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Umklapp scattering as the origin of T-linear resistivity in the normal state of high-T_c cuprate superconductors

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The high-temperature normal state of the unconventional cuprate superconductors has resistivity linear in temperature T, which persists to values well beyond the Mott-Ioffe-Regel upper bound. At low-temperature, within the pseudogap phase, the resistivity is instead quadratic in T, as would be expected from Fermi liquid theory. Developing an understanding of these normal phases of the cuprates is crucial to explain the unconventional superconductivity. We present a simple explanation for this behavior, in terms of umklapp scattering of electrons. This fits within the general picture emerging from functional renormalization group calculations that spurred the Yang-Rice-Zhang ansatz: umklapp scattering is at the heart of the behavior in the normal phase.

Introduction.— The anomalous temperature and frequency dependence of the electrical d.c. conductivity in the pseudogap (PS) phase of underdoped cuprates has attracted special attention.[1–3] While there is agreement that the PS phase is a precursor to the Mott insulator (MI), a coherent description of its unexpected features is still a challenge. Many of these are in momentum space, e.g. the breakup of the Fermi surface by energy gaps near the antinodes with superconductivity only near the nodal Fermi pockets.[4] At high temperatures the energy gap in the PS phase disappears and the resistivity increases linearly with temperature T to anomalously large values.[3] This radical departure from a standard metal has led to the label of 'strange metal' for this phase. In this letter, we argue that the explanation lies in dominant umklapp (U) scattering, i.e. elastic scattering processes that directly transfer momentum between the conduction electron sea and the underlying lattice.

The parent undoped cuprates are MIs[5] – a state where strong electron-electron interactions cause conduction electrons to condense onto ions, forming an insulating lattice of neutral atoms. In one dimension, however, a Mott state appears already at weak interactions, driven by U-scattering across the Fermi surface.[6] This causes momentum transfer between the conduction electron sea and the ionic lattice in units of reciprocal lattice vectors and leads to the insulating ground state. A case of particular interest to us is the $^1/_2$-filled two-leg Hubbard ladder (2LHL), and its so-called d-MI state, a state that contains seeds of d-wave superconductivity.[7,8] At high temperatures, a one-dimensional (1D) bosonization analysis finds that the resistivity due to umklapp scattering (see the processes in Fig. 1) rises linearly with T, the anomalous form also observed in the 'strange metal'.[9,10]

The approach that we follow here is to start from the bosonization solution of the 2LHL, and treat the generalization to 2D in the spirit of a k-space factorization within a finite patch approximation. This was introduced by Ossadnik,[11] who argued that short range order yields

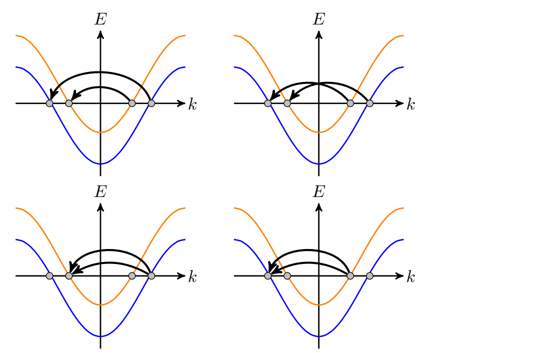

FIG. 1. Umklapp scattering processes ($RR \to LL$) for right-moving electrons at the Fermi points (grey circles) in the one-dimensional $^1/_2$-filled two-leg Hubbard ladder.

k-space correlations with a finite width, which justifies a wave packet analysis where one breaks k-space into finite width patches. When combined with the Yang-Rice-Zhang (YRZ) distortion of the Fermi surface,[12] this yields a reasonable description of the physics observed within ARPES in high-T_c cuprates,[13] as well as the behavior of the high-field Hall effect within the PS phase.[14] Combined with the rather different approach of,[15] these works provide a formal link between the physics of fermionic ladders and 2D doped MIs. The antinodal regions are mapped onto mutually perpendicular $^1/_2$-filled ladders, which interact only weakly with the nodal regions (to first approximation, the two regions can be treated as decoupled). In the present letter, we show that this picture captures the behavior of transport within the PS phase of the high-T_c cuprates, with the correct form of the longitudinal transport $\varrho(T)$ being found, as well as explaining the Hall angle as a function of temperature.

