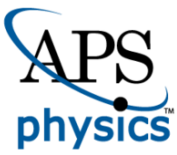

Hydrodynamic flows of non-Fermi liquids: Magnetotransport and bilayer drag

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Phys. Rev. B **96**, 205417 — Published 9 November 2017

DOI: 10.1103/PhysRevB.96.205417

Hydrodynamic flows of non-Fermi liquids: magnetotransport and bilayer drag

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(Dated: October 19, 2017)

We consider a hydrodynamic description of transport for generic two dimensional electron systems that lack Galilean invariance and do not fall into the category of Fermi liquids. We study magnetoresistance and show that it is governed only by the electronic viscosity provided that the wavelength of the underlying disorder potential is large compared to the microscopic equilibration length. We also derive the Coulomb drag transresistance for double-layer non-Fermi liquid systems in the hydrodynamic regime. As an example, we consider frictional drag between two quantum Hall states with half-filled lowest Landau levels, each described by a Fermi surface of composite fermions coupled to a $U(1)$ gauge field. We contrast our results to prior calculations of drag of Chern-Simons composite particles and place our findings in the context of available experimental data.

I. INTRODUCTION

Hydrodynamic flow of electrons can occur in solid state systems provided that the microscopic length scale of momentum-conserving electron-electron collisions is sufficiently short [1]. Under this condition the electron liquid attains local equilibrium and can be described in terms of slow variables associated with conserved quantities such as momentum and energy. However, this transport regime was hard to realize experimentally as typically electron-impurity scattering degrades electron momentum, whereas electron-phonon collisions violate both momentum and energy conservations of the electron liquid. Early evidence for the so-called hydrodynamic Gurzhi effect, related to the negative temperature derivative of resistivity, was reported in thin potassium wires [2], and later in the electrostatically defined wires in the two dimensional electron gas of (Al,Ga)As heterostructures [3]. The recent surge of experiments devoted to revealing hydrodynamic regimes of electronic transport is mainly focused on measurements conducted on graphene [4,5].

In the context of transport theories, a hydrodynamic description is powerful as it accurately describes most liquids. All microscopic details of the system at hand are then encoded into a handful of kinetic coefficients such as viscosities and thermal conductivity. In certain cases the latter can be controllably derived from the linearized Boltzmann kinetic equation by following the perturbative Chapman-Enskog procedure developed originally for gases. However, we have examples now where this kind of microscopic approach has to be substantially revisited. Deriving hydrodynamics for linearly dispersing electronic excitations in graphene represents an interesting example where this standard computation scheme had to be redone from scratch [6–10]. An even more dramatic example is given by strongly correlated electron liquids [11], where the effects of interactions are nonperturbative, and thus a Boltzmann-like description may not be applicable. Yet the hydrodynamic picture still holds [12] and has to be viewed as a phenomenology that enables one to express various transport observables in terms of pristine kinetic coefficients of the electron liquid and certain thermodynamic quantities. This is our motivation to consider a hydrodynamic description of transport for strongly correlated electron liquids where we do not assume Fermi liquid-like behavior. We also do not assume Galilean invariance to be present. In this study we focus on magnetotransport and frictional drag transresistance in bilayers.

II. HYDRODYNAMIC FORMALISM

The general linearized set of equations that govern nonrelativistic magnetohydrodynamic transport in two dimensional charged fluids are given by [13–17] (i) the force equations (repeated indices imply summation throughout this work)

$$\begin{aligned}&\partial_t(Mv_i)+\partial_j\mathcal{T}_{ij}=QE_i+S\xi_i+B\epsilon_{ij}J_j,\\&\mathcal{T}_{ij}=P\delta_{ij}-\eta(\partial_iv_j+\partial_jv_i)-(\zeta-\eta)\partial_kv_k\delta_{ij},\end{aligned}\tag{1}$$

which relate the rate of change of the momentum density to pressure, viscous, thermoelectric and Lorentz forces. M serves as an effective "mass density" and Q is the effective charge density of the fluid. η and ζ respectively are the shear and bulk viscosities. E_i and ξ_i represent the electric field and thermal gradient. Fluctuations in the fluid pressure P are given by $dP=Qd\mu+SdT$, where S is the entropy density and μ is the local screened chemical potential per unit charge. (ii) The equations for charge and heat currents read respectively as

