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Filling-enforced non-symmorphic Kondo semimetals in two dimensions

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We study the competition between Kondo screening and frustrated magnetism on the non-symmorphic Shastry-Sutherland Kondo lattice at a filling of two conduction electrons per unit cell. This model is known to host a set of gapless partially Kondo screened phases intermediate between the Kondo-destroyed paramagnet and the heavy Fermi liquid. Based on crystal symmetries, we argue that (i) both the paramagnet and the heavy Fermi liquid are *semimetals* protected by a glide symmetry; and (ii) partial Kondo screening breaks the symmetry, removing this protection and allowing the partially-Kondo-screened phase to be deformed into a Kondo insulator via a Lifshitz transition. We confirm these results using large-N mean field theory and then use non-perturbative arguments to derive a generalized Luttinger sum rule constraining the phase structure of 2D non-symmorphic Kondo lattices beyond the mean-field limit.

Introduction.— The interplay between the Kondo effect and magnetism in heavy fermion materials is a paradigmatic setting for competing electronic order [1]. In these rare-earth intermetallic compounds a lattice of local moments from strongly correlated d or f orbitals can hybridize with itinerant conduction electrons to form a heavy Fermi liquid (FL), with a 'large' Fermi surface that incorporates *both* constituents. Intermoment exchange induced by the Ruderman-Kittel-Kasuya-Yosida (RKKY) mechanism can suppress Kondo screening in favor of magnetic order, and much theoretical and experimental effort has focused on studying the intervening quantum critical point [2–8]. Magnetic frustration adds complexity to this scenario [9, 10] by introducing quantum fluctuations that favor local-moment singlet formation over magnetic order [11]. Several unconventional phases result from this competition, including a metallic valence bond solid (VBS) in $YbAl_3C_3$ [12], partial magnetic order in $CePd_{1-x}Ni_xAl$ [13, 14], and quantum criticality without tuning in CeRhSn [15]. Other proposed possibilities, such as fractionalized quantum spin liquids (QSLs) [16], remain experimentally elusive.

A classic instance of geometrical frustration due to lattice structure is furnished by the Shastry-Sutherland lattice [17] (SSL; Fig. 1) relevant to a class of heavy fermion materials such as Yb_2Pt_2Pb, Ce_2Pt_2Pb, and Ce_2Ge_2Mg [18]. Recently, the phase structure of the Shastry-Sutherland Kondo lattice (SSKL) model was analyzed using large-N techniques and mean field arguments [19–21], revealing several distinct phases. For weak Kondo coupling these include ordered antiferromagnets (at low frustration) and paramagnetic valence-bond solids (at strong frustration), whereas a heavy Fermi liquid (HFL) phase was identified at strong Kondo coupling, for a range of fillings. Here, we show that these results are intimately connected to constraints imposed on the phase structure by lattice symmetry. We focus on a filling of half an electron per site; at this filling the four-site SSL unit cell contains exactly two conduction electrons ($\nu_c = 2$) and — since there is a single local f-moment on each site — four spins-1/2 ($N_s = 4$). In the fully Kondo-screened phase, both of these must be included the Luttinger count — the total number of electrons and local moments per unit cell, modulo those which can be incorporated into fully filled bands. This may be derived via a periodic-lattice generalization of the Friedel sum rule [22], or using topological arguments [23]. Since $\nu \equiv \nu_c + N_s = 6$, the Fermi surface of the Kondo-screened phase encloses zero net volume ($V_F = 0$), as any even charge may be accommodated in filled, hybridized bands or in equal-volume electron and hole pockets. As we show below, the SSKL at $\nu_c = 2$ is *gapless* for large Kondo coupling J_K. A naive expectation based on Luttinger's theorem and the fact that $V_F = 0$ is that the gaplessness is 'accidental' and can be removed via a symmetry-preserving Lifshitz transition to a Kondo insulator (KI).

