Modeling all-electrical detection of the inverse Edelstein effect by spin-polarized tunneling in a topological-insulator/ferromagnetic-metal heterostructure

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The spin-momentum locking of the surface states in a three dimensional topological insulator (TI) allows a charge current on the surface of the TI induced by an applied spin current onto the surface, which is known as the inverse Edelstein effect (IEE), that could be achieved either by injecting pure spin current by spin-pumping from a ferromagnetic metal (FM) layer or by injecting spin-polarized charge current by direct tunneling of electrons from the FM to the TI. Here, we present a theory of the observed IEE effect in a TI-FM heterostructure for the spin-polarized tunneling experiments. If an electrical current is passed from the FM to the surface of the TI, because of density of states polarization of the FM, an effective imbalance of spin-polarized electrons occurs on the surface of the TI. Due to the spin-momentum helical locking of the surface states in the TI, a difference of transverse charge accumulation appears on the TI surface in a direction orthogonal to the direction of the magnetization of the FM, which is measured as a voltage difference. Here, we derive the two-dimensional transport equations of electrons on the surface of a diffusive TI, coupled to a FM, starting from quantum kinetic equation, and analytically solve the equations for a rectangular geometry to calculate the voltage difference.

Three dimensional (3D) topological insulators (TIs) possessing spin-momentum locking in their gapless two dimensional (2D) surface states provide a suitable platform for various spintronic applications due to efficient charge current to spin current conversion and vice versa[1,2]. The generation of spin current from the charge flow on the surface of the TI was used effectively to torque a ferromagnetic metal (FM) in proximity to the TI[3–6]. Another interesting effect is the spin current to charge current conversion on the surface of the TI, which is known as the inverse Edelstein effect (IEE)[7]. The IEE effect in the TI was supported by spin-pumping experiments on various representative TI-FM heterostructures[8–12]. In the spin-pumping experiment, as shown in Fig. 1(a), a pure spin current (I_s) is injected to the TI from a FM layer in contact with the TI, and the injected spin current tends to produce a charge current on the surface of the TI because of the spin-momentum locked band structure of the surface states of the TI, and an open circuit voltage develops to oppose the flow of the induced charge current. The IEE effect observed in spin pumping experiment on the surface of a TI is derived theoretically in literature[13,14] for a pure spin bias applied to the TI.

To characterize the IEE effect, a different kind of spin-tunneling experiment was performed by Liu *et. al.*[15], as shown schematically in Fig. 1(b) in which a charge current (I_c) was flowed from the FM to the surface of the TI, and a voltage drop was measured at the two ends of the TI surface in the transverse direction which is orthogonal to the FM magnetization direction. As the FM has different density of states for the majority and the minority electrons, a spin-polarized charge current flows from the FM to the TI, creating a spin imbalance on the surface of the TI. Because of spin-momentum helical locking on the TI surface, the spin-polarized electrons coming from the FM flow accordingly on the surface of the TI, and the imbalance of spin-polarized electrons creates a difference of charge accumulation along the direction perpendicular to the direction of magnetization in the FM. The difference of charge accumulation is measured as a voltage difference at the two ends of the TI surface. An order of estimate for the observed voltage drop is provided by Liu *et. al.* with a simpli-

fying assumption of uniform current density on the surface of the TI[15]. However, the current density on the TI surface could be non-uniform, and one needs to solve the diffusion and the continuity equation on the TI surface, considering current injection from the FM along with proper boundary condition on the TI surface to obtain the distribution of electrochemical potential on the TI surface. In this paper, we derive the transport equation on the surface of the TI, coupled to the FM, starting from quantum kinetic theory and obtain a second order partial differential equation for the electrochemical potential on the TI surface with a source term due to the tunneling from the FM. Then, we solve the differential equation analytically in a rectangular geometry with net current flowing along one boundary and no currents at other boundaries to calculate the transverse voltage drop that is measured in experiment. We show that the voltage drop depends on the interface conductance and becomes small if the interface conductance is high, which implies the importance of a tunnel barrier in such spin-polarized tunneling experiment.

