Distinguishing XY from Ising electron nematics

S. Basak and E. W. Carlson

Phys. Rev. B **96**, 081303 — Published 11 August 2017

DOI: 10.1103/PhysRevB.96.081303

Distinguishing XY from Ising Electron Nematics

S. Basak[1] and E. W. Carlson[1,*]

[1]*Department of Physics, Purdue University, West Lafayette, IN 47907, USA*

At low temperatures in ultraclean GaAs-AlGaAs heterojunctions, high Landau levels near half-integral filling break rotational symmetry, leading to increasingly anisotropic transport properties as temperature is lowered below ∼150mK. While the onset of transport anisotropy is well described by an XY model of an electron nematic in the presence of a weak uniform symmetry-breaking term, the low temperature behavior deviates significantly from this model. We find that inclusion of interactions between the electron nematic and the underlying crystalline lattice in the form of a 4-fold symmetry breaking term is sufficient to describe the entire temperature dependence of the transport anisotropy at $\nu = 9/2$. This implies that this electron nematic is in the Ising universality class. We propose new experimental tests that can distinguish whether any two-dimensional electron nematic is in the XY or Ising universality class.

Strong electron correlations can drive systems into a variety of novel electronic phases of matter. Electronic liquid crystals[1–3] form when electronic degrees of freedom partially break the symmetries of the host crystal. Like their molecular counterparts, electron nematic phases break rotational symmetry, while retaining liquidity. Such oriented electronic liquids have been observed in a variety of systems, including strontium ruthenates[4], iron superconductors[5–7], cuprate superconductors[8,9], the (111) surface of bismuth[10,11], and high fractional Landau levels. The key signature in the quantum Hall regime is a pronounced transport anisotropy that develops at low temperature.[12–16]

At high fractional Landau levels, uniform quantum Hall phases are unstable to the formation of stripe and bubble phases, with the stripe phases being preferred near high half-filling.[17] Several stripe phases are possible, including (insulating) stripe crystals, as well as (compressible) electronic liquid crystal phases like nematic or smectic.[2] In ultraclean GaAs-AlGaAs heterojunctions near high half-integral fillings $\nu \geq 9/2$[13,14], longitudinal transport anisotropy spontaneously develops at low temperature. This state has been identified as a nematic quantum Hall metal (NQHM), an oriented, compressible stripe phase of interleaved integer quantum Hall states.[18,19] Fradkin *et al.*[18] developed an order parameter theory of the nematic to describe the temperature evolution of the resistivity anisotropy as it develops. Using symmetry to map the resistivity anisotropy to the nematic order parameter, they showed that the temperature evolution of the resistivity anisotropy in the $\nu = 9/2$ state is well described by a classical 2D XY model, with a weak uniform symmetry-breaking term, through the onset of the resistivity anisotropy as temperature is lowered below ∼150mK, with deviations from the theory beginning below ∼55mK. This model places the transition in the BKT universality class[20,21].

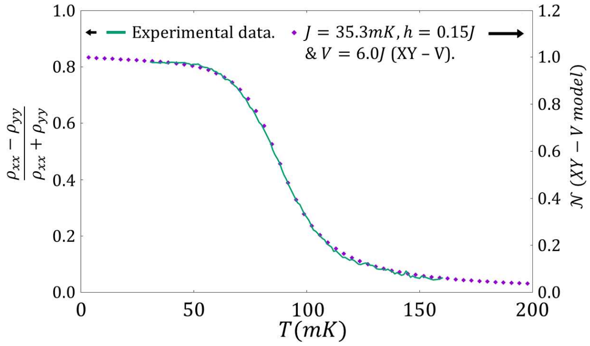
One difficulty with this identification is that a true BKT transition does not break symmetry, and in fact in that model long-range order of a nematic is forbidden at finite temperature. However, as stressed in Ref. 18, the nematic susceptibility is sufficiently strong in the BKT phase that net nematicity can develop anyway in the presence of even a weak uniform orienting field. Note that without the development of net nematicity, the resistivity anisotropy would be zero.