Contrary to this expectation, we demonstrate that this gapless Kondo-screened phase is a filling-enforced [24] Kondo semimetal (here and below, *semimetal* will denote any gapless system with $V_F = 0$), protected by a non-symmorphic glide symmetry (reflection combined with a half-lattice translation) of the SSL: it cannot become insulating without breaking this symmetry, or triggering fractionalization. While similar results are known for purely electronic systems [24–28], those are not directly applicable to the Kondo lattice. At intermediate J_K, glide symmetry is spontaneously broken, leading to a partially Kondo screened insulator (PKSI) that alleviates magnetic frustration [29, 30] by modulating hybridization between local moments and conduction electrons while preserving translational symmetry. The PKSI is thus distinct from conventional KIs that the SSL hosts at a filling of one electron per site, $\nu_c = 4$ [21], that preserve all symmetries. Previous work [20] found an intermediate-

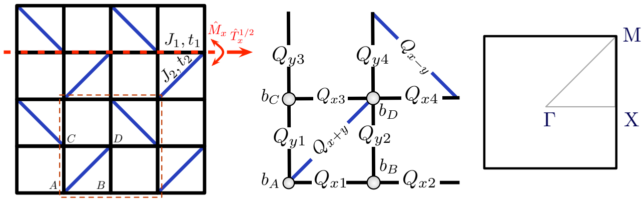

FIG. 1. SSL, mean-field parameters, and reduced Brillouin zone. One of the glide planes is depicted, involving a reflection ($\hat{M}_x$) followed by a half-lattice-translation ($\hat{T}_x^{1/2}$).

J_K 'partially Kondo screened' gapless phase with similar broken symmetries at $\nu_c = 2$; unlike the large J_K Kondo semimetal, this *is* connected to the PKSI via a symmetry-preserving Lifshitz transition. We substantiate these claims within a large-N mean-field study of the phase diagram of the SSKL, and discuss transitions between the PKSI and its proximate semimetals. We unify these results by identifying a generalized 'Luttinger invariant' for Kondo lattice models at even integer filling, and discuss its possible extensions.

Model and $SU(N)$ Mean-Field Theory.— The SSKL is described by the Kondo-Heisenberg Hamiltonian,

$$H = \sum_{(i,j),\sigma} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + \sum_{(i,j)} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j + J_K \sum_i \mathbf{s}_i \cdot \mathbf{S}_i. \quad (1)$$

This describes conduction electrons ($c_{i\sigma}$) with hopping amplitude t_{ij}, and spin-1/2 moments ($\mathbf{S}_i$), coupled by an on-site antiferromagnetic Kondo coupling $J_K > 0$. $\mathbf{s}_i = c^\dagger_{i\alpha}(\boldsymbol{\sigma}_{\alpha\beta}/2)c_{i\beta}$ is the conduction electron spin density and the sum on (i,j) ranges over nearest (NN; t_1, J_1) and next-nearest (NNN; t_2, J_2) neighbors on the SSL.

As a first step, we rewrite (1) in terms of fermionic spinons $\mathbf{S}_i = f^\dagger_{i\alpha}(\boldsymbol{\sigma}_{\alpha\beta}/2)f_{i\beta}$, subject to the constraint $\sum_\sigma f^\dagger_{i\sigma} f_{i\sigma} = 1$. We then solve the problem via a large-N mean-field approach, generalizing the spin symmetry from $SU(2)$ to $SU(N)$, so that saddle-point results become exact for $N \to \infty$. Decoupling the interactions via Hubbard-Stratonovich transformations that introduce fields b_i and Q_{ij} respectively and allowing these to condense (i.e., acquire a non-zero saddle-point value), we arrive at the mean-field Hamiltonian [11, 16, 20]

$$\begin{aligned} H_{MF} = E &- \sum_{(i,j),\sigma} (Q^*_{ij} f^\dagger_{i\sigma} f_{j\sigma} + \text{h.c.}) + \sum_{i,\sigma} \lambda_i f^\dagger_{i\sigma} f_{i\sigma} \quad (2) \\ &+ \sum_{(i,j),\sigma} t_{ij}(c^\dagger_{i\sigma} c_{j\sigma} + \text{h.c.}) - \sum_{i,\sigma} (b^*_i c^\dagger_{i\sigma} f_{i\sigma} + \text{h.c.}). \end{aligned}$$