To derive the quantum kinetic equation, we follow the approach given in Ref. [16-18]. The quantum kinetic equation will be obtained from the Keldysh component of the Wigner transformed left-right subtracted Dyson equation after lowest order gradient expansion[16–19], and is given by

$$
\begin{aligned}
&\partial_T G^K + \frac{1}{2}\{\mathbf{v}\cdot\nabla_{\mathbf{R}}, G^K\} + \frac{i}{\hbar}\left[H(\mathbf{p}), G^K\right] \\
&= -i\left[\left(\Sigma^R G^K - G^K \Sigma^A\right) - \left(G^R \Sigma^K - \Sigma^K G^A\right)\right],
\end{aligned}
\tag{1}
$$

where the linear Dirac cone Hamiltonian for the surface states of a TI is $H(\mathbf{p}) = \hbar v_F(\mathbf{p}\times\hat{\mathbf{z}})\cdot\boldsymbol{\sigma}$, the velocity operator for the surface states is $\mathbf{v} = \frac{1}{\hbar}\partial_{\mathbf{p}} H(\mathbf{p}) = v_F(\hat{\mathbf{z}}\times\boldsymbol{\sigma})$, v_F is the Fermi velocity of the TI surface states, and $\hbar$ is the reduced Planck constant. Here, $G^{R,A,K}$ and $\Sigma^{R,A,K}$ are the retarded (R), advanced (A) and Keldysh (K) component of the Wigner transformed Green's functions (G's) of the TI surface states and the self-energies (Σ's) in terms of the variable $(\mathbf{R}, T; \mathbf{p}, \epsilon)$, where $(\mathbf{R}, T)$ are the center-of-mass position and time co-ordinates, and $(\mathbf{p}, \epsilon)$ are the Fourier transformed momentum and energy for the relative position and time co-ordinates. The
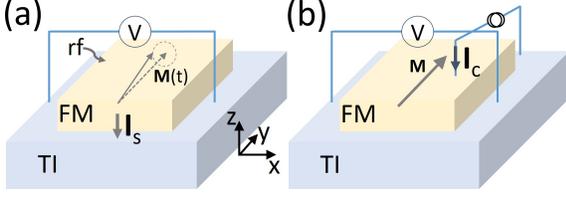
FIG. 1. Schematics for measuring inverse Edelstein effect (IEE) on the surface of a TI in a TI-FM heterostructure by (a) spin-pumping, and (b) spin-polarized tunneling experiments.

self-energy consists of both disorder and tunneling contributions, i.e. $\Sigma = \Sigma_{\text{dis}} + \Sigma_{\text{tun}}$. We consider spin-independent short-ranged randomly distributed impurity potential, and the impurity averaged self-energy for disorder is

$$\Sigma_{\text{dis}}^{\text{R,A,K}}(\mathbf{R}, T; \mathbf{p}, \epsilon) = \frac{1}{\pi N \tau_{\text{p}}} \int \frac{d^2 p'}{(2\pi)^2} G^{\text{R,A,K}}(\mathbf{R}, T; \mathbf{p}', \epsilon), \tag{2}$$

where τ_{p} is the momentum relaxation time in the TI, N is the density of states of the TI surface states at the Fermi energy ϵ_{F}. We introduce the quasi-classical Green's functions (g's) as

$$g^{R,A,K}(\mathbf{R}, T; p_F \hat{\mathbf{p}}, \epsilon) = \frac{i}{\pi} \int d\xi_p \, G^{R,A,K}(\mathbf{R}, T; \mathbf{p}, \epsilon), \tag{3}$$

where p_F is the Fermi momentum and $\xi_p = \hbar v_F p - \epsilon_F$. We consider that ϵ_F lies in the conduction band of the TI. In the quasi-classical limit, ϵ_F is the largest energy scale in the problem and ξ_p integration is performed near ϵ_F, so only the projections to the conduction band of the retarded and advanced Green's functions of the TI surface states will be relevant to the transport[20–22]. The conduction band projections of the retarded/advanced Green's functions of the TI surface state are

$$G^{\text{R,A}}(\mathbf{p}, \epsilon) = \frac{\frac{1}{2}[\sigma_0 + (\hat{\mathbf{p}} \times \hat{\mathbf{z}}) \cdot \boldsymbol{\sigma}]}{\epsilon - \xi_p \pm i0^+}. \tag{4}$$

So, the retarded and advanced quasi-classical Green's functions become $g^{\text{R,A}} = \pm[\sigma_0 + (\hat{\mathbf{p}} \times \hat{\mathbf{z}}) \cdot \boldsymbol{\sigma}]/2$ and the disorder self-energies are given by $\Sigma_{\text{dis}}^{\text{R,A}} = \mp(i/2\tau_{\text{p}})\sigma_0$. Similarly, we can assume G^K is peaked at the Fermi energy, i.e. $G^K = -i\pi g^K \delta(\xi_p)$, and the Keldysh component of the disorder self-energy can be written as $\Sigma_{\text{dis}}^K = -(i/\tau_{\text{p}})\langle g^K \rangle$, where $\langle ... \rangle$ denotes angular averaging in the $\mathbf{p}$ space.

