Suppression of the Landau-Zener transition probability by weak classical noise

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Phys. Rev. B **96**, 075419 — Published 15 August 2017

DOI: 10.1103/PhysRevB.96.075419

Suppression of the Landau-Zener transition probability by a weak classical noise

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When the drive, which causes the level crossing in a qubit, is slow, the probability, P_{LZ}, of the Landau-Zener transition is close to 1. In this regime, which is most promising for applications, the noise due to the coupling to the environment, reduces the average P_{LZ}. At the same time, the survival probability, $1 - P_{LZ}$, which is exponentially small for a slow drive, can be completely dominated by noise-induced correction. Our main message is that the effect of a weak classical noise can be captured analytically by treating it as a perturbation in the Schrödinger equation. This allows us to study the dependence of the noise-induced correction to P_{LZ} on the correlation time of the noise. As this correlation time exceeds the bare Landau-Zener transition time, the effect of noise becomes negligible. On the physical level, the mechanism of enhancement of the survival probability can be viewed as an absorption of the "noise quanta" across the gap. With characteristic energy of the quantum governed by the noise spectrum, the slower is the noise, the less is the number of quanta for which the absorption is allowed energetically. We consider two conventional realizations of noise: gaussian noise and telegraph noise.

PACS numbers: 73.40.Gk, 05.40.Ca, 03.65.-w, 02.50.Ey

I. INTRODUCTION

Theoretical papers on coherent manipulation of the quantum states of a qubit can be divided into two groups. At the focus of the first group, see e.g. Refs. 1–9, is a quest for "superadiabaticity", which is an optimal protocol of drive-induced crossing of the energy levels. Following this protocol, at the end of the evolution, the final state of a qubit is as close as possible to the adiabatic ground state. If the time variation of the energy levels is linear, $\pm vt/2$, where v is the drive velocity, the degree of adiabaticity is given by the celebrated Landau-Zener (LZ) formula[10,11]

$$P_{LZ} = 1 - Q_{LZ}, \qquad Q_{LZ} = \exp\left\{-\frac{2\pi J^2}{v}\right\}, \tag{1}$$

where J is the tunnel splitting of the levels at the crossing point. The meaning of P_{LZ} is the probability to find the system, which is in $\uparrow$ state at $t \to -\infty$, in the state $\downarrow$ at $t \to \infty$. Correspondingly, the meaning of Q_{LZ} is the "survival" probability to find the system in the initial state.

The value P_{LZ} serves as an estimate of the degree of adiabaticity achievable when a two-level system is forced through an avoided crossing. In this regard, "superadiabatic" protocol minimizes the survival probability.

In the papers of the second group, see e.g. Refs. [12–25], the drive is assumed to be strictly linear. The subject of study is the effect of coupling of the qubit levels to the environment on the probability of the Landau-Zener transition. Within this group one can distinguish two subgroups: "noise-driven" LZ transition and the LZ transition modified by the environment. The first subgroup, see e.g. Refs. [12–16], deals with the situation when the average coupling, J, is zero, so that the P_{LZ} is entirely due to random noise-induced $\delta J(t)$. On the contrary, in the papers of the second subgroup[17–25] the average J is assumed to be much bigger than $\delta J(t)$. The question studied is how the coupling to the environment (thermal bath) modifies the transition probability.

A common approach in the papers of the second subgroup is to add to the Hamiltonian of the two-level system the Hamiltonian of the bath and the Hamiltonian of the linear coupling of the bath to the two-level system. After that, the equations of motion for the density matrix are cast in the form of master equations. This is achieved by generalizing the Lindblad approach of Bloch-Redfield approach developed for stationary two-level systems to the case of time-dependent Hamiltonian. The resulting closed system of master equations is solved numerically.[17–25] This numerics sometimes reveals a peculiar dependence[20] of the dynamics of the LZ transition on the noise frequency and intensity or, more precisely, on temperature.

The message of the present paper is that the effect of a weak classical noise can be studied analytically by treating it as perturbation in the Schrödinger equation. This allows to express the noise-induced correction to P_{LZ} in terms of the noise correlation function and study the dependence of this correction on the noise correlation time. The situation when this correction plays a crucial role is the strong-coupling limit, $J \gg v^{1/2}$, when the bare LZ transition probability is exponentially close to 1. In this limit, the bare survival probability, Q_{LZ}, is exponentially small. We will show that the correction to P_{LZ} is negative and does not contain the exponential factor $\exp\left[-(2\pi J^2)/v\right]$. Thus, even a weak noise can dominate Q_{LZ}. The physics behind the noise-induced correction is the absorption of the "noise quanta" across the gap, $2J$, in the course of the LZ transition. When the noise correlation time is shorter than J^{-1}, this absorption violates the adiabaticity. We analyze the noise-induced correction for the two realizations of the noise: gaussian noise and the telegraph noise.