where $E/N = \sum_i \left(|b_i|^2/J_K - \lambda_i/2\right) + \sum_{(i,j)} |Q_{ij}|^2/J_{ij}$. Self-consistency requires that

$$b_i = \frac{J_K}{N} \sum_\sigma \langle c^\dagger_{i\sigma} f_{i\sigma}\rangle \text{ and } Q_{ij} = \frac{J_{ij}}{N} \sum_\sigma \langle f^\dagger_{i\sigma} f_{j\sigma}\rangle, \quad (3)$$

and we take b_i to be real. We restrict ourselves to translationally-invariant mean-field solutions but do not enforce any symmetry within the unit cell. This permits us to access states that break lattice point-group symmetries but preserve translations. Such solutions may be parametrized in terms of 18 independent complex parameters: for each of the four sites in the unit cell λ_i enforces the constraint $\sum_\sigma \langle f^\dagger_{i\sigma} f_{i\sigma}\rangle = 1$, while b_i measures the hybridization on the site, and on each of the ten inequivalent bonds Q_{ij} measures the strength of the singlet order (Fig. 1). Previous work [20, 21] has exclusively studied the regime $2t_1 > t_2$; here we consider the opposite regime with $2t_1 < t_2$ (we fix $t_2/t_1 = 2.5$ without loss of generality). As noted, we restrict ourselves to a filling of half an electron per site ($\nu_c = 2$) and fix $J_2/J_1 = 1.6$ so that the Heisenberg part of the Hamiltonian is in the VBS phase within the mean field approach we have used here [20]. We numerically solve (3) and track the solution as a function of J_K; the results, shown in Figs. 2 and 3, are as follows.

(1) The $J_K = 0$ phase is stable for a finite range of $J_K \lesssim 1.5t_1$. In this case the only nonzero Q_{ij} are $Q_{x+y} = Q_{x-y}$ so that spinon bands remain flat; meanwhile, the hybridization $b_i = 0$ on all sites, so we can treat the spinon and conduction electron bands as decoupled. Since $N_s = 4$, the lower pair of the four spinon bands are fully filled. For $\nu_c = 2$, the chemical potential intersects the lower pair of the four total conduction electron bands. The spin-degenerate bands 'stick' in pairs due to glide symmetry [24–28] across the X-M face of the Brillouin zone (BZ; labeled as in Fig. 1) and cannot be detached without breaking symmetry, though additional perturbations may reduce the sticking along X-M to nodes [31]. The resulting Fermi surface has an electron pocket centered at X and hole pockets centered at Γ, M, that enclose zero net charge: it is a semimetal. As the singlet bonds are identical to those of the pure Heisenberg model [17], we label this the VBS phase; this preserves all symmetries of the SSL, including the glide symmetry responsible for the band sticking. As the VBS phase is an *exact* spin-gapped $J_K = 0$ ground state [17], we expect it is stable for $J_K > 0$ even beyond mean-field.

(2) For large $J_K \gtrsim 2.1t_1$, we find a symmetry-preserving Kondo-screened phase with $\lambda_i = \lambda$ and $b_i = b \neq 0$ on all sites of the unit cell, and $Q_{xi} = Q_{yi}$ also nonzero and distinct from $Q_{x+y} = Q_{x-y}$. The spinon and conduction electron bands are hybridized; as glide symmetry remains unbroken, the spin-degenerate hybridized bands again stick in pairs along the X-M face, as in the VBS phase. The bands near the Fermi energy split linearly at X and quadratically at M (Fig. 3), leading to hole and electron pockets centered at these points in the BZ. For filling $\nu_c + N_s = 6$ the chemical potential intersects the second-lowest pair of bands, again leading to a semimetal whose electron and hole pockets enclose zero net volume. We dub this $V_F = 0$ phase a Kondo semimetal (KSM), since although a finite density of states at the Fermi energy (as here) cannot be ruled out, sym-