Umklapp scattering in doped cuprates.— The importance of U-scattering in the high-T_c cuprate superconductors away from $^1/_2$-filling has been emphasized for some time now, see e.g..Honerkamp *et al.*[16] The 2D Hubbard

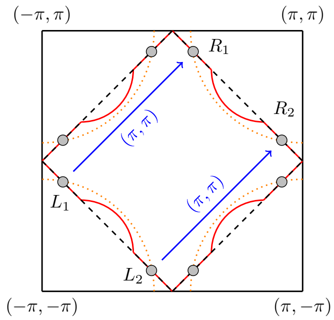

FIG. 2. The non-interacting Fermi surface (dotted) intersects the umklapp surface (dashed lines, connecting the antinodes) at eight isolated points within the Brillouin zone, known as hotspots (gray circles). In the presence of strong interactions the Fermi surface may deform in order to increasing the nesting (depicted in red), leading to the formation of the well-known 'pockets' close to the nodes. One umklapp scattering process, $L_1, L_2 \to R_1, R_2$, is illustrated.

model was studied within the functional renormalization group (FRG) framework; at small dopings away from 1/2-filling U-scattering flows to strong coupling. Thus any electrons on the "umklapp surface" (the square surface joining the antinodal points, see Fig. 2) experience strong U-scattering. On the Fermi surface, eight such isolated points exist, known as hotspots. The FRG analysis also showed clear hallmarks of d-wave superconductivity and antiferromagnetism,[16,17] in agreement with exact diagonalization studies.[18]

The presence of the U-surface, on which scattering is particular strong, motivated Yang, Rice and Zhang to propose their phenomenological ansatz for the Green's function in the PS phase.[12] This puts the U-surface at the center of the physics, with the pairing gap opening on this surface, rather than the Fermi surface. Strong U-scattering then increases the energy gap. The ansatz has successfully described a variety of experiments, from angle-resolved photoemission spectroscopy[13] to the change in the Hall effect upon entering the PS phase,[19] and resonant inelastic x-ray scattering.[20]

The important role of U-scattering in the cuprates was again emphasized recently by Liu *et al.*[14] and Wu *et al.*[21] They argued that U-scattering causes the emergence of the PS state, turning the superconducting gap at overdoping into an insulating PS at lower doping due to gaps in both single particle and pair spectra[22]. The presence of strong U-interactions can deform the Fermi surface to stabilize commensurate nesting, which in turn maximizes the interactions, leading to enhanced gaps (similar to Cr alloys[23]) on the YRZ Fermi surface, see Fig. 2 and Refs. [14,15,21]. This deformation of the Fermi surface to run along the U-surface allows a straightforward generalization of the 1D 2LHL analysis, leading to T-linear resistivity, to 2D.

The normal phases of the cuprates.— The normal phases of the cuprate superconductors are enigmatic and enduring mysteries in contemporary condensed matter physics.[4,12,24] A theory of these phases is crucial to understand the mechanism of high-T_c superconductivity, as these are the states from which superconductivity emerges. The 'strange metal' exhibits linear resistivity $\varrho(T) \propto T$, in striking contrast to the conventional $\varrho(T) \propto T^2$ behavior of Fermi liquid theory.[1–3] Whilst the resistivity may saturate at sufficiently high temperature,[2,25] it is clear that it can largely violate the Mott-Ioffe-Regel limit[26] On the other hand, the PS phase shows the conventional $\varrho(T) \propto T^2$ behavior at low-temperature, with a gradual crossover to linear at the transition to the 'strange metal', T^*.[3]

The transition temperature T^* between the 'strange metal' and PS phases is still the subject of much study. The order parameter characterizing the transition, if it exists, is hotly debated. Recent optical studies found a change in symmetry on passing through T^*.[27] Angle-resolved photo emission spectroscopy,[28] tunneling spectroscopy,[29] and Raman[30] studies lend support to the suggestion T^* can be related to the PS Δ_{PG} analogously to the superconducting gap Δ_{BCS} and T_c in a BCS superconductor, $2\Delta_{PG} \simeq 4.3 k_B T^*$.

The two fluid model.— The results of Refs. [14,15] present a picture of the PS phase where antinodal and nodal states are well separated. This sets the stage for the *two fluid model* of the conductivity $\sigma(T)$ in the normal phases[31]

$$\sigma(T) \approx \sigma_{\text{nodes}}(T) + \sigma_{\text{antinodes}}(T), \tag{1}$$

where each contribution comes from nodal or antinodal states. We will see that U-scattering of electrons in the antinodal regions at high temperatures leads to $\sigma_{\text{antinodes}}(T) \sim 1/T$ (i.e., $\varrho(T) \propto T$), while conventional scattering around the nodal regions leads to the Fermi liquid form $\sigma_{\text{nodes}}(T) \sim 1/T^2$ (i.e., $\varrho(T) \propto T^2$).