Here, we propose an order parameter model of the NQHM which solves both the problem of the deviation of the low temperature resistivity anisotropy data from the order parameter theory, as well as the issue of long-range order. The nematic order parameter is a headless vector, which depends on overall orientation but not the direction. (That is, it is symmetric with respect to a 180^o rotation.) If the interaction between the electron nematic and the host crystal is sufficiently weak, the nematic is free to form in any direction, and an XY model of the development of nematicity is appropriate.[18] We first consider this case, but in the presence of lattice effects which ultimately at low temperature lock the nematic to a crystalline axis:

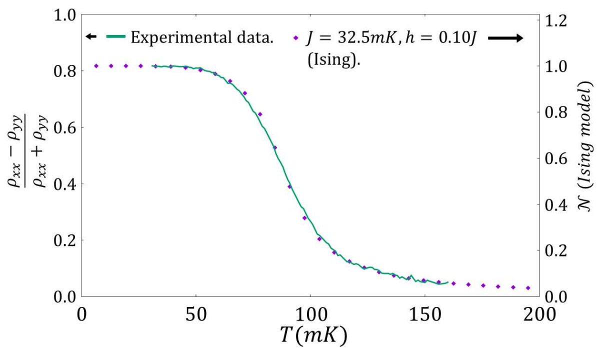
$$H = -J \sum_{\langle i,j \rangle} \cos\left(2(\theta_i - \theta_j)\right) - h \sum_i \cos\left(2(\theta_i - \phi)\right) - V \sum_i \cos\left(4\theta_i\right), \quad (1)$$

where J represents the tendency of neighboring regions to align their stripe orientation due to Coulomb interactions. The orienting field h can arise from intrinsic orienting effects such as an interaction with the sample boundaries, or from an external field tuned via, *e.g.*, an applied orienting field such as an in-plane magnetic field or strain, among other things.[22,23] The interaction of the electrons with a four-fold symmetric bandstructure gives rise to the V term, where the continuous U(1) symmetry is broken by the C_4 symmetry of the lattice.[24]

The "nematicity" (order parameter of the nematic) in this model is $\mathcal{N} = \langle e^{2i\theta} \rangle$.[18] Because the (normalized) macroscopic transport anisotropy ρ_a transforms under rotations in the same way as the nematicity, the two are related as $\rho_a \equiv \left[(r+1)/(r-1)\right]\left(\rho_{xx} - \rho_{yy}\right)/\left(\rho_{xx} + \rho_{yy}\right) = f(\mathcal{N})$ where $f(\mathcal{N})$ is an odd function of $\mathcal{N}$, and $r \equiv \rho_{xx}(\mathcal{N} \to 1)/\rho_{yy}(\mathcal{N} \to 1)$ is what the ratio of macroscopic resistivities would be in a fully oriented state. For small $\mathcal{N}$, $f(\mathcal{N}) = \mathcal{N}$.[18,25]



(a) XY model with moderate 4-fold symmetry breaking term V, Eqn. 1



(b) Ising Model, Eqn. 2

FIG. 1. Monte Carlo simulations (purple dot) on a lattice of 100x100 sites, compared to experimental data (green line) of resistivity anisotropy $\frac{\rho_{xx}-\rho_{yy}}{\rho_{xx}+\rho_{yy}}$ from Lilly *et al.*[13]. The theoretical comparison is to: (a) an XY model with a moderate four-fold symmetry breaking field V and uniform orienting field h, and (b) an Ising model with uniform orienting field h. Note that within an XY description, a moderate 4-fold symmetry breaking term $V \neq 0$ is required to capture the low temperature dependence of the resistivity anisotropy, which changes the universality class of the electron nematic from XY to Ising. The resistivity anisotropy $\frac{\rho_{xx}-\rho_{yy}}{\rho_{xx}+\rho_{yy}}$ is from the experimental data of Lilly *et al.*[13].