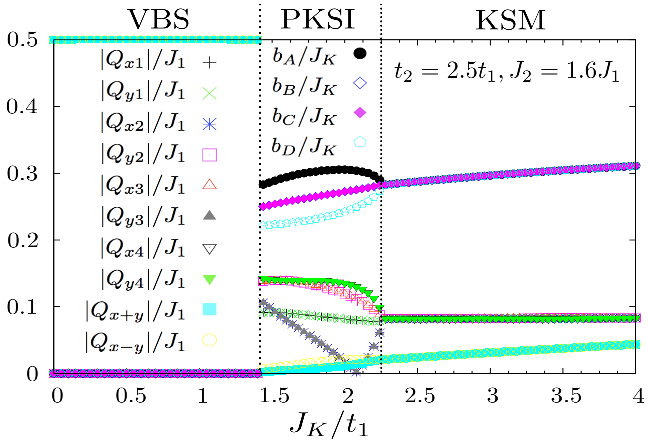

FIG. 2. Evolution of mean-field solutions with Kondo coupling J_K at low temperature $T = t_1/100$, with $t_2/t_1 = 2.5$ and $J_2/J_1 = 1.6$. A VBS phase, where unscreened conduction electrons form a semimetal, exists for $J_K \lesssim 1.5t_1$. The Kondo semimetal (KSM) found for $J_K \gtrsim 2.1t_1$ cannot be trivially gapped while preserving glide symmetry. At intermediate J_K glide is spontaneously broken, leading to a partially-Kondo-screened insulator (PKSI).

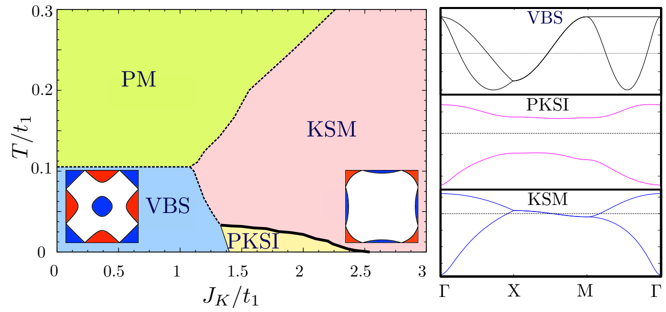

FIG. 3. Large-N Mean-field phase diagram of SSKL in the temperature-Kondo coupling ($T - J_K$) plane. Thick (thin) solid lines represent second- (first-)order transitions, while dashed lines are crossovers; the fermiology of the semimetallic KSM/VBS phases are inset, with electron (hole) pockets in red (blue). Right: band structures in the three $T = 0$ phases. (only spin-degenerate bands nearest Fermi energy shown).

metry *guarantees* that point or line nodes connect the pair of bands that intersect the Fermi energy. In contrast, the HFL has $V_F \neq 0$, and is hence a distinct phase.

(3) At intermediate Kondo coupling, the mean-field solution no longer preserves glide symmetry: for instance, the hybridization is modulated within the unit cell, $b_A \neq b_B = b_C \neq b_D$, and so is not invariant under the pairwise exchange $A \leftrightarrow B$, $C \leftrightarrow D$ generated by acting with the glide. Similarly, the singlet pattern breaks the glide symmetry reducing the symmetry from $Q_{xi} = Q_{yi}$ to $Q_{x1} = Q_{y1}$, $Q_{x3} = Q_{y2}$, $Q_{x2} = Q_{y3}$, and $Q_{x4} = Q_{y4}$. As a consequence of the broken symmetry the pairwise sticking of hybridized bands is no longer guaranteed. Accordingly, at filling $\nu_c + N_s = 6$ we see that the Fermi surface lies in a gap (Fig. 3). As the system screens unequally on different sublattices and is gapped, we identify this non-magnetic ground state as a partially Kondo screened insulator (PKSI). This is, to our knowledge, the first example of a Kondo insulator where screening spontaneously breaks lattice symmetry; absent symmetry breaking the only other route to opening a gap is to trigger topological order, a case we do not consider here. It may be possible to probe glide symmetry breaking via scattering experiments, where it is signaled by the reappearance of spectral weight at Bragg peaks 'systematically extinguished' by the glide symmetry. Although inaccessible in large-N, ordered phases that descend from PKSI (but only in a distinct J_2/J_1 regime than that studied here) will likely also break this symmetry. Previous studies of this regime [20] with $2t_1 > t_2$ found partially-Kondo-screened gapless phases for $0 < \nu_c < 4$. For $\nu_c = 2$, this is now identified as an 'accidental' semimetal with broken glide symmetry and electron and hole pockets with $V_F = 0$. This is obtained from PKSI through a Lifshitz transition where its bands intersect the Fermi energy. We show below how glide symmetry can distinguish accidental semimetals from the filling-enforced KSM and VBS semimetals via a generalized Luttinger sum rule.