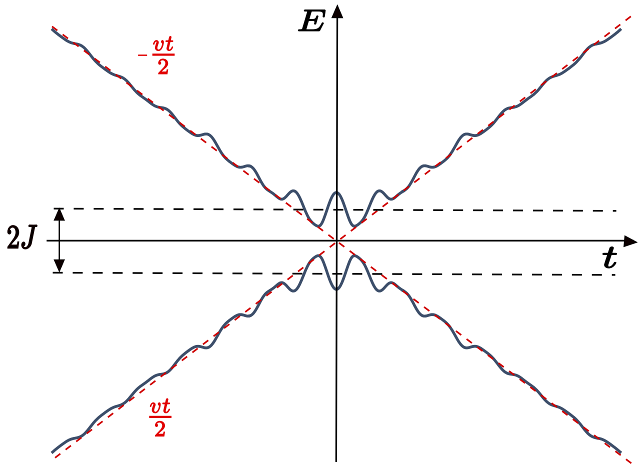

FIG. 1: (Color online) Schematic illustration of the Landau-Zener transition in the presence of a weak transverse noise. The noise causes random fluctuations of the gap, $2J$. When the gap is much bigger than $v^{1/2}$, where v is the sweep velocity, the LZ transition is almost fully adiabatic, so that the "survival" probability, Q_{LZ}, to stay on the initial diabatic level is exponentially small. Then, even a weak noise yields a dominant contribution to Q_{LZ}.

II. PERTURBATIVE SOLUTION OF THE SCHRÖDINGER EQUATION IN THE PRESENCE OF NOISE

Denote with $a_\uparrow$, $a_\downarrow$ the amplitudes to find a driven system in the $\uparrow$ and $\downarrow$ states, respectively. In the presence of random $\delta J(t)$, modeling the noise, these amplitudes satisfy the following system of equations

$$\begin{cases} i\dot{a}_\uparrow = \frac{vt}{2}a_\uparrow + [J+\delta J(t)]a_\downarrow, \\ i\dot{a}_\downarrow = -\frac{vt}{2}a_\downarrow + [J+\delta J(t)]a_\uparrow. \end{cases} \tag{2}$$

In the absence of noise, two linearly independent solutions of the system Eq. (2) have the form

$$\begin{cases} a_\uparrow^{(1)} = D_\nu(z), \\ a_\downarrow^{(1)} = -i\sqrt{\nu}D_{\nu-1}(z), \end{cases} \tag{3}$$

$$\begin{cases} a_\uparrow^{(2)} = D_\nu(-z), \\ a_\downarrow^{(2)} = -i\sqrt{\nu}D_{\nu-1}(-z), \end{cases} \tag{4}$$

where $D_\nu(z)$ is the parabolic cylinder function[26] of the argument, z, defined as $z = \sqrt{v}e^{i\pi/4}t$, while the index ν is given by

$$\nu = -\frac{iJ^2}{v}. \tag{5}$$

The solution Eq. 3 satisfies the "right" initial condition $a_\downarrow^{(1)}(-\infty) = 0$, i.e. that the system is initially in the state $\uparrow$.

In the presence of noise, we search for the corrections to the amplitudes, $a_\uparrow^{(1)}$ and $a_\downarrow^{(1)}$, in the form of the linear combination

$$\begin{pmatrix} \delta a_\uparrow \\ \delta a_\downarrow \end{pmatrix} = c_1(t)\begin{pmatrix} a_\uparrow^{(1)} \\ a_\downarrow^{(1)} \end{pmatrix} + c_2(t)\begin{pmatrix} a_\uparrow^{(2)} \\ a_\downarrow^{(2)} \end{pmatrix}. \tag{6}$$

Substituting $a_\uparrow^{(1)} + \delta a_\uparrow$, $a_\downarrow^{(1)} + \delta a_\downarrow$ into the system Eq. (2) and keeping only $a_\uparrow^{(1)}$, $a_\downarrow^{(1)}$ in the terms proportional to δJ, we arrive to the following linear system of equations for $\dot{c}_1(t)$ and $\dot{c}_2(t)$

$$\begin{cases} i\left(\dot{c}_1(t)a_\uparrow^{(1)} + \dot{c}_2(t)a_\uparrow^{(2)}\right) = \delta J(t)a_\downarrow^{(1)}, \\ i\left(\dot{c}_1(t)a_\downarrow^{(1)} + \dot{c}_2(t)a_\downarrow^{(2)}\right) = \delta J(t)a_\uparrow^{(1)}. \end{cases} \tag{7}$$

Taking into account that, being the corrections, c_1, c_2 satisfy the initial conditions $c_1(-\infty) = 0$ and $c_2(-\infty) = 0$, we find the expressions for c_1 and c_2

$$c_1(t) = -i\int_{-\infty}^{t} dt'\delta J(t')\frac{a_\downarrow^{(1)}(t')a_\downarrow^{(2)}(t') - a_\uparrow^{(1)}(t')a_\uparrow^{(2)}(t')}{a_\uparrow^{(1)}a_\downarrow^{(2)} - a_\uparrow^{(2)}a_\downarrow^{(1)}}, \tag{8}$$

$$c_2(t) = -i\int_{-\infty}^{t} dt'\delta J(t')\frac{[a_\downarrow^{(1)}(t')]^2 - [a_\uparrow^{(1)}(t')]^2}{a_\uparrow^{(1)}a_\downarrow^{(2)} - a_\uparrow^{(2)}a_\downarrow^{(1)}}. \tag{9}$$