$$\begin{aligned}J_i&=Qv_i-\sigma^Q_{ij}(\partial_j\mu-E_j-B\epsilon_{jk}v_k)-\alpha^Q_{ij}(\partial_jT-\xi_j),\\J^H_i&=TSv_i-T\bar{\alpha}^Q_{ij}(\partial_j\mu-E_j-B\epsilon_{jk}v_k)-\bar{\kappa}^Q_{ij}(\partial_jT-\xi_j),\end{aligned}\tag{2}$$

where σ^Q, α^Q and $\bar{\kappa}^Q$ are microscopic "incoherent" conductivities [18], and (iii) the continuity equations are

$$\partial_tQ+\partial_iJ_i=\partial_t(TS)+\partial_iJ^H_i=0.\tag{3}$$

Onsager reciprocity requires $\alpha^Q_{ij}(B) = \bar{\alpha}^Q_{ji}(-B)$. The incoherent conductivities, viscosities and thermodynamic properties are derived from correlation functions of the underlying microscopic field theory of the non-Fermi liquid [19–22]. This is a generalization of the usual theory of hydrodynamics to systems without Galilean invariance.

A. Magnetotransport in a single layer

We consider the steady state solutions of these equations in the presence of a disordered chemical potential $\mu(\mathbf{x})$. In the absence of applied electric fields and temperature gradients, we can apply a background electric field $\bar{E}_i = \partial_i \mu$ to nullify currents and fluid motion, assuming a uniform temperature. We then look for steady state solutions when this background is perturbed by an infinitesimal uniform electric field δE_i in linear response [10,16]. The difference between the unperturbed and perturbed set of equations gives

$$\begin{aligned}
&\partial_i J_i = \partial_i(Qv_i - \sigma^Q_{ij}(\partial_j\delta\mu - \delta E_j - B\epsilon_{jk}v_k) - \alpha^Q_{ij}\partial_j\delta T)\\
&=\partial_i(TSv_i - T\bar{\alpha}^Q_{ij}(\partial_j\delta\mu - \delta E_j - B\epsilon_{jk}v_k) - \bar{\kappa}^Q_{ij}\partial_j\delta T) = 0,\\
&Q(\partial_i\delta\mu - \delta E_i) + S\partial_i\delta T - \partial_j(\eta(\partial_i v_j + \partial_j v_i))\\
&- \partial_i((\zeta - \eta)\partial_k v_k) = B\epsilon_{ij}J_j, \qquad (4)
\end{aligned}$$

where the delta-quantities represent deviations from the background values generated by the applied electric field ($v_i \sim O(\delta)$). We read off transport coefficients by looking at the change of the uniform components of their respective currents with respect to the applied electric field. For example, $\sigma_{xx} = \delta J_x(q = 0)/\delta E_x$ and $\sigma_{yx} = \delta J_y(q = 0)/\delta E_x$. The equations (4) need to be given periodic boundary conditions in order to ensure a unique solution; otherwise one may shift v by a constant and cancel the effects by appropriately shifting $\delta\mu$ by a function that has a constant gradient [16]. We can consider the solution of (4) while treating disorder perturbatively [10,16]. Against a uniform background chemical potential, it is easy to see that the only response is a finite uniform velocity field $v^{(0)}_i = \epsilon_{ij}\delta E_j/B$, which implies $\sigma^{(0)}_{ij} = \epsilon_{ij}Q^{(0)}/B$, where $Q^{(0)}$ is the uniform charge density in the absence of any disorder. Introducing a small parameter ϵ to parameterize the strength of the disorder, we expand $\mu(\mathbf{x}) = \sum_{n=1}^{\infty}\epsilon^n\mu^{(n)}(\mathbf{x})$. All responses, densities, viscosities and microscopic conductivities may also be expanded in powers of ϵ. For example $v_i(\mathbf{x}) = \sum_{n=0}^{\infty}\epsilon^n v_i^{(n)}(\mathbf{x})$ and $Q(\mathbf{x}) = \sum_{n=0}^{\infty}\epsilon^n Q^{(n)}(\mathbf{x})$.