Finite-Temperature Phase Diagram.— Usually, Kondo screening onsets via a crossover, as no broken symmetry distinguishes the $T \to 0$ HFL from the high-temperature paramagnet (PM). A similar argument applies to the VBS state, since the valence bond pattern preserves symmetry (magnetic order would require a true phase transition). Accordingly, we identify the PM-VBS and PM-KSM lines as crossovers, although they appear as transitions within mean-field theory (Fig. 3). The intermediate PKSI phase, however, breaks a discrete glide symmetry, and hence can form via a finite-temperature transition in $d = 2$. Note that the transition temperature T_c is an order of magnitude lower than the screening scale T_K; hence, the relevant degrees of freedom are hybridized fermions rather than bare electrons (typically, $T_K \sim 10$K so T_c is experimentally accessible). At mean-field level, the PKSI-VBS transition appears first order, though this may be an artifact of the large-N approach. The $T = 0$ PKSI-KSM transition appears continuous within mean-field theory, and is worthy of further study [32].

Generalized Luttinger Invariant.— The SSKL illustrates general symmetry constraints for non-symmorphic Kondo lattices in $d = 2$, that we now derive . Let us consider a lattice of $L_x \times L_y$ unit cells with periodic boundary conditions (i.e. a torus) described by a non-symmorphic space group $\mathcal{G}$. $\mathcal{G}$ includes at least one glide reflection $\hat{G}_x$, involving a mirror reflection $\hat{M}_x$ about the x axis followed by a translation through a half-lattice vector in the mirror plane: $\hat{G}_x = \hat{T}_x^{1/2}\hat{M}_x : (x, y) \to (x + 1/2, -y)$. We restrict ourselves to Hamiltonians with $SO(3)$ spin rotation symmetry so that we can separately couple to "up" and "down" spins relative to a fixed magnetization axis and take reflections to act trivially on spin. A single quantum Φ_0 of Aharonov-Bohm flux that couples only to one spin species is threaded through one of the non-contractible loops of the torus, e.g by adiabatically increasing a uniform time-dependent vector potential from $A_x = 0$ to $A_x = \frac{2\pi}{L_x}$ (we take $\hbar = e = 1$ so $\Phi_0 = 2\pi$). Under this process, the ground state $|\Psi_0\rangle$ evolves into a (possibly distinct) state $|\Psi_0'\rangle$; since the Hamiltonian with the vector potential A_x commutes with the glide symmetry operator at all times, both states share the same glide eigenvalue, $\hat{G}_x|\Psi_0\rangle = e^{ig}|\Psi_0\rangle, \hat{G}_x|\Psi_0'\rangle = e^{ig}|\Psi_0'\rangle$. However, $|\Psi_0'\rangle$ is an eigenstate of a Hamiltonian $H(\Phi_0)$ with an inserted flux; we may return to $H(0)$ by implementing a large gauge transformation, so that $|\Psi_0'\rangle \to |\tilde{\Psi}_0\rangle = U_\sigma|\Psi_0'\rangle$ ($\sigma = \pm$ denotes spin, and we define $\hat{n}^c_{j\sigma} = c^\dagger_{j\sigma}c_{j\sigma}$), with

