i = (\psi_{ix}^2 - \psi_{iy}^2)$ where $\psi_{ix}$ and $\psi_{iy}$ are formed by averaging $\psi_{ix} = \rho_i S_i(-1)^{x_i}$, and $\psi_{iy} = \rho_i S_i(-1)^{y_i}$ over $2 \times 2$ plaquette number $i$.

Figure 8 illustrates the emergence of the domains in a system of linear size $L = 100$ at $g = 1$ and $T = 0.55$ as we increase the concentration $p$ of impurities. For impurity concentration $p = 1/8$, the local order parameter fluctuates only slightly, i.e., the entire system belongs to a single domain. For the more disordered sample, $p = 1/4$, the characteristic domain size has fallen below the system size. The figure now shows random-field induced domain walls percolating throughout the sample, thus leading to the destruction of long-range stripe order.

18. The Ashkin-Teller model consists of two identical Ising
models coupled locally via their energy densities. Its transition from the low-temperature ordered phase to the high-temperature paramagnetic phase is continuous but with non-universal continuously varying exponents. This is precisely the scenario found in the clean frustrated Ising model for $g > g^*$. 


J. Cardy (Ed), Finite-size scaling (North Holland, Amsterdam, 1988).

We denote the reduced sum of squared errors of the fit (per degree of freedom) by $\bar{\chi}^2$ to distinguish it from the susceptibility $\chi$.


Strictly, this arguments holds at $T = 0$ only. At finite temperatures, random fields may be generated via entropic effects. However, at low temperatures they are expected to be extremely weak and likely unobservable in experiments and simulations.


As for the Binder cumulant, logarithmic corrections to the clean Ising behavior are expected. Our system sizes are too small, however, to reliably extract these logarithms.