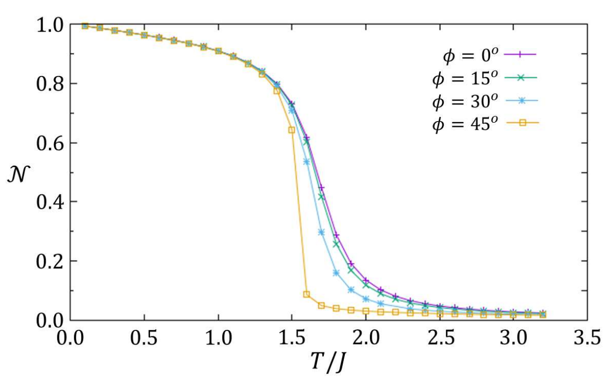


FIG. 2. Monte Carlo simulations on a lattice of 100x100 sites of an XY model with 4-fold symmetry breaking term, for various angles ϕ of the orienting field h. This shows the increasing steepness of the nematic-to-isotropic transition when ϕ goes from $0^o \to 15^o \to 30^o \to 45^o$. All of these simulation are done with $h = 0.05J$ and $V = 1.0J$.

Results for the model of Eqn. 1 are shown in Fig. 1(a). As shown in Fig. 2 of Ref. 18, the experimental data of Ref. 13 can be matched reasonably well for $T \gtrsim 55$mK in the presence of a weak uniform orienting field h and $V = 0$, but with significant deviation below 55mK. We find that the entire temperature evolution can be captured in the presence of both nonzero h and nonzero V, as shown in Fig. 1(a). In the Figure, we use uniform orientational field $h = .15J$ along with four-fold symmetry breaking term $V = 6J$, and $J = 35.3$mK. Smaller values of V have too steep of a slope at low temperature. For larger values of V, the higher temperature behavior ($100-150$mK) can no longer be captured. For the parameters of Fig. 1(a), the absolute strength of the interaction J is about half that of Ref. 18. Because the value of V that we use is not small with respect to J, the universality class of the transition is now Ising, not XY. For a pure XY model with $h = 0$ and $V = 0$, the transition temperature is $T_{\rm KT} = .89J$.[21], but in Fig. 1(a) the onset of nematicity is happening closer to the (2D) Ising transition temperature of $T_c = 2.27J$, consistent with this shift of universality class. (See Supplemental Material[26] for results with other values of parameters.)

The effect of rotating the uniform orienting field h away from a crystalline axis is explored in Fig. 2, where the angle ϕ between h and the crystalline axes is varied. Note that up until $\phi \approx 30^o$ the impact on the temperature evolution is negligible. However, at the high symmetry point $\phi = 45^o$, there is a true symmetry breaking transition, and the temperature onset is quite sudden.[26]

We have found that within an XY description (Eqn. 1), moderate values of V/J are required to capture the entire temperature dependence of the resistivity anisotropy in the NQHM at $\nu = 9/2$. This naturally leads to the question of how well a simple Ising model can account for the data:

$$H = -J \sum_{\langle i,j \rangle} \sigma_i \sigma_j - h \sum_j \sigma_i \ . \qquad (2)$$

Here, we make the assumption that the electron nematic, once it develops, tends to lock to a favorable lattice direction. In a crystal with 4-fold rotational symmetry, because the director of the nematic is a headless vector, the order parameter of the nematic is explicitly in the Ising universality class with the two possible orientations of the nematic being mapped to $\sigma = \pm 1$. A uniform orienting field (whether intrinsic or applied) is modeled by h.

The nematic order parameter in this case is $\mathcal{N} = (1/N)\sum_i \sigma_i$. As in the case of an XY model of an electron nematic, the (normalized) macroscopic resistivity anisotropy ρ_a maps to the macroscopic order parameter in the Ising description as $\rho_a = g(\mathcal{N})$ where $g(\mathcal{N})$ is an odd function of $\mathcal{N}$ and to first order in $\mathcal{N}$, $g = f$.