It is easy to see that the denominator in Eqs. (8), (9) is a time independent constant. This is the consequence of the relation

$$J\left(a_\uparrow^{(1)}a_\downarrow^{(2)} - a_\uparrow^{(2)}a_\downarrow^{(1)}\right) = i\left(\dot{a}_\uparrow^{(1)}a_\uparrow^{(2)} - \dot{a}_\uparrow^{(2)}a_\uparrow^{(1)}\right), \tag{10}$$

which straightforwardly follows from the system Eq. (2). The expression in the right-hand side is a Wronskian, the value of which is known[26]

$$D_\nu(z)\frac{d}{dz}D_\nu(-z) - D_\nu(-z)\frac{d}{dz}D_\nu(z) = \frac{(2\pi)^{1/2}}{\Gamma(-\nu)}. \tag{11}$$

Here $\Gamma(-\nu)$ is the Gamma-function.

With survival probability defined as $|a_\uparrow(\infty)|^2$ given that the initial state is $\uparrow$, we can express this probability, with noise taken into account to the lowest order, via the bare survival probability. With the help of Eq. (6) one finds

$$Q_{LZ} = |1 + c_1(\infty)|^2 e^{-2\pi|\nu|} + 2\mathrm{Re}\big[(1 + c_1(\infty))^* c_2(\infty)\big]e^{-\pi|\nu|} + |c_2(\infty)|^2. \tag{12}$$

The latter expression illustrates our main point, namely, when the bare survival probability is exponentially small, the net survival probability is dominated by the noise-induced correction, $|c_2(\infty)|^2$. The analytical expression for this correction follows from Eq. (9). It should be averaged over the noise realizations. This averaging is carried out in the next Section.

III. AVERAGING OVER THE NOISE REALIZATIONS

The strength and the correlation time of the noise are encoded in the correlator defined as

$$\langle \delta J(t_1)\delta J(t_2)\rangle = (\delta J)^2 K(t_1 - t_2), \tag{13}$$

where δJ is the r.m.s. noise magnitude and $K(0) = 1$.

Using Eqs. (10), and (11), the average survival probability, $\langle Q_{LZ}\rangle = \langle |c_2(\infty)|^2\rangle$, can be expressed via the correlator as follows

$$\langle |c_2(\infty)|^2\rangle = \frac{(\delta J)^2}{2\sinh \pi|\nu|}\int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 K(t_1-t_2)\left\{\left[a_\downarrow^{(1)}(t_1)\right]^2 - \left[a_\uparrow^{(1)}(t_1)\right]^2\right\}\left\{\left[a_\downarrow^{(1)}(t_2)\right]^2 - \left[a_\uparrow^{(1)}(t_2)\right]^2\right\}^*, \tag{14}$$

where we used the identity $|\Gamma(-\nu)|^2 = \pi/|\nu|\sinh(\pi|\nu|)$.

To evaluate the double integral, we take advantage of the fact that, without noise, the transition probability is close to 1, which implies that the parameter $|\nu|$ is big, $|\nu| \gg 1$. This, in turn, justifies using the semiclassical asymptotes for the parabolic cylinder functions not only for big, but, in fact, for *all* values of the argument. The asymptotic forms of the parabolic cylinder functions valid at large $|\nu|$ and arbitrary t can be found in Ref. 27. Using these asymptotes, for the combination $\left[a_\downarrow^{(1)}(t)\right]^2 - \left[a_\uparrow^{(1)}(t)\right]^2$ which enters into Eq. (14), one obtains

$$\left[a_\downarrow^{(1)}(t)\right]^2 - \left[a_\uparrow^{(1)}(t)\right]^2 = D_\nu^2\left(\sqrt{v}e^{i\frac{\pi}{4}}t\right) + \nu D_{\nu-1}^2\left(\sqrt{v}e^{i\frac{\pi}{4}}t\right)$$
$$\approx \frac{\frac{vt}{2}\exp\left(\frac{\pi|\nu|}{2}\right)}{\left(J^2 + \frac{v^2t^2}{4}\right)^{1/2}}\exp[-2i\Phi(t)], \tag{15}$$

where $\Phi(t)$ is the semiclassical phase

$$\Phi(t) = \int_0^t dt'\left[J^2 + \frac{v^2t'^2}{4}\right]^{1/2}. \tag{16}$$

Due to $|\nu|$ being large, the term corresponding to $\exp(2i\Phi(t))$ in Eq. (15) is exponentially suppressed. The denominator in the prefactor is conventional for semiclassics. Appearance of t in the numerator can be simply illustrated by substituting $a_\downarrow^{(1)}(t) \propto \exp(-i\Phi(t))$ into the system Eq. (2). This will yield the relation

$$\left[a_\downarrow^{(1)}(t)\right]^2 - \left[a_\uparrow^{(1)}(t)\right]^2 \approx -\frac{vt}{2J}\left[a_\uparrow^{(1)}(t)\right]^2. \tag{17}$$

For the further evaluation of the double integral in Eq. (14), it is convenient to switch from time domain to the frequency domain, as it is illustrated in the next section.