Order by order in ϵ, there are 4 unknowns $\delta\mu^{(n)}$, $\delta T^{(n)}$, $v_i^{(n)}$ and 4 equations in (4), so a unique solution is possible. This expansion in disorder strength while keeping B finite implies the assumption that the magnetic field relaxes momentum faster than the disordered potential (see [23] for when both relaxation rates are comparable). The expression for the uniform charge current at $\mathcal{O}(\epsilon^2)$ is (in momentum space)

$$\begin{aligned}
&BJ_i^{(2)}(k=0) = \epsilon_{ij}Q^{(2)}(k=0)E_j - i\epsilon_{ij}\\
&\times\int_{\mathbf{k}}(Q^{(1)}(-\mathbf{k})\delta\mu^{(1)}(\mathbf{k}) + S^{(1)}(-\mathbf{k})\delta T^{(1)}(\mathbf{k}))k_j. \qquad (5)
\end{aligned}$$

Thus, solving the equations at $\mathcal{O}(\epsilon)$ gives all the information needed to obtain the uniform conductivities up to $\mathcal{O}(\epsilon^2)$.

In general the densities, viscosities and incoherent conductivities depend on B, and their functional forms can be deduced from the underlying quantum critical theory, which is beyond the scope of hydrodynamics. However, for small values of B these dependences can be neglected as the dominant effect on magnetoresistance arises from the long-range modulations of the equilibrium density (see Refs. [24–26] for other large B effects). This contribution exceeds the one due to the B-dependence of the kinetic coefficients of the liquid by a parametrically large factor controlled by the ratio of disorder wavelength to electron equilibration length. We hence set the off-diagonal components of the quantum critical transport to zero. We assume that $\partial Q/\partial\mu \neq 0$ and $\partial S/\partial\mu \neq 0$, so $Q^{(1)} \neq 0$ and $S^{(1)} \neq 0$. The solutions of (4) are provided in the supplementary material [27].

Using (5) to read off the uniform charge current, we see that $\sigma_{xx,yy}$ are $\mathcal{O}(\epsilon^2)$, whereas σ_{xy} is $\mathcal{O}(1)$. Hence the symmetrized electrical resistance is given by Tr $\rho \approx (\sigma_{xx} + \sigma_{yy})/\sigma^2_{xy}$. This is in general a very complicated function, with a potentially complicated temperature dependence due to the temperature dependences of all the microscopic coefficients. However, if we assume that the disorder is very long wavelength, thus retaining only the leading contribution in the inverse disorder wavelength in the diagonal conductivity, we find a rather simple result

$$\begin{aligned}
\sigma_{ij} &= \sigma^{(0)}_{ij} + \epsilon^2\sigma^{(2)}_{ij} = \epsilon_{ij}\frac{Q^{(0)} + \epsilon^2 Q^{(2)}(k=0)}{B}\\
&+ \frac{\epsilon^2}{\eta^{(0)}}\epsilon_{il}\epsilon_{jm}\int_{\mathbf{k}}\frac{k_l k_m|Q^{(1)}(\mathbf{k})|^2}{k^4}, \qquad (6)
\end{aligned}$$

which is consistent with Onsager reciprocity $\sigma_{ij}(B) = \sigma_{ji}(-B)$. All corrections from the microscopic incoherent conductivities appear at higher orders in the inverse disorder wavelength (for details see supplementary information [27]). For the second term of (6) to be smaller than the first, so the perturbative structure is consistent, we must have $(\partial Q/\partial \ln\mu) \ll (\eta^{(0)}Q^{(0)})/(\lambda^4_\mu B))^{1/2}$, where λ_μ is a characteristic wavelength of the disorder. To leading order in ϵ, one gets the symmetrized magnetoresistance at leading order in the inverse disorder wavelength

$$\text{Tr }\rho(B) - \text{Tr }\rho(0) = \frac{\epsilon^2 B^2(\partial Q/\partial\mu)^2_T}{Q^{(0)2}\eta^{(0)}}\int_{\mathbf{k}}\frac{|\mu^{(1)}(\mathbf{k})|^2}{k^2}. \qquad (7)$$