Note that for $h = 0$, the magnetization for 2D-Ising model is

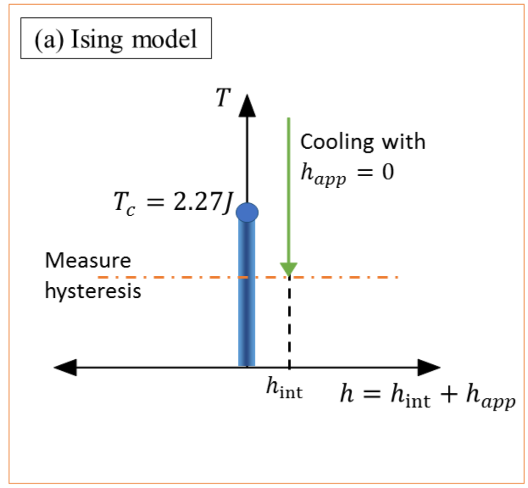
$$M(T, h = 0) = \left[1 - sinh^{-4}\left(\frac{2J}{T}\right)\right]^{\frac{1}{8}} \ , \ T < T_c \ . \qquad (3)$$

Unlike the XY model where the low temperature nematicity can only develop for nonzero h and is linear in temperature T, the Ising magnetization can develop even with $h = 0$ and is flat at low temperatures. Our comparison of the experimental resistivity anisotropy to an Ising model is shown in Fig.1(b). We find that the data can be well described throughout the entire temperature range within a simple Ising model, with $J = 32.5mK$ and $h = 0.1J$.

Remarkably, we find that the entire temperature range of the resistivity anisotropy ρ_a can be captured quite well within an Ising model in the presence of a weak uniform orienting field. Within this context, the low temperature saturation of $(\rho_{xx}-\rho_{yy})/(\rho_{xx}+\rho_{yy})$ to a value $\approx .818 \neq 1$ could have several origins:[18,25] (i) Taken at face value, the saturation implies that the bare "nematogens" represented by each Ising variable have an intrinsic resistivity anisotropy which persists down to the lowest temperatures, $r = \rho_{xx}/\rho_{yy} \approx 10$. This could be attributable to quantum fluctuations within a bare nematogen. (ii) Similar saturation effects could also arise from even a small amount of quenched disorder, since the critical (random field type) disorder strength is zero in a two-dimensional Ising model. (iii) Nonlinear terms in the function $g(\mathcal{N})$ can lead to $g \neq 1$ as $\mathcal{N} \to 1$ at low temperature.

Experimental test of universality class: We propose that low temperature hysteresis measurements can distinguish whether any electron nematic (including the NQHM at high fractional filling discussed here) is in the Ising or XY universality class. Fig. 3 shows the equilibrium phase diagram for both models, as a function of temperature and total *orienting field* h. Note that because the nematic order parameter switches sign upon rotating 90^o, the orienting field is related to the applied in-plane magnetic field by $h \propto B_x^2 - B_y^2$.[22] Other external perturbations also contribute to an orienting field, such as strain.[22,27] In both models, a phase transition only exists at zero orienting field, $h = 0$. In the Ising case, the phase transition is into a low-temperature, long-range ordered nematic phase which spontaneously breaks rotational symmetry. For the 2D XY model, the phase transition is in the BKT universality class, and the low temperature phase is critical throughout the temperature range, with no long range order, and therefore no net nematicity $\mathcal{N}$, measurable by $\mathcal{N} \propto (\rho_{xx} - \rho_{yy})/(\rho_{xx} + \rho_{yy})$. Upon field cooling in any weak h, *both models* will develop a net nematicity below a crossover temperature which is close to the phase transition temperature, whether $T_c = 2.27J$ in the Ising case, or $T_{KT} = .89J$ in the XY case.

However, hysteresis can clearly distinguish between these universality classes. The hysteresis protocol we propose (shown in Fig. 3) is the following: Cool in an orienting field $h > 0$ such as in-plane magnetic field (see Ref. 22 and 23 for a list of orienting fields), and go to low temperature, well within the nematic region. Then, reduce h to zero, and sweep it to negative values $h < 0$. Using, *e.g.*, in-plane magnetic field as an orienting field, this is equivalent to cooling with an in-plane field configuration of $\vec{B}_{\text{in-plane}} = (B_x > 0, B_y = 0)$, then holding the temperature fixed, decreasing B_x to zero, then immediately increasing the field B_y from zero while holding $B_x = 0$ so as to end with an in-plane field configuration of $\vec{B}_{\text{in-plane}} = (B_x = 0, B_y > 0)$. Indeed, quantum Hall stripes can be reoriented via application of in-plane field.[28] At low temperature in the Ising case, there is hysteresis in the net nematicity $\mathcal{N}$ as the in-plane field is