Linear resistivity for $T > T^*$.— At the center of our argument for linear resistivity $\varrho(T) \propto T$ at high temperature is the mapping of electrons along the U-surface (i.e., in the antinodal region and the vicinity of the hotspots) to effective ladder models, see Refs. [14,15]. U-scattering of such electrons dominates the resistivity,[32] both due to the propensity of U-scattering to dissipate momentum and the strength of such terms (which flow to strong coupling under the FRG[16]). Linear resistivity then follows in a straightforward way; at an elementary level, this can be seen from the naïve Boltzmann argument presented earlier. At a more quantitative level, we can consider the resistivity generated by presence of a U-term $H_U = \int \mathrm{d}x\, \mathcal{H}_U$ in the low-energy effective theory[7,32,33] (for details we refer the reader to[15]),

$$\mathcal{H}_U = u\Big[\cos(\sqrt{8\pi}\Phi_{1,c}) + \cos(\sqrt{8\pi}\Phi_{2,c})\Big], \tag{2}$$

where u is the U-interaction strength and $\Phi_{1,c}$, $\Phi_{2,c}$ are charge fields in the low-energy description of the ladder.

Their two-point correlation function is

$$\langle\Phi_{a,c}(x,t)\Phi_{a,c}(0,0)\rangle = \frac{K_c}{4\pi}\ln\left[\frac{\pi T}{\sinh\Big(\pi T(t-x-i0)\Big)}\frac{\pi T}{\sinh\Big(\pi T(t+x-i0)\Big)}\right], \quad (3)$$

where K_c is the Luttinger parameter for the charge degrees of freedom.[33]

The charge field in the the presence of a constant current J, denoted $\tilde{\Phi}_{i,c}$, can be related to the zero current field through $\Phi_{i,c}(x,t) = \tilde{\Phi}_{i,c}(x,t) - \pi J t$. The resistivity can then be computed following the approach of Ref. [34]. The voltage drop V across the system is related to the time-derivative of the dual charge fields $\Theta_{a,c}(x,t)$

$$eV = \sum_{a=1,2}\left\langle \frac{\mathrm{d}}{\mathrm{d}t}\left[\Theta_{a,c}\left(\frac{L}{2},t\right) - \Theta_{a,c}\left(-\frac{L}{2},t\right)\right]\right\rangle, \quad (4)$$

We then compute the time derivative of the dual field

$$\begin{aligned}\dot{\Theta}_{a,c}(x,t) &= i[H,\Theta_{a,c}(x,t)] \\ &= -i\pi u\int_{-\frac{L}{2}}^{x}\mathrm{d}y\sin\left(\sqrt{8\pi}\Phi_{a,c}(y,t)\right).\end{aligned} \quad (5)$$

Combining Eqs. (4) and (5), we derive the expression for the voltage drop across the system:

$$V = Lu^2\sum_{i=1,2}\int \mathrm{d}x\int_{-\infty}^{\infty}\mathrm{d}t\ \sin(\pi J t)\langle\mathcal{H}_U(x,t)\mathcal{H}_U(0,0)\rangle. \quad (6)$$

To zeroth order in the coupling u, the finite temperature T correlation function follows from Eq. (3)

$$\left\langle\cos\left(\sqrt{8\pi}\Phi_{i,c}(x,t)\right)\cos\left(\sqrt{8\pi}\Phi_{i,c}(0,0)\right)\right\rangle = \left[\frac{\pi T}{\sinh\Big(\pi T(t-x-i0)\Big)}\frac{\pi T}{\sinh\Big(\pi T(t+x-i0)\Big)}\right]^{2K_c}.$$

Inserting this expression into (6) we find the resistivity

$$\varrho(T) \sim u^2 T^{-3+4K_c}. \quad (7)$$

In this simple manner we reproduce the well-known result of Ref. [9]. Linear resistivity is recovered in the limit $K_c \to 1$. Expanding Eq. (6) further in the coupling u, we arrive at the scaling law

$$\varrho(T) = Tf\left(\frac{T}{T^*}\right), \quad (8)$$

where $T^* \sim u^{1/2(1-K_c)}$, and the scaling function satisfies $f(x) \sim x^{-4(1-K_c)}$ for $x \gg 1$ and $f(x) \sim \exp(\alpha/x)$ for $x \ll 1$, where α is a constant of order one. The low-temperature limit is determined by the presence of a gap. From Eq. (8) we conclude that the contribution to the conductivity has a maximum around T^*.