The temperature dependence of the magnetoresistance is controlled only by the viscosity in this long-wavelength

disorder limit, as was the case in [28] for the special case of Galilean-invariant fluids ($\sigma^Q = \alpha^Q = 0$). However, there the magnetoresistance was controlled only by the viscosity regardless of the spectrum of the disorder. Since we do not expect most non-Fermi liquid metals to be Galilean-invariant, this is an important strengthening of the previous result. It can additionally be shown that the long-wavelength disorder result (7) is also insensitive to the Hall viscosity [29] and vorticity susceptibility [30], which are new parity-odd microscopic transport coefficients that can appear in the presence of a magnetic field.

The above result could enable the extraction of the temperature dependence of the viscosity of the electron liquid as $\delta\rho(B)/\rho(0) \propto 1/\eta^{(0)}$ and thus allow for testing theoretical models of potential non-Fermi liquid states in the hydrodynamic regime. In the supplementary material we also provide results for the magnetothermal resistance.

B. Drag transport in bilayers

For drag type transport [31], we use our hydrodynamic equations for each layer of the bilayer system, with $E = B = \xi = 0$. Drag is generated by intrinsic hydrodynamic fluctuations encoded in fluctuating noise terms [32–35] added to T_{ij}, J_i, J_i^H that are uncorrelated between the layers ($T_{ij}^{1,2} \to T_{ij}^{1,2} + s_{ij}^{1,2}$, $J_i^{1,2} \to J_i^{1,2} + r_i^{1,2}$, $J_i^{H\ 1,2} \to J_i^{H\ 1,2} + g_i^{1,2}$)

$$\begin{aligned}
&\langle s_{ij}^{1,2}(\mathbf{k},\omega) s_{lm}^{1,2}(\mathbf{k}',\omega')\rangle = 2T(\eta^{(0)}(\delta_{il}\delta_{jm}+\delta_{im}\delta_{jl})\\
&+(\zeta^{(0)}-\eta^{(0)})\delta_{ij}\delta_{lm})\delta(\mathbf{k}+\mathbf{k}')\delta(\omega+\omega'), \qquad (8)\\
&\langle r_i^{1,2}(\mathbf{k},\omega) r_j^{1,2}(\mathbf{k}',\omega')\rangle = 2T\sigma^{Q(0)}\delta_{ij}\delta(\mathbf{k}+\mathbf{k}')\delta(\omega+\omega'),\\
&\langle g_i^{1,2}(\mathbf{k},\omega) g_j^{1,2}(\mathbf{k}',\omega')\rangle = 2T^2\bar{\kappa}^{Q(0)}\delta_{ij}\delta(\mathbf{k}+\mathbf{k}')\delta(\omega+\omega'),\\
&\langle r_i^{1,2}(\mathbf{k},\omega) g_j^{1,2}(\mathbf{k}',\omega')\rangle = 2T^2\alpha^{Q(0)}\delta_{ij}\delta(\mathbf{k}+\mathbf{k}')\delta(\omega+\omega'),
\end{aligned}$$

with all other correlators of the sources being zero. These fluctuations induce fluctuations in the charge and entropy densities in the layers ($Q_{1,2}^{(0)}, S_{1,2}^{(0)} \to Q_{1,2}^{(0)} + \delta Q_{1,2}, S_{1,2}^{(0)} + \delta S_{1,2}$). The fluctuations of chemical potential and temperature are expressed in terms of the charge and entropy fluctuations

$$\delta\mu_{1,2} = \left(\frac{\partial\mu}{\partial Q}\right)_S \delta Q_{1,2} + \left(\frac{\partial\mu}{\partial S}\right)_Q \delta S_{1,2}, \qquad (9)$$

and likewise for $\delta T_{1,2}$. We must add to the pressure term in each layer the effects of intra and inter-layer Coulomb forces generated by the fluctuations in the charge densities (the layers are separated by a distance d)

$$\delta P_{1,2} \to \delta P_{1,2} + \frac{2\pi Q^{(0)}}{k}(\delta Q_{1,2} + e^{-kd}\delta Q_{2,1}). \qquad (10)$$