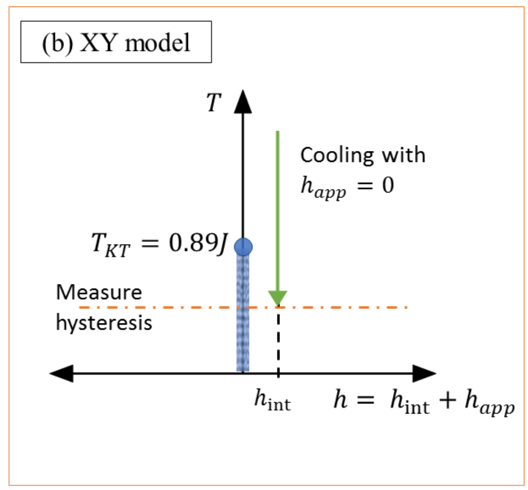


FIG. 3. Equilibrium phase diagram for (a) two-dimensional Ising model and (b) two-dimensional XY model. In both cases, a low temperature phase transition occurs only for orienting field $h = 0$. In the Ising case, the low temperature phase has long-range nematic order, and in the XY case the low temperature phase only has topological order but no long-range nematic order. The experimental hysteresis test we propose begins by (i) cooling (green arrow) with or without applied field h_{app}, followed by (ii) sweeping the orienting field h_{app} so as to move the system back and forth across the low temperature phase (orange dotted line). Refer to Fig. 4 for the experimental prediction of the response of the nematicity $\mathcal{N}$ as a function of applied orienting field.

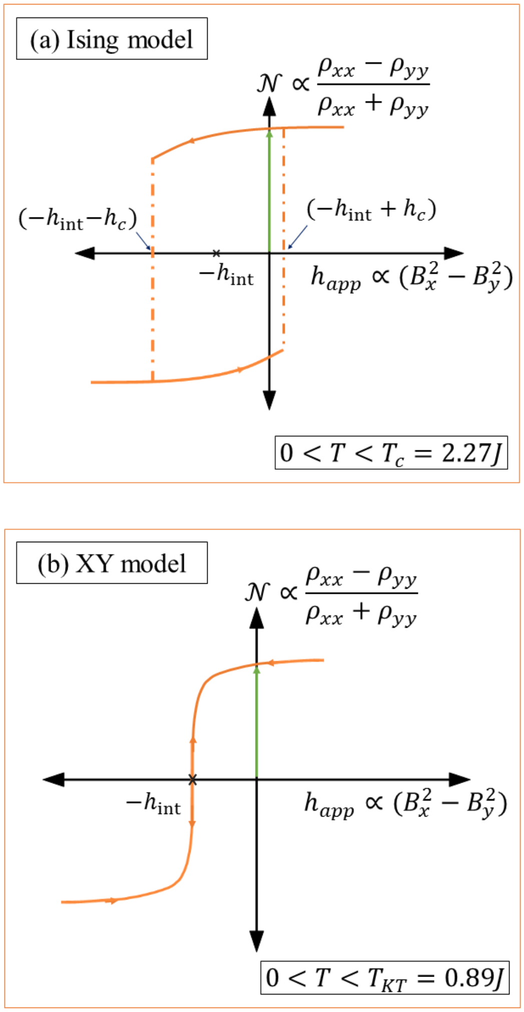


FIG. 4. Predicted result of hysteresis test for (a) an Ising nematic and (b) an XY nematic. Cooling (green arrow) the system below T_c (Ising) or T_{KT} (XY) gives rise to a net nematicity in the presence of any orienting field h, including the case of no applied orienting field, since then $h = h_{\text{int}} \neq 0$. Subsequently sweeping the in-plane orienting field gives rise to either hysteresis in the Ising case, or no hysteresis in the XY case.

swept so as to take h from positive to negative and back again, or *vice versa*. Therefore in the Ising case, the net nematicity should remain in an oriented state, until the coercive field strength $h_c \neq 0$ is reached.