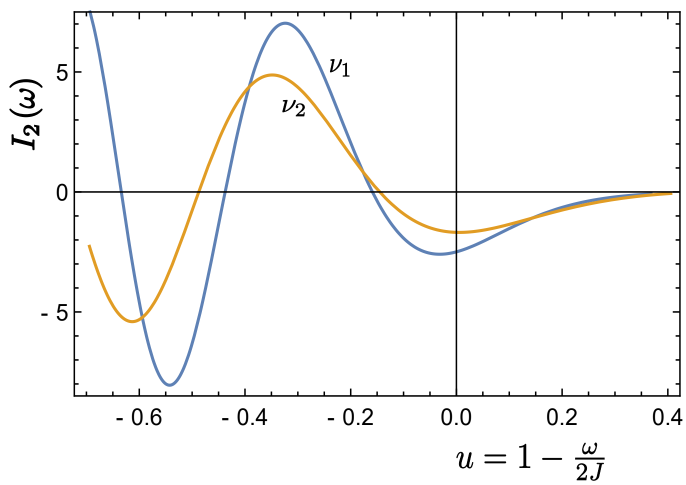

FIG. 2: (Color online) The integral $I_2(\omega)$ (in the units $4i(2\pi)^{1/2}/J^3$) is plotted from Eq. (25) versus the dimensionless variable, $u = (1-\omega/2J)$, for two values of the dimensionless parameter ν: $\nu = 3$ (orange) and $\nu = 3.5$ (blue). For negative u, $I(\omega)$ oscillates and reproduces the semiclassical result Eq. (22) after the first maximum. For positive u it falls off exponentially. Despite the $u > 0$-tail is slim, it is responsible the survival probability when the noise is slow.

IV. CALCULATION OF Q_{LZ} IN THE FREQUENCY DOMAIN

Denote with $\tilde{K}$ the Fourier transform of the correlator Eq. (13)

$$K(t) = \int_{-\infty}^{\infty} \tilde{K}(\omega)e^{i\omega t}d\omega. \tag{18}$$

Upon substituting Eq. (18) into Eq. (14), the integrations over t_1 and t_2 get decoupled and we obtain

$$\langle |c_2(\infty)|^2\rangle = \frac{e^{\pi|\nu|}}{2\sinh \pi|\nu|}(\delta J)^2\int_{-\infty}^{\infty} d\omega \tilde{K}(\omega)|I(\omega)|^2, \tag{19}$$

where the function $I(\omega)$ is defined as

$$I(\omega) = \int_{-\infty}^{\infty} dt \, \frac{\frac{vt}{2}}{\left(J^2 + \frac{v^2t^2}{4}\right)^{1/2}} \exp\left[i\Big(\omega t - 2\Phi(t)\Big)\right]. \quad (20)$$

Analytical form of $I(\omega)$ depends on the frequency domain. For high ω one can use the steepest descent method. The exponent in Eq. (20) has two extrema at $t = \pm t_\omega$, where

$$t_\omega = \frac{2}{v}\left[\frac{\omega^2}{4} - J^2\right]^{1/2}. \quad (21)$$

Expanding the exponent near these extrema and taking into account that $\partial^2\Phi/\partial t^2 = v^2t/4(J^2+v^2t^2/4)^{1/2}$, after combining the two contributions, we obtain

$$I(\omega)\Big|_{\omega>2J} = I_1(\omega) = i\frac{2^{3/2}\pi^{1/2}t_\omega^{1/2}}{|\omega|^{1/2}} \sin\left[\omega t_\omega - 2\Phi(t_\omega) + \frac{\pi}{4}\right]. \quad (22)$$

The above result applies when the argument of sine is big. For $|\nu| \gg 1$ this requirement is already satisfied when ω exceeds $2J$ only slightly. Indeed, the criterion $\omega t_\omega \gg 1$ can be cast in the form

$$(\omega - 2J) \gg \frac{J}{|\nu|^2}. \quad (23)$$

Physically, this criterion means that the direct absorption (emission) of a noise "quantum", say, a phonon, if the noise is due to lattice vibrations, is allowed.

For frequencies $\omega < 2J$ the behavior of $I(\omega)$ exhibits a sharp cutoff as the difference $2J - \omega$ grows. It appears that, in order to capture this cutoff, it is sufficient to replace $\Phi(t)$ by its small-t expansion, namely

$$\Phi(t) \approx Jt + \frac{v^2t^3}{12J}. \quad (24)$$

One can also neglect $v^2t^2/4$ in the denominator of Eq. (20). After that, $I(\omega)$ reduces to the derivative of the Airy function, namely

$$I(\omega)\Big|_{\omega<2J} = I_2(\omega) = i\frac{2^{4/3}\pi}{J^{1/3}v^{1/3}} Ai'\left[\left(\frac{4J}{v^2}\right)^{1/3}(2J-\omega)\right]. \quad (25)$$