We consider a spin conserving and site-to-site (local) instantaneous tunneling at the interface for both the case of a rough interface and a smooth interface. In case of a rough interface, the tunneling can be modeled by randomly distributed tunneling centers and momentum randomization happens in the tunneling process. The tunneling self-energy is obtained after averaging over random distribution of the tunneling centers similar to the case of impurity scattering, and will be given in terms of the Green's functions of the FM at the interface,

$$\Sigma_{\text{tun}}^{\text{R,A,K}} = \frac{\gamma}{\pi} \int \frac{d^3 k'}{(2\pi)^3} G_{\text{FM}}^{\text{R,A,K}}(\mathbf{R}, z = 0, T; \mathbf{k}', \epsilon), \tag{5}$$

where γ represents the strength of the tunneling rate, $G_{\text{FM}}^{\text{R,A,K}}$ is the retarded, advanced and Keldysh component of the Green's function in the FM, and $\mathbf{k}$ is the 3D momentum in the FM band structure. However, in case of a smooth interface, the tunneling can be modeled by a constant tunneling potential throughout the interface, and as a result the in-plane momentum will be conserved in the tunneling process ($\mathbf{k}_\parallel = \mathbf{p}$). The tunneling self-energies is given by (with tunneling rate γ')

$$\Sigma_{\text{tun}}^{\text{R,A,K}} = \frac{\gamma'}{\pi} \int \frac{dk_z'}{2\pi} G_{\text{FM}}^{\text{R,A,K}}(\mathbf{R}, z = 0, T; \mathbf{p}, k_z', \epsilon). \tag{6}$$

The retarded/advanced Green's functions for FM are

$$G_{\text{FM}}^{\text{R,A}}(\mathbf{k}, \epsilon) = P_\uparrow \frac{1}{\epsilon - \xi_{k\uparrow} \pm i0^+} + P_\downarrow \frac{1}{\epsilon - \xi_{k\downarrow} \pm i0^+}. \tag{7}$$

where $\xi_{k\uparrow,\downarrow} = \hbar^2 k^2/2m_{\text{c}} + \epsilon_{\text{b}} \pm \Delta_{\text{ex}} - \epsilon_{\text{F}}$ for the two spin splitted bands in the FM and $P_{\uparrow,\downarrow} = (\sigma_0 \pm \hat{\mathbf{m}} \cdot \boldsymbol{\sigma})/2$ are the projection operators to these bands. Here, m_{c} is the effective mass for both the conduction bands in the FM, ϵ_{b} is the band offset relative to the Dirac point in the TI, Δ_{ex} is the effective strength of exchange interaction between the conduction s-electrons and the localized d-electrons in the FM, and $\hat{\mathbf{m}} = m_x \hat{\mathbf{x}} + m_y \hat{\mathbf{y}} + m_z \hat{\mathbf{z}}$ is the unit vector along the direction of magnetization in the FM. We consider the Keldysh component of the Green's function in the FM consisting of incoherent superposition of the Keldysh components of the quasi-classical Green's functions for the majority and minority electrons, i.e., $g_{\uparrow,\downarrow}(\mathbf{R}, z = 0, T; k_{F\uparrow,\downarrow}\hat{\mathbf{k}}_{\uparrow,\downarrow}, \epsilon)$, peaked at the corresponding Fermi momentum $k_{F\uparrow,\downarrow}$ of the each spin-splitted band[18]. Similar to the retarded and advanced Green's functions given in Eq. (7), we can write

$$G_{\text{FM}}^K(\mathbf{R}, z = 0, T; \mathbf{k}, \epsilon) = -i\pi \left[P_\uparrow g_\uparrow \delta(\xi_{k\uparrow}) + P_\downarrow g_\downarrow \delta(\xi_{k\downarrow}) \right]. \tag{8}$$