The drag resistance measures the sensitivity of the electric field induced by the dragging force in the open-circuit passive layer to the current flowing in the driven layer. It is given by

$$\begin{aligned}
&\rho_D \equiv \frac{E_{2D}}{J_1} = \frac{F_{12}(v_x) - F_{12}(0)}{Q^{(0)2} v_x}, \qquad (11)\\
&F_{12}(v_x) = -i\int_{\mathbf{k},\omega} \frac{2\pi}{k} k_x e^{-kd}\langle \delta Q_1(\mathbf{k},\omega)\delta Q_2(-\mathbf{k},-\omega)\rangle.
\end{aligned}$$

The derivation of these force and pressure relations only requires a straightforward application of Coulomb's law. In additional to the noise sources, we also linearize in the velocity v_x (the driven layer is driven by this uniform velocity field, not by an electric field). Note that $J_1 = Q^{(0)} v_x$ is valid even for non-Galilean invariant fluids as the noise terms themselves cannot induce any uniform current flow due to averaged inversion and time-reversal symmetries. Thus J_1 must vanish when $v_x = 0$, and renormalizations of $Q^{(0)}$ due to the noise terms are subleading.

We neglect the effects of thermal currents: they produce only subleading effects at large spatial separations (see supplementary information for further details [27]). Switching to the basis defined by $\delta Q_\pm = \delta Q_1 \pm \delta Q_2$, $s_{ij}^\pm = s_{ij}^1 \pm s_{ij}^2$, $r_i^\pm = r_i^1 \pm r_i^2$, the hydrodynamic equations can be reduced to the form

$$\begin{aligned}
&\Pi_\pm \delta Q_\pm + \frac{M}{2} k_x (i(D_\sigma + D_\eta)k^2 + \omega) v_x (\delta Q_+ + \delta Q_-) =\\
&\frac{M}{2} k_x v_x k_i (r_i^+ + r_i^-) - iM(D_\eta k^2 - i\omega) k_i r_i^\pm - Q^{(0)} k_i s_{ij}^\pm k_j,\\
&\Pi_\pm = M(D_\eta k^2 - i\omega)(D_\sigma k^2 - i\omega)\\
&+ kQ^{(0)}(2\pi Q^{(0)}(1 \pm e^{-kd}) + b_1 k), \qquad (12)
\end{aligned}$$

where $D_\sigma = \sigma^{Q(0)}(\partial\mu/\partial Q)_S$, $D_\eta = (\eta^{(0)} + \zeta^{(0)})/M$ and $b_1 = Q^{(0)}(\partial\mu/\partial Q)_S$. The solutions to these equations are linearized in v_x: $\delta Q_\pm = \delta Q_\pm^{(0)} + \delta Q_\pm^{(1)} v_x$. Since the v_x-less configuration obeys averaged inversion and time-reversal symmetry and v_x always appears as $v_x k_x$ which is odd under inversion, $\delta Q_\pm^{(0)}$ is even under $\mathbf{k} \to -\mathbf{k}$ whereas $\delta Q_\pm^{(1)}$ is odd. The dragging force may be written as

$$\begin{aligned}
&\frac{F_{12}(v_x) - F_{12}(0)}{i\pi} = \int_{\mathbf{k},\omega} \frac{k_x v_x}{k e^{kd}} (\langle \delta Q_+^{(0)}(\mathbf{k},\omega)\delta Q_-^{(1)}(-\mathbf{k},-\omega)\rangle\\
&- \langle \delta Q_-^{(0)}(\mathbf{k},\omega)\delta Q_+^{(1)}(-\mathbf{k},-\omega)\rangle). \qquad (13)
\end{aligned}$$