The behavior of $I(\omega)$ near $\omega = 2J$ is illustrated in Fig. 2. For $\omega < 2J$, the function $I(\omega)$ falls off exponentially as $\exp\left[-\frac{2^{7/2}|\nu|}{3}(1-\omega/2J)^{3/2}\right]$ when the difference $2J - \omega$ exceeds $J/|\nu|^{2/3}$. For $\omega > 2J$ the asymptote Eq. (25) oscillates and merges with the asymptote Eq. (22) after the first maximum. It follows from the plot that, numerically, the small-ω tail is relatively slim. Still, we will keep it, since it captures the behavior of Q_{LZ} for long correlation times of the noise. For arbitrary correlation time, it is sufficient to use the asymptote Eq. (22) for $\omega > 2J$ and the asymptote Eq. (25) for $\omega < 2J$. Then the expression

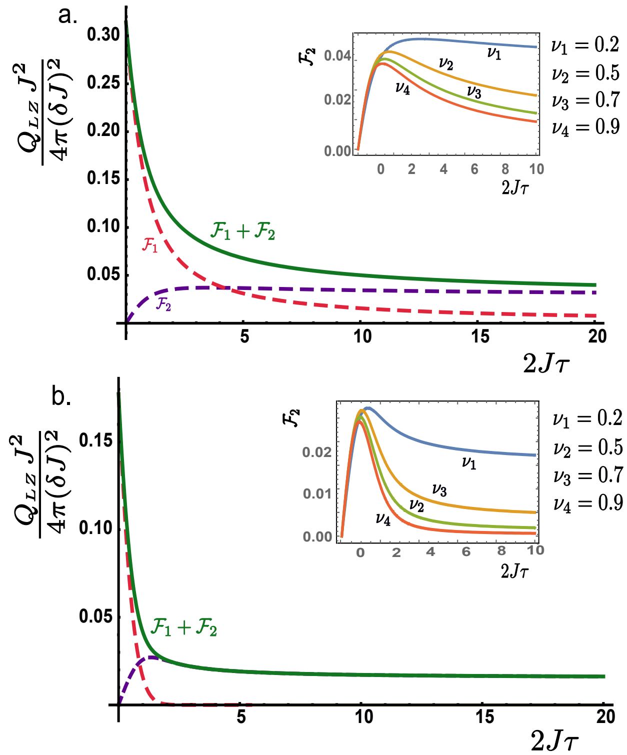

FIG. 3: (Color online) Survival probability, Q_{LZ}, is plotted from Eqs. (27)-(29) versus the dimensionless noise correlation time for the telegraph noise (a) and for the gaussian noise (b). The contribution from the absorption of the high-frequency "noise quanta" ($\omega > 2J$) are shown with red dashed lines ($\mathcal{F}_1$), while the contributions from the absorption with $\omega < 2J$ are shown with purple dashed lines ($\mathcal{F}_2$). The insets illustrate the evolution of the low-frequency contribution with increasing $\nu = J^2/v$. The smaller is the gap, the stronger is the absorption of the sub-gap quanta. Characteristic correlation time being $\tau \sim 1/J$ which is much shorter than the time J/v of the LZ transition time indicates that it is "fast" noise that suppresses the adiabaticity.

Eq. (19) for the average survival probability takes the form

$$\langle Q_{LZ}\rangle = (\delta J)^2\left[\int_0^{2J} d\omega \tilde{K}(\omega)|I_2(\omega)|^2 + \int_{2J}^{\infty} d\omega \tilde{K}(\omega)|I_1(\omega)|^2\right], \quad (26)$$

where we have replaced $\sinh(\pi|\nu|)$ by $\exp(\pi|\nu|)/2$, since $|\nu|$ is big. Eq. (26) is our main result. While the dependence of Q_{LZ} on the on the noise magnitude is obvious, the dependence on the noise correlation time, predicted by Eq. (26) is nontrivial. We analyze this dependence in the next section.

V. DEPENDENCE OF $\langle Q_{LZ} \rangle$ ON THE NOISE CORRELATION TIME

If the correlation time of the noise is τ, then $\frac{1}{\tau}\tilde{K}(\omega)$ is a dimensionless function of the argument $\omega\tau$. Since the frequency scale of both $I_1(\omega)$ and $I_2(\omega)$ is the gap $2J$, the two contributions to Q_{LZ} are the dimensionless functions of the argument $2J\tau$. Correspondingly, we rewrite Eq. (26) in the form

$$\langle Q_{LZ} \rangle = 4\pi \frac{(\delta J)^2}{J^2}\Big[\mathcal{F}_1(2J\tau) + \mathcal{F}_2(2J\tau)\Big], \tag{27}$$

where the functions $\mathcal{F}_1$ and $\mathcal{F}_2$ are defined as

$$\mathcal{F}_1 = |\nu| \int\limits_{2J}^{\infty} d\omega \tilde{K}(\omega)\Big(1 - \frac{4J^2}{\omega^2}\Big)^{1/2}, \tag{28}$$

$$\mathcal{F}_2 = 2^{2/3}\pi|\nu|^{2/3} \int\limits_{0}^{2J} d\omega \tilde{K}(\omega) Ai'^2\left[2^{5/3}|\nu|^{1/3}\left(1 - \frac{\omega}{2J}\right)\right]. \tag{29}$$