In case of momentum randomizing tunneling, the tunneling self-energies are calculated to be $\Sigma_{\text{tun}}^{\text{R,A}} = \mp i\gamma(N_\uparrow P_\uparrow + N_\downarrow P_\downarrow)$ and $\Sigma_{\text{tun}}^K = -i\gamma(N_\uparrow P_\uparrow \langle g_\uparrow \rangle + N_\downarrow P_\downarrow \langle g_\downarrow \rangle)$, where $N_{\uparrow,\downarrow}$ are the 3D DOS of the majority and minority electrons in the FM at ϵ_{F}, and $\langle ... \rangle$ denotes angular averaging in the $\mathbf{k}$ space. In case of in-plane momentum conservation, assuming that $g_{\uparrow,\downarrow}$ are independent of the solid angle in the $\mathbf{k}$ space[23], we obtain same relation for the tunneling self-energy with γ replaced by γ', and $N_{\uparrow,\downarrow}$ replaced by the corresponding one-dimensional (1D) DOSs for majority and minority electrons in the FM calculated at ϵ_{F} with the constraint of in-plane momentum conservation.

The quantum kinetic equation in terms of the quasi-classical Green's function g^K of the TI surface states is obtained after doing ξ_p integration of Eq. (1),

$$\partial_T g^K + \frac{v_F}{2} \{\hat{\mathbf{z}} \times \boldsymbol{\sigma} \cdot \nabla_{\mathbf{R}}, g^K\} + iv_F p_F [(\hat{\mathbf{p}} \times \hat{\mathbf{z}}) \cdot \boldsymbol{\sigma}, g^K]$$
$$= -\frac{g^K}{\tau_{\text{p}}} + \frac{\langle g^K \rangle}{\tau_{\text{p}}} + \frac{1}{2\tau_{\text{p}}} \{(\hat{\mathbf{p}} \times \hat{\mathbf{z}}) \cdot \boldsymbol{\sigma}, \langle g^K \rangle\}$$
$$- \gamma \big(\{(N_\uparrow P_\uparrow + N_\downarrow P_\downarrow), g^K\} + N_\uparrow P_\uparrow \langle g_\uparrow \rangle + N_\downarrow P_\downarrow \langle g_\downarrow \rangle\big)$$
$$+ \frac{\gamma}{2} \{(\hat{\mathbf{p}} \times \hat{\mathbf{z}}) \cdot \boldsymbol{\sigma}, (N_\uparrow P_\uparrow \langle g_\uparrow \rangle + N_\downarrow P_\downarrow \langle g_\downarrow \rangle)\}. \tag{9}$$

The Fermi energy ϵ_F is the largest energy scale compared to all other relevant energy scales available in the system (ϵ_M being the maximum of them), so the lowest order solution (with respect to the perturbation parameter ϵ_M/ϵ_F) of g^K will commute with $[(\hat{\mathbf{p}} \times \hat{\mathbf{z}}) \cdot \boldsymbol{\sigma}]^{20-22}$ and we can write the following *ansatz*: $g^K = g_0(\hat{\mathbf{p}}, \epsilon)[\sigma_0 + (\hat{\mathbf{p}} \times \hat{\mathbf{z}}) \cdot \boldsymbol{\sigma}]$. As we will consider the FM to be magnetized in-plane (i,.e. $m_z = 0$), there will be no gap opening of the TI surface state as well as the coefficient of σ_z in G_{FM}^K will be zero, so the perturbation expansion and the *ansatz* for g^K will be valid[24]. Schwab *et. al.*[18] derived the charge continuity equation in the TI using the *ansatz* in Eq. (9), taking trace over the spin space and doing the integration over the angle in $\mathbf{p}$ space and energy ϵ. However, the associated diffusion equation for the charge current density on the TI surface was not derived by Schwab *et. al.*[18]. On the other hand, using semi-classical drift-diffusion model with tunneling calculated from the Fermi's golden rule, Yokoyama *et. al.*[25] showed that both the continuity and the diffusion equation in the TI are modified due to the tunneling from the FM. Here, we complete the derivation of the modified continuity equation as well as the modified diffusion equation in the TI starting from the quantum kinetic equation.