All other terms vanish upon momentum/frequency integration due to even/odd cancellations. Inserting the solutions of (12), we obtain $\rho_D = \rho_D^\sigma + \rho_D^\eta$, where ρ_D^σ is generated by the charge fluctuations $r^\pm$ and ρ_D^η is generated by the viscous fluctuations $s^\pm$:

$$\begin{aligned}
&\rho_D^\sigma = M^3 T \sigma^{Q(0)} \int_0^\infty dk \int_\omega \frac{(D_\sigma + D_\eta)k^7(\omega^2 - D_\eta^2 k^4)}{e^{2kd}|\Pi_+|^2|\Pi_-|^2},\\
&\rho_D^\eta = MQ^{(0)2} T(\eta^{(0)} + \zeta^{(0)}) \int_0^\infty dk \int_\omega \frac{(D_\sigma + D_\eta)k^9}{e^{2kd}|\Pi_+|^2|\Pi_-|^2}.
\end{aligned} \qquad (14)$$

This yields a complicated integral expression for ρ_D. We can however make simplifications in the regimes of "large" and "small" d. The model of Fermi surface coupled to $U(1)$ gauge field has roughly the following properties [20,21,36] for dynamical critical exponent $z = 3$ (m is the effective fermion mass), corresponding to the case of short-ranged interactions of composite fermions [36]:

$$Q^{(0)} \propto e k_F^2, \quad M \propto m k_F^2, \quad \left(\frac{\partial \mu}{\partial Q}\right)_S \propto \frac{\hbar^2}{e^2 m}, \quad b_1 \propto \frac{E_F}{e},$$
$$\sigma^{Q(0)} \propto \left(\frac{E_F}{k_B T}\right)^{2/3} \frac{e^2}{\hbar}, \quad \eta^{(0)} \sim \zeta^{(0)} \propto \left(\frac{E_F}{k_B T}\right)^{2/3} \hbar k_F^2. \tag{15}$$

d is said to be "large" when $d^3 \gg d_c^3 \equiv M(D_\eta + D_\sigma)^2/Q^{(0)2}$. This gives

$$(k_F d)^3 \gg \left(\frac{E_F}{k_B T}\right)^{4/3} \frac{\varepsilon E_F}{e^2 k_F}. \tag{16}$$

We have set the electrostatic permittivity $\varepsilon = 1$ so far in the paper but restored it in the last equation. We also demand $d \gg d_e \equiv \hbar^2 \varepsilon/(e^2 m)$, which is trivially achieved as d_e is typically a very small distance scale ($\mathcal{O}(10^{-10})$ m for $m \sim m_e/4$).

For $d \gg d_c$ we obtain the leading contributions

$$\rho_D^\sigma \sim \frac{\hbar}{e^2}\left(\frac{k_B T}{E_F}\right)^{1/3} \frac{\ln^4\left(\frac{d k_F^{2/3}}{d_e^{1/3}}\left(\frac{k_B T}{E_F}\right)^{4/9}\right)}{(k_F d)^4},$$
$$\rho_D^\eta \sim \frac{\hbar}{e^2}\left(\frac{k_B T}{E_F}\right)^{1/3} \frac{\ln^5\left(\frac{d k_F^{2/3}}{d_e^{1/3}}\left(\frac{k_B T}{E_F}\right)^{4/9}\right)}{(k_F d)^5/(k_F d_e)}. \tag{17}$$

ρ_D^σ and ρ_D^η have the same temperature scaling up to logarithms. However, ρ_D^η falls off faster with d than ρ_D^σ. This results should be contrasted to that obtained earlier for Fermi liquids [34]. Note that even though the power dependence on temperature is $T^{1/3}$, there is a $\ln^4(T)$ correction, which will make the temperature dependence appear faster than $T^{1/3}$ but slower than T, which is consistent with the data of Refs. [37,38] at large separations.

At small separations $d \ll d_c$, all contributions to ρ_D scale as $T^{n>1}$ (see further details in supplementary materials). This is again consistent with [38], which shows an apparent crossover from positive to negative curvature in $\rho_D(T)$ as a function of T as d is increased. In Fig. 1 we show $\rho_D(T)$ obtained by numerically evaluating the integrals without the above approximations that confirm the qualitative behaviors we discussed. It should be carefully noted that in Fig. 1 the line corresponding to $d = 150$ nm appears superficially above the line of $d = 15$ nm plot which is due to the choice of the normalization factor $\rho_0(d)$. Drag is obviously a decaying function of inter-layer separation d as is clear from (17).