The first and the second terms describe the absorption of “above-gap” and “below-gap” noise quanta, respectively. Note, that the integrand in Eq. (28) does not contain the parameter ν. In Fig. 3(a),(b) we plotted Q_{LZ} for the telegraph noise with $\tilde{K}(\omega) = \frac{\tau}{1+\omega^2\tau^2}$ and for the gaussian noise with $\tilde{K}(\omega) = \tau\exp(-\omega^2\tau^2)$, respectively. The contributions $\mathcal{F}_1$ can be evaluated analytically for both cases. Namely, for the telegraph noise the calculation yields

$$\mathcal{F}_1(2J\tau) = \frac{\pi|\nu|}{2}\left(\frac{1}{2J\tau + (4J^2\tau^2+1)^{1/2}}\right), \tag{30}$$

while for gaussian noise the result reads

$$\mathcal{F}_1(2J\tau) = |\nu|\left[\frac{\pi^{1/2}}{2}\exp\left(-4J^2\tau^2\right) - \pi J\tau \ \mathrm{Erfc}(2J\tau)\right], \tag{31}$$

where $\mathrm{Erfc}(x)$ is the error function. The contributions $\mathcal{F}_1$ dominate Q_{LZ} in the small-τ domain, which corresponds to the fast noise. In fact, the contribution $\mathcal{F}_2$ turns to zero for $J\tau \ll 1$. The behavior of the contributions $\mathcal{F}_1$ at small τ is $\mathcal{F}_1(2J\tau) \approx \frac{\pi|\nu|}{2}(1 - 2J\tau)$ for the telegraph noise and $\mathcal{F}_2(2J\tau) \approx |\nu|\big(\frac{\pi^{1/2}}{2} - 2J\tau\big)$ for the gaussian noise. The slopes are related as $2/\pi^{1/2}$, i.e. they are close. The fact that for short correlations times the prefactor in Q_{LZ} is proportional to $\frac{(\delta J)^2}{J^2}|\nu|$ reflects a simple physics that the absorption of the high-frequency noise quanta does not depend on J. Indeed, J drops out from the combination $|\nu|/J^2$.

The difference between the two noise realizations manifests itself in the contributions $\mathcal{F}_2$. It is seen from Fig. 3 that for the telegraph noise, this contribution falls off with τ much slower than for the gaussian noise. In fact, the slow decay of $\mathcal{F}_2$ can be estimated qualitatively[27]. Indeed, subsequent jumps of the gap width with magnitude (δJ) take place at time moments, t, separated by τ. A jump results in the absorption only if $t \lesssim J/v$, since J/v is the LZ transition time. The probability that $t \lesssim J/v$ is $\sim \frac{J}{v\tau}$. This suggests that $\mathcal{F}_2$ contribution falls off as $1/J\tau$. A nontrivial feature of the $\mathcal{F}_2$ contribution is that it passes through a maximum at $2J\tau \approx 1$.

VI. LONGITUDINAL NOISE

Throughout the paper we assumed that the noise is transverse, i.e. it is described by the Hamiltonian $\delta J(t)\hat{\sigma}_x$. In this section we briefly outline the changes to be made in the result Eq. (27) if the noise is longitudinal with the Hamiltonian $\delta\varepsilon(t)\hat{\sigma}_z$. The steps of the perturbative derivation of $\langle |c_2(\infty)|^2\rangle$ leading to Eq. (14) for the longitudinal noise are completely similar to the transverse noise. Naturally, $(\delta\varepsilon)^2$ instead of $(\delta J)^2$ appears in the prefactor. In the integrand, the combination $\Big[a_\downarrow^{(1)}(t_1)\Big]^2 - \Big[a_\uparrow^{(1)}(t_1)\Big]^2$ gets replaced by $2\Big[a_\downarrow^{(1)}(t_1)a_\uparrow^{(1)}(t_1)\Big]$. The absolute value of the former combination has a meaning of $|S_z(t)|$, which is the absolute value of the polarization. Correspondingly, the absolute value of the product $2\Big[a_\downarrow^{(1)}(t_1)a_\uparrow^{(1)}(t_1)\Big]$ corresponds to $|S_x(t)|$. For $|\nu| \gg 1$, this quantity is calculated in the Appendix. Then the modification of Eq. (20) amounts to the replacement of $vt/2$ by J in the numerator of the integrand. As a result, for $\omega > 2J$ the result Eq. (22) gets modified as

$$I_1(\omega) \to i\frac{2^{5/2}\pi^{1/2}J}{v\left(\omega t_\omega\right)^{1/2}}\sin\Big[\omega t_\omega - 2\Phi(t_\omega) + \frac{\pi}{4}\Big]. \tag{32}$$

Due to this modification, the integral $\mathcal{F}_1$ in the expression for the survival probability assumes the form

$$\mathcal{F}_1 \to 4J^2|\nu| \int\limits_{2J}^{\infty} d\omega \frac{\tilde{K}(\omega)}{\omega^2}\Big(1 - \frac{4J^2}{\omega^2}\Big)^{-1/2}. \tag{33}$$

For the telegraph noise, the evaluation of this integral yields

$$\mathcal{F}_1(2J\tau) \to \frac{\pi|\nu|}{2}\left[\frac{2J\tau}{(4J^2\tau^2+1)^{1/2}}\right]\left(\frac{1}{2J\tau + (4J^2\tau^2+1)^{1/2}}\right), \tag{34}$$

We see that the result differs from the corresponding expression Eq. (30) only by an additional factor in the square brackets. We conclude that the effect of longitudinal noise on Q_{LZ} is suppressed, compared to the transverse noise, in the limit $J\tau \ll 1$, i.e. in the limit of the fast noise.