In the limit of diffusive transport of the quasi-particles, $g_0(\hat{\mathbf{p}}, \epsilon)$ can be expanded through the zeroth and the first harmonics in 2D[20-22], i.e. $g_0(\hat{\mathbf{p}}, \epsilon) = g_s(\epsilon) + \hat{\mathbf{p}} \cdot \mathbf{g}_a(\epsilon)$. The continuity equation and the diffusion equation in the TI will be obtained from Eq. (9) by separating the zeroth and the first harmonic components, respectively. Substituting the *ansatz* for g^K with the expression of $g_0(\hat{\mathbf{p}}, \epsilon)$ in Eq. (9), and doing the integration over the angle in the $\mathbf{p}$ space, we obtain

$$\partial_T g_s + \frac{v_F}{2} \nabla_{\mathbf{R}} \cdot \mathbf{g}_a = \frac{1}{2}\gamma(N_\uparrow \langle g_\uparrow \rangle + N_\downarrow \langle g_\downarrow \rangle)$$
$$- \gamma N_+ g_s + \frac{1}{2}\gamma N_- \mathbf{g}_a \cdot (\hat{\mathbf{m}} \times \hat{\mathbf{z}}), \quad (10)$$

where we define $N_\pm = N_\uparrow \pm N_\downarrow$. To separate out the anisotropic part, we multiply Eq. (9) by $\hat{\mathbf{p}}$ and do the integration over the angle in the $\mathbf{p}$ space to get

$$\partial_T \mathbf{g}_a + v_F \nabla_{\mathbf{R}} g_s = -\left(\frac{1}{2\tau_p} + \gamma N_+\right)\mathbf{g}_a$$
$$+ \gamma N_- g_s \, \hat{\mathbf{m}} \times \hat{\mathbf{z}} - \frac{\gamma}{2}\big(N_\uparrow \langle g_\uparrow \rangle - N_\downarrow \langle g_\downarrow \rangle\big)\hat{\mathbf{m}} \times \hat{\mathbf{z}}. \quad (11)$$

The charge density n and the charge current density $\mathbf{J}$ on the surface of the TI are given by $n = (eN/2) \int d\epsilon \, g_s(\epsilon)$ and $\mathbf{J} = (eN/2) \int d\epsilon \, (v_F/2)\mathbf{g}_a(\epsilon)$[18]. We define the electrochemical potential μ in the TI as $n = e^2 N \mu$, and the electrochemical potential $\mu_{\uparrow,\downarrow}$ for the majority and minority electrons in the FM as $\mu_{\uparrow,\downarrow} = (1/4e) \int d\epsilon \, g_{\uparrow,\downarrow}(\epsilon)$[18]. The charge density n_+ and the spin density n_- in the FM can be written as $n_\pm = n_\uparrow \pm n_\downarrow$, where $n_{\uparrow,\downarrow} = e^2 N_{\uparrow,\downarrow}\mu_{\uparrow,\downarrow}$. Integrating Eq. (10) over the energy ϵ and using these definitions, in the steady state the following continuity equation in the TI is obtained

$$\nabla_{\mathbf{R}} \cdot \mathbf{J} = -\gamma N_+ n + \gamma N n_+ + \frac{\gamma N_-}{v_F}\mathbf{J} \cdot (\hat{\mathbf{m}} \times \hat{\mathbf{z}}). \quad (12)$$

Similarly, from Eq. (11) we obtain the diffusion equation

$$\mathbf{J} = \frac{1}{(1+\xi)}\left[-D\nabla_{\mathbf{R}} n + \frac{\gamma v_F \tau_{tr}}{2}\big(N_- n - N n_-\big)(\hat{\mathbf{m}} \times \hat{\mathbf{z}})\right], \quad (13)$$

where $\tau_{tr} = 2\tau_p$ is the transport relaxation time and $D = v_F^2 \tau_{tr}/2$ is the diffusion constant in the TI without tunneling, and the term $(1+\xi)^{-1}$ is the modification of the the the diffusion constant due to tunneling, where $\xi = \gamma N_+ \tau_{tr}$ is a dimensionless parameter which is proportional to the interface conductance and denotes the strength of the tunneling. The diffusion equation Eq. (13) and the continuity equation Eq. (12) constitute the transport equations in the TI coupled to a FM, which agrees with those derived by Yokoyama *et. al.*[25].