$$\hat{U}_\pm = \exp\left[\frac{2\pi i}{L_x}\sum_{\vec{r}} x(\hat{n}^c_{\vec{r},\pm} \pm \hat{S}_{z,\vec{r}})\right], \tag{4}$$

where $\hat{S}^z_{\vec{r}}$ keeps the Kondo coupling invariant. Then, $\hat{U}_\pm^{-1}\hat{T}_x\hat{U}_\pm = \hat{T}_x e^{\frac{2\pi i}{L_x}\{\hat{N}^{\rm tot}_{c\pm} \pm \hat{S}^{\rm tot}_z + \frac{N_s}{2}L_xL_y\}}$ where $\frac{N_s}{2}$ is a boundary term and $\hat{N}^{\rm tot}_{c\sigma}, \hat{S}^{\rm tot}_z$ are the total number of spin-σ electrons and total magnetization [23]. Since $\hat{G}_x = \hat{T}_x^{1/2}\hat{M}_x$ and $[\hat{M}_x, \hat{N}^{\rm tot}_{c\sigma}] = [\hat{M}_x, \hat{S}^{\rm tot}_z] = 0$, $\hat{U}_\pm^{-1}\hat{G}_x\hat{U}_\pm = \hat{G}_x \exp\frac{\pi i}{L_x}\{\hat{N}^{\rm tot}_{c\pm} \pm \hat{S}^{\rm tot}_z + \frac{N_s}{2}L_xL_y\}$. From this, we find

$$\hat{G}_x|\tilde{\Psi}_0\rangle = e^{i\tilde{g}}|\tilde{\Psi}_0\rangle \text{ with } e^{i(\tilde{g}-g)} = e^{i\pi[\nu^\sigma_c + N_s(\frac{1}{2}\pm m)]L_y}, \tag{5}$$

where we used charge conservation and $U(1)$ spin symmetry to set $\hat{N}^{\rm tot}_{c\sigma} = \nu^\sigma_c L_xL_y, \hat{S}^{\rm tot}_z = N_sL_xL_ym$ where m is the average magnetization per spin. Now, assume that the system is described by Fermi liquid theory. Flux insertion corresponds to shifting every quasiparticle state via $k_x \to k_x + 2\pi/L_x$, exciting quasiparticles/quasiholes on opposite sides of the Fermi surface. This is equivalent (e.g., by applying Stoke's theorem in the BZ) to a shift of all filled states by $2\pi/L_x$, and therefore the momentum change is $\Delta P_x = \frac{2\pi}{L_x}N^{(L)}_{F,\sigma}$, where $N^{(L)}_{F,\sigma}$ is the total number of filled spin-σ states in the finite system. Since the glide involves a half-translation but does not mix spin projections, the change in the glide quantum number is

$$e^{i(\tilde{g}-g)} = e^{i\frac{\pi}{L_x}N^{(L)}_{F,\sigma}}. \tag{6}$$

Comparing Eqs. (5) and (6) and setting $L_x = L_y = L$, we find that $(\chi^{(L)}_{F,\sigma} - \nu_\sigma)L = 2p$ where p is an integer, and we defined $\nu_\sigma \equiv \nu^\sigma_c + N_s(\frac{1}{2} \pm m)$ and $\chi^{(L)}_{F,\sigma} \equiv N^{(L)}_{F,\sigma}/L^2$. A consistent thermodynamic limit for L odd then requires

$$\chi^\sigma_F \equiv \nu_\sigma (\text{mod } 2) \tag{7}$$

where $\chi^\sigma_F \equiv \lim_{L\to\infty}\chi^{(L)}_{F,\sigma}$ is the new (spin filtered) Luttinger invariant. A similar computation with $\hat{T}_x$ replacing $\hat{G}_x$, can be used to constrain the Fermi volume, via $\frac{V^\sigma_F}{(2\pi)^2} = \nu_\sigma$ (mod 1) [23]. Let us now examine the behavior of these invariants in the spin symmetric case, where $m = 0$, $\nu^\uparrow_c = \nu^\downarrow_c = \frac{\nu_c}{2}$, $V_F = V^\uparrow_F = V^\downarrow_F$, and $\chi^\uparrow_F = \chi^\downarrow_F$. Consider a filling $\nu = 4p + 2$, where p is an integer (as in the example above); then, $\nu_\sigma = \frac{\nu}{2} = 2p + 1$, and