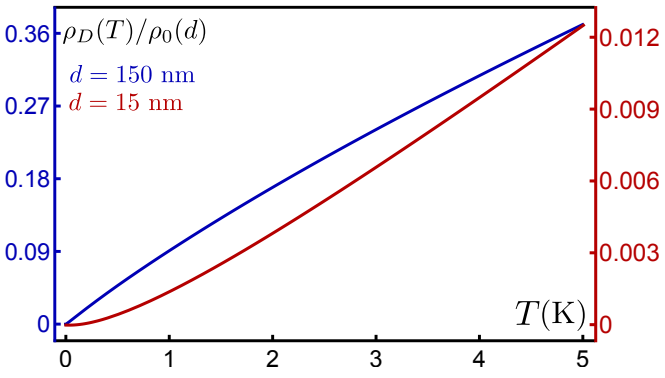

FIG. 1: Normalized drag resistance $\rho_D(T)/\rho_0(d)$. $\rho_D(T) = \rho_D^\sigma(T) + \rho_D^\eta(T)$ is obtained by numerically evaluating (14) for two different spatial separations. $\rho_0(d) = (\hbar/e^2)/(k_F d)^4$. Note the crossover from positive to negative curvature as d is increased. This feature holds for other values of the dynamical critical exponent $2 < z < 3$ as well that can appear in the theory of Ref. [36]. We use $T_F \sim 50$ K and $m \sim m_e/4$ ($d_c \sim 10$ nm at $T \sim 5$ K). We set all constants of proportionality in (15) to 1. Numerical values should be treated as order-of-magnitude estimates only.

III. DISCUSSION

The most extensively studied example of transresistance in the case of non-Fermi liquids corresponds to inter-layer frictional Coulomb drag between bilayers of half-filled Landau levels [37–40]. The theoretical approach that has proved most useful for understanding the filling fraction $\nu = 1/2$ state is the fermion Chern-Simons field theory, which is based in turn on the composite-fermion picture [36]. Previous calculations [41–43] showed that the dominant low-temperature behavior for ρ_D scales with temperature as $T^{4/3}$ (see supplementary material for a brief summary of this result [27]). This unique power exponent can be traced back to a special momentum dependence of the electronic longitudinal conductivity, as can be deduced from surface acoustic wave measurements. Indeed, in the composite-fermion picture, at $\nu = 1/2$, the density response at small frequencies and small wave-vectors is of the form $\propto (k^3 - 8\pi i \chi \omega k_F)^{-1}$, which can be viewed as slow diffusion with an effective diffusion constant that vanishes linearly with k (where χ is the thermodynamic compressibility of the $\nu = 1/2$ state). Since the typical frequency is set by temperature $\omega \sim T$, the pole structure of long-wavelength density fluctuations sets a characteristic scale for momentum transfer between the layers $k \propto T^{1/3}$ that then carriers over to drag resistance $\rho_D \propto T^{4/3}$. This should be contrasted the Fermi liquid prediction $\rho_D \propto T^2$ at lowest temperatures, and our prediction $\rho_D \propto T^{1/3}$. In our current understanding, the results of Ref. [41] correspond to the "collisionless" regime of transport with respect to intra-layer collisions, namely a long equilibration length as mediated by interactions with the gauge field.

We considered the opposite collision-dominated regime where this length scale is assumed to be short. This should explain the difference between the power exponents 4/3 and 1/3 between two limiting cases. We hope that understanding different transport regimes and corresponding temperature dependencies will be of help for the interpretation of future experiments, as it also deepens our current understanding of the existing transport data and corresponding theories.

Acknowledgements

We thank A. Andreev, L. V. Delacretaz, B. Halperin, P. Kim, A. Lucas, and S. Sachdev for helpful discussions. A.A.P. acknowledges support by NSF Grant DMR-1360789. R.A.D. is supported by the Gordon and Betty Moore Foundation Grant GBMF-4306. The work of A.L. was financially supported in part by NSF CAREER Grant DMR-1653661, and by the Office of the Vice Chancellor for Research and Graduate Education with funding from the Wisconsin Alumni Research Foundation.

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