In the spin tunneling experiment[15] with the TI-FM heterostructure as shown in Fig 1(b), a charge current was applied through the FM and extracted at one end of the TI, and a transverse voltage drop was measured on the surface of the TI as shown in Fig. 2(a). We can consider the electrochemical potential in the FM to be uniform throughout the FM, i.e. $\partial_x \mu_{\uparrow,\downarrow} = \partial_y \mu_{\uparrow,\downarrow} = 0$, as the FM conductivity is much higher than the TI conductivity. We also consider that there is no spin electrochemical potential μ_s in the FM, i.e. $\mu_s = \mu_\uparrow - \mu_\downarrow = 0$ and the electrochemical potential for both the majority and minority electrons in the FM to be equal to the charge electrochemical potential μ_c, i.e. $\mu_\uparrow = \mu_\downarrow = \mu_c$. Substituting Eq. (13) into Eq. (12) and inserting these conditions, we get

$$0 = \partial_x^2 \mu + \partial_y^2 \mu - \frac{2\xi\eta}{l}m_y \partial_x \mu + \frac{2\xi\eta}{l}m_x \partial_y \mu$$
$$+ \left[\frac{\xi^2\eta^2}{l^2}(m_x^2 + m_y^2) - \frac{2\xi(1+\xi)}{l^2}\right](\mu - \mu_c). \quad (14)$$

Here, $\eta = \frac{N_-}{N_+}$ is the DOS polarization of the FM and $l = v_f \tau_{tr}$ is the mean free path in the TI. To solve equation Eq. (14), we make the substitution $(\mu - \mu_c) \to \mu$, which amounts to calculating the electrochemical potential in the TI with respect to that of the FM. The resulting differential equation is a second order homogeneous partial differential equation in μ with no cross derivative $\partial_x \partial_y \mu$, so it can be separated into two second order ordinary differential equations by the separation of variables, i.e., setting $\mu(x, y) = \mu_X(x)\mu_Y(y)$. The domain of the solution is a rectangular region $[0, L_x] \times [0, L_y]$, as shown in Fig. 2(a), which is a product of intervals of the two independent variables x and y. After finding solutions for each second order ordinary differential equations with unknown coefficients, the boundary conditions on the rectangular domain are used to find the values of the unknown coefficients. By the existence and uniqueness of solution of a second order partial differential equation with a given boundary condition, the solution found in this way will be *the* solution. After separating out the functions of the variables x and y, we obtain

$$0 = d_x^2 \mu_X - 2b_0 m_y d_x \mu_X + (b_0^2 m_y^2 - c_X^2)\mu_X, \quad (15a)$$
$$0 = d_y^2 \mu_Y + 2b_0 m_x d_y \mu_Y + (b_0^2 m_x^2 - c_Y^2)\mu_Y, \quad (15b)$$

where $b_0 = \xi\eta/l$, the constants c_X, c_Y satisfy the constraint $c_X^2 + c_Y^2 = c_0^2$ and we define $c_0 = \sqrt{2\xi(1+\xi)}/l$. The current
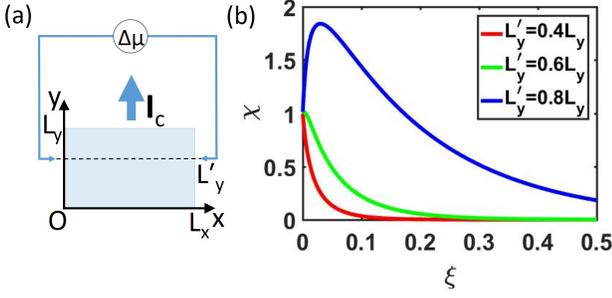
FIG. 2. (a) The rectangular region on the TI surface showing current I_c flowing out of the boundary $y = L_y$ and electrochemical potential drop $\Delta\mu$ measured at $y = L'_y$. (b) Variation of χ with ξ at different $L'_y = fL_y$ for $f = 0.4, 0.6, 0.8$ and $L_y/l = 20$.

density can be written as $\mathbf{J} = j_x \hat{\mathbf{x}} + j_y \hat{\mathbf{y}}$, where

$$j_x = -\sigma'[d_x \mu_X - b_0 m_y \mu_X]\mu_Y, \tag{16a}$$

$$j_y = -\sigma'[d_y \mu_Y + b_0 m_x \mu_Y]\mu_X. \tag{16b}$$

Here, $\sigma' = \sigma/(1 + \xi)$ is the modified conductivity of the TI due to tunneling with $\sigma = e^2 ND$ being the conductivity of the TI without tunneling.