$$V^\sigma_F = 0 \text{ and } \chi^\sigma_F = 1, \tag{8}$$

i.e., the Fermi volume vanishes, while the generalized Luttinger invariant is non-zero. A complementary derivation, for the periodic Anderson model, is given in [31], and extends our results to the mixed valence setting.

The non-zero Luttinger invariant indicates a nontrivial spectral flow (reflected by the change in glide quantum numbers) under flux insertion [33], which can be satisfied either by the presence of gapless excitations of the ground state or by the existence of a fractionalized topological quasiparticle. A Kondo insulator — which is a gapped, non-fractionalized phase — cannot respond to the insertion of a flux by changing its glide quantum number; hence, it cannot have a nonzero Luttinger invariant. However, the Fermi volume is zero [Eq.(8)]. As we do not consider fractionalized phases, the only possibility consistent with these two requirements is for the system to be a semimetal with band crossings protected by glide symmetry. (For $\chi^\sigma_F = 1$ bands must cross an *odd* number of times along the glide-symmetric line.) As long as glide symmetry is preserved, the electron and hole Fermi pockets can be shrunk to point nodes but cannot be completely removed — as in the symmetric phases (VBS, KSM) identified in our study. Breaking glide symmetry allows $\chi^\sigma_F = 0$ permitting a gapped phase (as in PKSI). A modification of this argument was presented for the $N_s = 0$ case in [33]. This generalized Luttinger sum rule may be 'topologically enriched' by allowing for the possibility of gapped, symmetry-preserving phases with fractionalized quasiparticles [16, 32, 34, 35].

Concluding Remarks.— We have examined the role of glide symmetries in determining the phase structure of a canonical 2D non-symmorphic Kondo lattice, the SSKL, and identified a filling-enforced Kondo semimetal. We have also demonstrated that competition with frustrated magnetism can lead to a broken-symmetry Kondo insulator. While we use a large-N approximation, our results are consistent with a non-perturbative Luttinger sum rule that applies well away from the mean-field limit. Our symmetry analysis provides a unified perspective on the Doniach diagram [36] of 2D nonsymmorphic Kondo lattices. For fillings $\nu = 4p + 2$, corresponding to vanishing Fermi volume, the nonzero Luttinger invariant requires that any symmetry-preserving phase either remains gapless or else has topological order. The former possibility — a symmetric semimetal — is likely at large and small J_K, where either magnetism or Kondo screening dominates. At intermediate coupling, competition leads to the opening of a gap; absent topological order, such a gapped phase must necessarily break glide symmetry, as in the PKSI we find here. For $\nu = 4p + 4$ [21], both the Fermi volume and the generalized invariant vanish and these constraints do not apply. Although spin-orbit coupling (SOC) is challenging to treat using flux insertion, if we apply existing results [24, 26, 27] on filling-enforced semimetals to the hybridized bands at these fillings our results remain unchanged if time-reversal symmetry is present. There is then also the additional interesting possibility that the PKSI may be a topological Kondo insulator, as topological insulators can emerge naturally from filling-enforced SOC semimetals upon breaking glide symmetry [24, 27, 37]. As glide is the only non-symmorphic symmetry in $d = 2$, this exhausts possible non-fractionalized symmetric phases at large- and small-J_K for 2D Kondo lattices at commensurate filling (i.e., $V_F = 0$). Our work suggests that non-symmorphic lattices are natural hosts for strongly correlated semimetals and descendant phases; in the future, we hope to extend our analysis to all 157 non-symmorphic 3D space groups [32].

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