Quadratic resistivity for $T \ll T^*$.— Let us now consider the low-T PS phase. At sufficiently low T, the PS has opened around the antinodes, leading to the well-known nodal pockets. In this limit, there are no single electron states in the vicinity of the U-surface, and resistivity is dominated by conventional scattering occurring at the pockets. This produces $\varrho(T) \propto T^2$ via the usual argument for a Landau-Fermi liquid. Namely, the number of scattering channels varies as T^2 when energy and momentum conservation is imposed, leading to $\varrho(T) \propto T^2$.

The transition region, $T \lesssim T^*$.— In the low-T PS phase, antinodal excitations are gapped and the nodal pockets give $\varrho(T) \propto T^2$. On the other hand, within the 'strange metal' $T > T^*$ the resistivity is dominated by U-scattering, leading to $\varrho(T) \propto T$. Our explanation naturally leads to a crossover between these, as follows.

There exist eight hotspots where the Fermi surface and the U-surface coincide. However, much of the Fermi surface is close to the U-surface and so, at finite temperature, a significant density of electrons resides on it. These electrons experience extremely strong U-scattering, as explained above, and dominate the resistivity, leading to $\varrho(T) \propto T$ at high T. As the temperature is lowered, the number of electrons that undergo U-scattering is reduced. At the cross over to the PS phase at $T = T^*$, a gap opens about the antinodes that increases with decreasing temperature. Eventually this gap *encompasses the U-surface in the antinodal regions*, and no electronic states exist in which U-scattering can occur. At this point, the resistivity is dominated by the pockets, leading to $\varrho(T) \propto T^2$. Between these two limits, a crossover occurs.

More formally this can be extracted from Eqs. (1) and (8). At a crude level, we can use an extrapolation formula that combines the antinodal contribution to the conductivity at low T, $\sigma_{\text{antinodes}}(T) \propto T^{-1}\exp(-\alpha T^*/T)$, with the nodal contribution, $\sigma_{\text{nodes}}(T) \propto 1/T^2$,

$$\varrho(T) = A_1 T\left[\exp\left(-\frac{\alpha T^*}{T}\right) + \frac{BT^*}{T}\right]^{-1}. \quad (9)$$

Here A_1, B and α are constants. It is worth emphasizing that this behavior is very unusual. Upon cooling, quasiparticle excitations within much of the Brillouin zone become gapped. It would then be natural to expect that the resistivity *increases*, yet, on the contrary, the resistivity drops from $\varrho(T) \propto T$ to $\varrho(T) \propto T^2$.

Lastly we compare our model to the detailed experiments of Barišić *et al.*,[3] who analyzed the temperature dependence of the resistivity of the normal state in a series of under- and overdoped cuprates. In Fig. 3 we compare the resistivity in $HgBa_2CuO_{4+\delta}$ with a fit to the crude scaling form (9). We see that there is excellent agreement.

Barišić *et al.*[3] found that the results for different cuprates could be coalesced into a single curve when they normalized their hole densities to holes per Cu_4O_4 plaquette. In the PS phase, $p < 0.18$, they fit their results

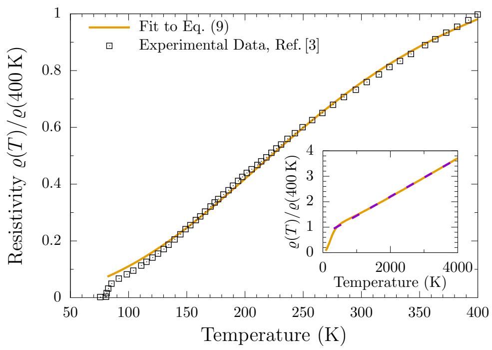

FIG. 3. The crude scaling form (9) fit to experimental data, extracted from Fig. 2 of Ref. [3], for the resistivity as a function of temperature in $HgBa_2CuO_{4+\delta}$ at $p = 11\%$ doping. Here $A_1/\varrho(400\,\mathrm{K}) \approx 8 \times 10^{-4}\,\mathrm{K}^{-1}$, $B \approx 0.25$, $\alpha \approx 2.8$, and we take $T^* = 280$ K. (Inset) The fit to Eq. (9) plotted to higher temperatures, showing linear in T behavior over many decades (dashed line $\propto T$ is a guide to the eye).