The solutions of $\mu_{X,Y}$ are given by $\mu_X = A_1 e^{r_1 x} + A_2 e^{r_2 x}$ and $\mu_Y = B_1 e^{s_1 x} + B_2 e^{s_2 x}$, if $r_1 \neq r_2$ and $s_1 \neq s_2$, where $r_{1,2} = b_0 m_y \pm c_X$ and $s_{1,2} = -b_0 m_x \pm c_Y$. Then we obtain $j_x = -\sigma' c_X (A_1 e^{r_1 x} - A_2 e^{r_2 x})\mu_Y(y)$ and $j_y = -\sigma' c_Y (B_1 e^{s_1 x} - B_2 e^{s_2 x})\mu_X(x)$. As shown in Fig. 2(a), the boundary conditions on the TI surface in the rectangular region $[0, L_x] \times [0, L_y]$ of the TI-FM heterostructure are $j_x(x = 0) = j_x(x = L_x) = 0, \forall y$, $j_y(y = 0) = 0, \forall x$ and $\int dx j_y(x, y = L_y) = I_c$. The solution of the boundary condition $j_x = 0, \forall y$ at $x = 0, L_x$ implies $r_1 = r_2 = b_0 m_y$, i.e. $c_X = 0, c_Y = c_0$. Hence, the solution of μ_X should be given by $\mu_X = (C + Dx)e^{b_0 m_y x}$, and $j_x = -\sigma' D e^{b_0 m_y x}\mu_Y(y)$. The boundary condition $j_x = 0, \forall y$ at $x = 0, L_x$ gives $D = 0$. The constant C can be absorbed in the constants B_1, B_2 of μ_Y, and we obtain $\mu_X = e^{b_0 m_y x}$. The boundary condition $j_y(y = 0) = 0, \forall x$ gives $B_1 = B_2 = B$, and the only unknown constant B is obtained from the remaining boundary condition that the total charge current out of the boundary $y = L_y$ is I_c. So, we find the solution of $\mu(x, y)$ to be

$$\mu(x, y) = -\frac{I_c}{\sigma'} \frac{b_0 m_y e^{b_0 m_y x}}{e^{b_0 m_y L_x} - 1} \frac{e^{-b_0 m_x y} \cosh(c_0 y)}{e^{b_0 m_x L_y} c_0 \sinh(c_0 L_y)}. \tag{17}$$

This is the main result of our paper. We consider two special cases: the magnetization of the FM along the x-direction and along the y-direction. For the FM magnetized in the x-direction, i.e. for $m_x = \pm 1, m_y = 0$, using the limit formula $\lim_{t \to 0} t/(e^t - 1) \to 1$, we obtain from Eq. (17)

$$\mu(x, y) = -\frac{I_c}{\sigma'} \frac{1}{L_x} \frac{e^{-b_0 m_x y} \cosh(c_0 y)}{e^{b_0 m_x L_y} c_0 \sinh(c_0 L_y)}, \tag{18}$$

which is independent of x, and so, no voltage drop will be measured at the two ends $x = 0$ and $x = L_x$ on the TI

surface. For the FM magnetized in the y-direction, i.e. for $m_x = 0, m_y = \pm 1$, we obtain from Eq. (17),

$$\mu(x, y) = -\frac{I_c}{\sigma} \frac{\eta m_y}{2} \frac{e^{\xi \eta m_y x/l}}{(e^{\xi \eta m_y L_x/l} - 1)} \frac{d_0 \cosh(d_0 y/l)}{\sinh(d_0 L_y/l)}, \tag{19}$$

where we have inserted the values of b_0, c_0 and defined $d_0 = c_0 l = \sqrt{2\xi(1 + \xi)}$. The difference of the electrochemical potential measured at some $y = L'_y$ and at the two ends $x = 0$ and $x = L_x$ on the TI surface is defined as $\Delta\mu(L'_y) = \mu(x = 0, y = L'_y) - \mu(x = L_x, y = L'_y)$. From Eq.(19),

$$\Delta\mu(L'_y) = \frac{I_c}{\sigma} \frac{\eta m_y}{2} \frac{l}{L_y} \frac{(d_0 L_y/l) \cosh(d_0 L'_y/l)}{\sinh(d_0 L_y/l)}. \tag{20}$$