However, in the XY case, there should be no hysteresis. This follows from the Mermin-Wagner-Hohenberg theorem, since decreasing an applied field h so as to end on the critical phase at $h = 0$ can leave no long range order, $\mathcal{N}(h \to 0) \to 0$ where $\mathcal{N}$ is the net nematicity. Because $h \to 0$ with $T < T_{\text{KT}}$ is critical, $\mathcal{N} \propto h^{(1/\delta)}$ as field is swept, where the critical exponent $\delta = (4/\eta) - 1$ varies from $\delta(T_{\text{KT}}) = 15$ to $\delta(T \to 0) \to \infty$.[29] This case is shown in Fig. 4(b).

It should also be noted that the test is clearest in clean samples, since addition of random field effects in the presence of a net orienting field h puts both models in the universality class of the random field Ising model,[30] which has hysteresis at low temperature. Whereas hysteresis of a clean Ising model has a net macroscopic jump in the nematicity, hysteresis of a random field Ising model is smooth in two dimensions.[31] At very weak but finite random field strength, the model predicts avalanches in the resistivity anisotropy around the hysteresis loop with power law behavior set by critical exponents characteristic of the 2D random field Ising model critical point.

Note that our simulations as well as those of Ref. 18 indicate the presence of a weak intrinsic orienting field, h_{int} in the sample, on the order of $h_{\text{int}} \approx 3 - 5\text{mK}$.[26] This means that to achieve $h = 0$ requires that some extrinsic orienting field, such as an in-plane magnetic field or uniaxial strain,[22,23] must be applied to compensate. Assuming this could be achieved, then zero-field cooling (ZFC) with $h = h_{\text{int}} + h_{app} = 0$ has stark differences in the two models. In the Ising case, ZFC gives rise to long-range order with net nematicity and macroscopic resistivity anisotropy, with Ising critical behavior at the onset of nematicity, and the direction of that nematicity can randomly switch upon repeated cooling at $h = 0$. In the XY case, ZFC can't produce long-range order or net nematicity, but the system would instead enter a topological phase with power-law nematic order, and accompanying critical phenomena.

In conclusion, we have shown that the entire temperature dependence of the observed resistivity anisotropy in NQHM at $\nu = 9/2$ can be well described by taking into account the discrete rotational symmetry of the underlying crystal.[32] Inclusion of such a symmetry-breaking term shifts the universality class of the electron nematic from the Kosterlitz-Thouless universality class of the two-dimensional XY model to the two-dimensional Ising universality class. We furthermore propose an experimental test for hysteresis that can clearly distinguish whether any 2D electron nematic is in the Ising or XY (Kosterlitz-Thouless) universality class.

ACKNOWLEDGMENTS

We acknowledge helpful conversations with G. A. Csathy, K. A. Dahmen, E. Fradkin, S. A. Kivelson, P. Muzikar, and H. Nakanishi. E.W.C. and S.B. acknowledge support from NSF Grant No. DMR-1508236, and Department of Education Grant No. P116F140459. This research was supported in part through computational resources provided by Information Technology at Purdue, West Lafayette, Indiana.