VII. DISCUSSION

A. Applicability of the perturbative treatment

In the absence of noise, the probability, $|a_\uparrow(t)|^2$, to stay in the state $\uparrow$ starts from $|a_\uparrow|^2 = 1$ at $t \to -\infty$ and approaches the exponentially small value Q_{LZ} at $t \to \infty$. In fact, the substantial fall-off of $|a_\uparrow(t)|^2$ to the value much smaller than 1 takes place during the LZ transition time $\sim J/v$, so that $|a_\uparrow(t)|^2$ becomes exponentially small at much longer times. More quantitatively, as we demonstrate in the Appendix, in the domain $t \sim J/v$ the "population inversion", $|a_\uparrow(t)|^2 - |a_\downarrow(t)|^2$, behaves as $-vt/\left(4J^2 + v^2t^2\right)^{1/2}$, and thus it approaches -1 as a power-law, not exponentially. The saturation of $|a_\uparrow(t)|^2$ at the exponentially small value Q_{LZ} at large times is accompanied by the rapid oscillations with magnitude which decreases slowly with time, as J/vt. In fact, it is this oscillations that induce a delicate interference leading to the population inversion being exponentially close to -1, i.e. the LZ transition being almost adiabatic.

The reason why we were able to find the noise-induced modification of this complex behavior is that, for large $\nu = J^2/v$, fast oscillations can be neglected in calculation of Q_{LZ} when Q_{LZ} is dominated by noise. This is because the value of Q_{LZ} in the presence of noise builds up during the time $\sim J/v$ when the bare $|a_\uparrow|^2$ is *not yet exponentially small*. Once the correction is formed, or, in other words, the "noise quantum" is absorbed, it leads to the dephasing of the oscillations of the $\uparrow$ and $\downarrow$ amplitudes, i.e. to the suppression of their interference. From here we conclude that the criterion of the applicability of the perturbative treatment, adopted in this paper, is $c_1(t)$, $c_2(t)$ are much smaller than 1, where c_1 and c_2 are the corrections to the amplitude $a_\uparrow$ given by Eqs. (8), (9). This corrections are proportional to $\delta J/J$, where δJ is the noise magnitude. With δJ being much smaller than J, the correction, $|c_2(\infty)|^2$, can still dominate the net survival probability when it exceeds the value $\exp\left(-2\pi J^2/v\right)$. This is because $c_2(t)$, being built up during the LZ time J/v, does not contain exponential smallness. Thus our results apply when $\delta J/J \gg \exp\left(-2\pi J^2/v\right)$. On the other hand, the condition $|c_2|^2 \ll 1$ limits the magnitude δJ from the above. Indeed, as follows from Eqs. (27), (28), for most relevant correlation time $\tau \sim J^{-1}$, the value Q_{LZ} can be estimated as $(\delta J/J)^2\,|\nu| = (\delta J)^2/v$. This value diverges when $v \to 0$. Physically, this means that for very slow drive, even a weak noise will equalize the $\uparrow$ and $\downarrow$ probabilities.

On the basis of the above arguments, we can quantify the criterion of applicability of the results obtained in the present paper as follows

$$\exp\left(-2\pi|\nu|\right) \ll \frac{(\delta J)^2}{J^2} \ll \frac{1}{|\nu|}. \qquad (35)$$

B. Comparison to the previous results

In this subsection we compare our results with the results of previous studies[18,22,23,28–33] of the effect of noise on the LZ transition.

(i). We calculated the survival probability for arbitrary noise correlation time assuming that the noise is weak, so that the bare survival probability is exponentially small. This domain of parameters corresponds to "high-fidelity" qubit and is most appealing for applications. In earlier analytical calculations Refs. 28–30 the noise intensity was not assumed to be weak, but the noise was assumed to be fast. Both longitudinal and transverse noise were treated on the same footing. The authors adopted a standard model of a bosonic bath consisting of harmonic oscillators. For the case of transverse noise (affecting only J) considered in the present paper the results of Refs. 28–30 can be summarized as follows. In the presence of noise $Q_{LZ} = \exp\left[-2\pi(J^2 + (\delta J)^2)/v\right]$, which suggests that the noise suppresses the survival probability in contrast to what we find. However, the comparison of our results to Refs. [28], [30] is impossible since the exact results obtained in these papers are valid strictly at zero temperature. However, in Ref. [29] the temperature was assumed to be finite. In subsequent detailed numerical studies.[18,22,23] the conclusion of Ref. [29] was questioned. The results of Refs. 18, 22, and 23 demonstrate that the Landau-Zener probability decreases with temperature, i.e. with noise magnitude, for all values of the bare LZ probability (all values of parameter $|\nu|$). An interesting observation made in these papers is that Q_{LZ}, modified by noise, is a non-monotonic function of ν.