The potential drop $\Delta\mu$ depends on the orientation of the magnetization in the y-direction and changes sign when the magnetization is reversed. We introduce the dimensionless parameter $\chi = (d_0 L_y/l) \cosh(d_0 L'_y/l)/\sinh(d_0 L_y/l)$, and write $\Delta\mu(L'_y) = \chi \eta \frac{I_c}{\sigma} \frac{l}{L_y} \frac{m_y}{2}$, and the expression agrees with that given by Liu et. al.[15]. The parameter η depends both on the FM and on the quality of the interface. For the rough and the smooth interface, the tunneling is momentum randomizing and in-plane momentum conserving, respectively, and the DOS polarization of the FM η will be given by the 3D and the 1D DOSs for the majority and minority electrons in the FM, respectively. The parameter χ is a function of ξ, where $\xi = \gamma N_+ \tau_{tr}$, γ depends on the nature of the interface, and N_+ will be given by the 3D and the 1D DOSs of the FM for the rough and the smooth interface, respectively. We also show that the parameter χ depends on the geometry of the problem, and in Fig. 2(b) we have plotted the variation of χ with ξ for different values of $L'_y = fL_y$, where $f = 0.4, 0.6, 0.8$ and $L_y/l = 20$. From Fig. 2(b), we observed that the value of χ reduces as ξ increases or the interface conductance increases. In the limit $\xi \to 0$, we obtain $\chi \to 1$ irrespective of the value of l, L'_y, L_y. In the experiment by Liu et. al.[15], a tunnel barrier was used between the TI and FM, and we show the importance of the tunnel barrier in such tunneling experiment. The band structure of a real TI material is complicated by a quadratic mass term, and/or a mass term due to gap opening at the Dirac point in a thin TI film, and/or a hexagonal warping term present in the Hamiltonian. However, if the Fermi level is away from the Dirac point and does not lie in the bulk of the TI, the primary contribution to the physics of the surface states of interest of a TI comes from the still linear Dirac-cone-like dispersion relation that we have used. The presence of other non-idealities will, of course, modify somewhat the charge transport on the TI surface, but the essential conclusions of this work remain.

In conclusion, starting from quantum kinetic equation we derive the diffusion and the continuity equations in the TI considering tunneling from the FM in a TI-FM heterostructure. From these transport equations, a second order partial differential equation is obtained for the electrochemical potential in the TI with a source term describing tunneling from the FM. Next, we solve the differential equation analytically in a rectangular geometry, for constant potential applied to the

FM and with net current flowing out of one boundary on the TI surface, to get the transverse potential drop at two other boundaries on the TI surface. A non-zero transverse potential drop orthogonal to the direction of the FM magnetization is calculated, which changes sign when the magnetization is reversed and agrees with the experiment to detect the IEE effect on the surface of a TI. We show that the potential drop depends on the quality of interface, and find the importance of the tunnel barrier in such spin-polarized tunneling experiment.

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[23] In the diffusive limit, $g_{\uparrow,\downarrow}$ can be expanded through the isotropic and anisotropic components, and the anisotropic component will be given by the gradient of the isotropic component, where the isotropic component is proportional to the density of majority and minority electrons in the FM or the corresponding electrochemical potentials. As we only consider uniform electrochemical potential applied in the FM, the anisotropic component will be zero and $g_{\uparrow,\downarrow}$ will be given by only the isotropic component.

[24] In general, $g^{K} = g'\sigma_0 + g''(\hat{\mathbf{p}} \times \hat{\mathbf{z}}) \cdot \boldsymbol{\sigma} + g_{\parallel}\hat{\mathbf{p}} \cdot \boldsymbol{\sigma} + g_z\sigma_z$. It can be shown[20–22] that $(g_{\parallel}, g_z) \sim \mathcal{O}(\frac{\epsilon_M}{\epsilon_F})F(g', g'', g_{\uparrow}, g_{\downarrow})$ for some linear function F, and $\epsilon_M = \max(\hbar/\tau_p, \hbar|\mathbf{q}|\, l/\tau_p, \hbar\omega, \hbar\gamma N_{\pm})$, where $(\mathbf{q}, l)$ is the Fourier transform variable of $(\mathbf{R}, T)$. So, $g_{\parallel}, g_z$ are less than g', g'' by a factor $(\epsilon_M/\epsilon_F) \ll 1$. After neglecting $g_{\parallel}, g_z$ in g^{K}, it can be shown[20–22] that $g' = g''$ will be a solution.

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