* ewcarlson@purdue.edu

1. S. A. Kivelson, E. Fradkin, and V. J. Emery, Nature **393**, 550 (1998).
2. E. Fradkin and S. A. Kivelson, Phys. Rev. B **59**, 8065 (1999).
3. S. A. Kivelson, I. P. Bindloss, E. Fradkin, V. Oganesyan, J. M. Tranquada, A. Kapitulnik, and C. Howald, Rev. Mod. Phys. **75**, 1201 (2003).
4. R. A. Borzi, S. A. Grigera, J. Farrell, R. S. Perry, S. J. S. Lister, S. L. Lee, D. A. Tennant, Y. Maeno, and A. P. Mackenzie, Science **315**, 214 (2007), http://science.sciencemag.org/content/315/5809/214.full.pdf.
5. C. Fang, H. Yao, W.-F. Tsai, J. Hu, and S. A. Kivelson, Phys. Rev. B **77**, 224509 (2008).
6. C. Xu, M. Müller, and S. Sachdev, Phys. Rev. B **78**, 020501 (2008).
7. I. I. Mazin and M. D. Johannes, Nat Phys **5**, 141 (2009).
8. Y. Ando, K. Segawa, S. Komiya, and A. N. Lavrov, Phys. Rev. Lett. **88**, 137005 (2002).
9. V. Hinkov, D. Haug, B. Fauqué, P. Bourges, Y. Sidis, A. Ivanov, C. Bernhard, C. T. Lin, and B. Keimer, Science **319**, 597 (2008), http://science.sciencemag.org/content/319/5863/597.full.pdf.
10. X. Li, F. Zhang, and A. H. MacDonald, Phys. Rev. Lett. **116**, 026803 (2016).
11. B. E. Feldman, M. T. Randeria, A. Gyenis, F. Wu, H. Ji, R. J. Cava, A. H. MacDonald, and A. Yazdani, Science **354**, 316 (2016), http://science.sciencemag.org/content/354/6310/316.full.pdf.
12. E. Fradkin, S. A. Kivelson, M. J. Lawler, J. P. Eisenstein, and A. P. Mackenzie, Annual Review of Condensed Matter Physics **1**, 153 (2010), http://dx.doi.org/10.1146/annurev-conmatphys-070909-103925.
13. M. P. Lilly, K. B. Cooper, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **82**, 394 (1999).
14. R. Du, D. Tsui, H. Stormer, L. Pfeiffer, K. Baldwin, and K. West, Solid State Communications **109**, 389 (1999).
15. Y. You, G. Y. Cho, and E. Fradkin, Phys. Rev. X **4**, 041050 (2014).
16. Y. You, G. Y. Cho, and E. Fradkin, Phys. Rev. B **93**, 205401 (2016).
17. M. M. Fogler, A. A. Koulakov, and B. I. Shklovskii, Physical Review B **54**, 1853 (1996).
18. E. Fradkin, S. A. Kivelson, E. Manousakis, and K. Nho, Phys. Rev. Lett. **84**, 1982 (2000).
19. D. A. Abanin, S. A. Parameswaran, S. A. Kivelson, and S. L. Sondhi, Physical Review B **82**, 035428 (2010).
20. V. L. Berezinskii, Sov Phys JETP (1971).
21. J. M. Kosterlitz and D. J. Thouless, Journal of Physics C: Solid State Physics **5**, L124 (2001).
22. E. W. Carlson and K. A. Dahmen, Nature Communications **2**, 379 (2011).
23. K. B. Cooper, *New Phases of Two-Dimensional Electrons in Excited Landau Levels*, Ph.D. thesis, California Institute of Technology, (2003).
24. N. Regnault, J. Maciejko, S. A. Kivelson, and S. L. Sondhi, ArXiv e-prints (2016), arXiv:1607.02178 [cond-mat.str-el].
25. E. W. Carlson, K. A. Dahmen, E. Fradkin, and S. A. Kivelson, Physical Review Letters **96**, 097003 (2006).
26. See Supplemental Material for the results of XY and Ising model simulations with other values of V and h, the effect of changing ϕ, and the origin of h_{int}.
27. S. P. Koduvayur, Y. Lyanda-Geller, S. Khlebnikov, G. Csathy, M. J. Manfra, L. N. Pfeiffer, K. W. West, and L. P. Rokhinson, Physical Review Letters **106**, 016804 (2011).
28. Q. Shi, M. A. Zudov, J. D. Watson, G. C. Gardner, and M. J. Manfra, Physical Review B **93**, 121404 (2016).
29. J. M. Kosterlitz, Journal of Physics C: Solid State Physics (1974).
30. R. da Silveira and M. Kardar, Physical Review E **59**, 1355 (1999).
31. K. Dahmen and J. P. Sethna, Phys. Rev. B **53**, 14872 (1996).
32. We expect similar temperature evolution to occur for $\nu > 9/2$ half integer fillings, and also for the $\nu = 5/2$ and $7/2$ states which display anisotropic resistivity upon application of a moderate in-plane field.