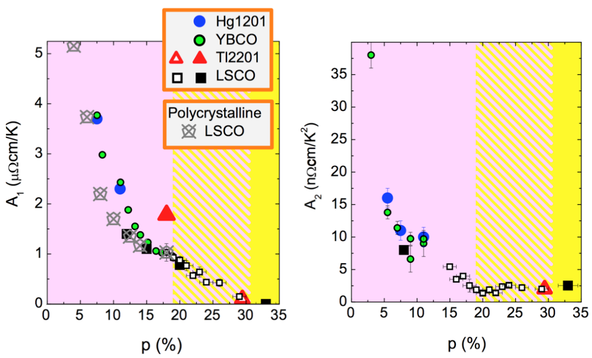

FIG. 4. The doping, p, dependence of coefficients A_1, A_2 of the resistivity $\varrho(T) = A_1(p)T$ [$\varrho(T) = A_2(p)T^2$] at high [low] temperatures in a number of cuprate superconductors. Figure adapted from Ref. [3], where further details can be found.

to a resistivity of the form $\varrho(T) = A_1(p)T$ at high temperatures and $\varrho(T) = A_2(p)T^2$ at low-temperatures; the doping-dependent coefficients $A_1(p)$, $A_2(p)$ are displayed in Fig. 4. Both coefficients increase dramatically as p decreases within the PS phase. This behavior is consistent with Eq. (7), as the PS state is approaching the MI – a state with a substantial single particle energy gap. The screening at low energies is reduced, leading to an increase in the U-interaction, u in Eq. (2). The length of the antinodal gapped region is also enhanced. $A_1(p)$ is finite but small in the overdoped region $p > 0.20$, consistent with the one-loop FRG calculations of Ref. [35].

The Hall angle.— Our scenario can also explain an old mystery, that of the two scattering rates observed in the longitudinal and Hall conductivities.[36] The magnetic field in transport coefficients scales with T^2, which we suggest is related to the scattering rate of the nodal quasiparticles, while the longitudinal conductivity suggests scattering rate linear in T. Such a separation of scattering rates naturally follows from our theory, where nodal and antinodal fermions interact weakly with each other. The nodal pockets are conventional Fermi liquids, giving rise to the T^2 transport coefficient in a magnetic field. Instead, the antinodal regions can be mapped onto an effective ladder model with *with zero interchain tunneling*.[15] The absence of interchain tunneling is implied by the exact degeneracy of the two bands and explains why these regions—which give T-linear contributions to the longitudinal resistivity—are unaffected by the magnetic field. In order for the magnetic field to influence the transport of antinodal quasiparticles, the quasiparticles must be able to move around a closed path that encloses magnetic flux. When interchain tunneling is absent, no such paths can be formed and hence the antinodal regions do not contribute to the Hall conductance. In [32] we use Ong's geometric analysis of the Hall effect in 2D metals[37] to explain the anomalous behavior of the Hall constant in the PS phase.

Conclusions.— In this work we have proposed a simple mechanism for the origin of the linear resistivity in the normal phase of high temperature cuprate superconductors. As with a multitude of other works on the normal phase, including the enigmatic PS, this mechanism places U-scattering at its heart. Elastic U-scattering within the 'strange metal' phase of the cuprates leads to $\varrho(T) \propto T$. As U-scattering flows to strong coupling under the FRG within microscopic models of the cuprates in the normal phase,[16] it is reasonable to expect that this is the dominant contribution to the resistivity. Decreasing the temperature, the PS opens and restricts the available U-scattering channels, leading to a crossover between the linear resistivity of the 'strange metal' and the traditional T^2 form of the scattering from the Fermi pockets.

The analogy between the physics of ladders and under doped cuprate superconductors explains phenomena beyond the transport discussed here.[38] The excitation spectrum of the fermionic ladders contains gapped collective modes that match those observed in the cuprates. The most obvious one is the spin $S = 1$ magnon,[39] while another is the cooperon – a gapped bound state of two electrons (holes).[8,40] Ongoing work suggests that the interaction of the cooperon with nodal quasiparticle pairs leads to a kink appearing within the spectrum of nodal quasiparticles.[41]

The path towards detailed microscopic calculations is clear. The wave packet approach to interacting fermions, introduced by Ossadnik,[11] allows one to map the full 2D problem to that of 1D (effective) two-leg ladders. This then opens the door to calculations that use the non-perturbative tools available for ladder systems, such as bosonization and refermionization[7,8,15,42] and integrability,[8,43] to further study the contribution of U-scattering to the resistivity.

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