(ii). Technically, our calculation is most close to the paper by Ao and Rammer Ref. 31. In our notations and, within a numerical factor, their result reads $Q_{LZ} = (\delta J)^2\,(J/v)\,\tilde{K}(2J)n(2J)$, where $n(\omega)$ is the Bose distribution. The above expression suggests that the noise-induced survival probability is dominated exclusively by the noise "quanta" with frequency $\omega = 2J$. This conclusion seems unphysical and contradicts our result Eq. (26), according to which all frequencies with $\omega > 2J$ contribute to Q_{LZ}. On the quantitative level, the difference can be traced to the use of the asymptotes of the parabolic cylinder functions in Ref. 31. The principle observation reported in Ref. 31 is that noise-induced Q_{LZ} can exceed the bare value, which is exponentially small. On the physical level, this observation was interpreted in Ref. 32 as a result of dephasing of the fast interference oscillations of the amplitudes $a_\uparrow$ and $a_\downarrow$ in the transition region.

Historically, the first analytical treatment of the effect of noise on Q_{LZ} was reported in Ref. 33. The authors arrived to the correct conclusion that Q_{LZ} grows with temperature. Concerning the expression for Q_{LZ} obtained in Ref. 33, it represents a product of the bare survival probability and the noise-induced exponential factor. This is contrast to Refs. 31, Ref. 32, and our result in which the enhancement of Q_{LZ} due to noise adds to the the bare Q_{LZ}.

(iii). Note finally, that for very strong noise $\delta J \gg J$ the LZ transition can be viewed as simply noise-driven. This limit was studied in a pioneering paper Ref. 12. In particular, for fast noise, with frequency much bigger than $v/\delta J$, the survival probability is given by $Q_{LZ} = \frac{1}{2}\Big[1 + \exp\big(-4\pi(\delta J)^2/v\big)\Big]$.

(iv). Throughout the paper we assumed that $|\nu|$ is big, i.e. the bare survival probability is small. It is interesting to note that, in the opposite limit of small enough ν, the dependence of survival probability on the noise magnitude can be *non-monotonic*. Below we illustrate this observation analytically assuming that the noise is slow.

It is known[12] that in the limit of infinite τ_c, the average probability of the transition should be calculated by averaging this probability of transition at a given J over the distribution of J.

For slow noise with correlation time much longer than J/v, the survival probability is given by

$$\langle Q_{LZ}\rangle = \int\limits_{-\infty}^{\infty} d\delta J \; P\left(\delta J\right) \exp\left[-2\pi|\nu|\left(1+\frac{\delta J}{J}\right)^2\right]. \tag{36}$$

For gaussian $P\left(\delta J\right) = \frac{1}{\pi^{1/2}J_0}\exp\left[-\left(\frac{\delta J}{J_0}\right)\right]^2$ the integration yields

$$\langle Q_{LZ}\rangle = \frac{1}{\left[1+\frac{2\pi}{v}J_0^2\right]^{1/2}}\exp\Big[-\frac{2\pi|\nu|}{1+\frac{2\pi}{v}J_0^2}\Big]. \tag{37}$$

Note that, for $|\nu| < 1/4\pi$, the survival probability is suppressed by noise while for $|\nu| > 1/4\pi$ it is enhanced by noise. This behavior is illustrated in Fig. 4. Fig. 4 suggests the following nontrivial effect of low-frequency environment[20] on the LZ transition. As the coupling to environment, parametrized by J_0, increases, the initially adiabatic transition becomes first less adiabatic, and then, more adiabatic.

(v). The noise spectrum, $\tilde{K}(\omega)$, depends on the concrete realization of the environment. In theoretical papers, see e.g. Refs. 18, 22, 23, the environment is usually modeled by a set of harmonic oscillators with the frequency distribution $g(\omega) \propto \omega\exp(-\omega/\omega_c)$ (Ohmic environment). Then $\tilde{K}(\omega)$ is proportional to $g(\omega)\coth(\omega/2T)$, where T is temperature.

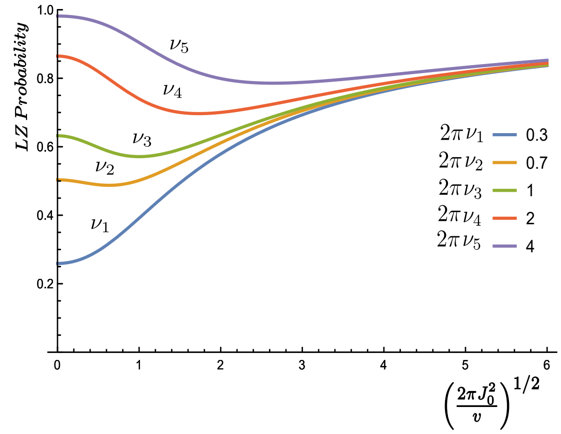

FIG. 4: (Color online) Landau-Zener probability in the presence of a slow noise is plotted from Eq. (37) as a function of the dimensionless noise magnitude for different values of parameter ν, which quantifies the bare survival probability $Q_{LZ} = \exp(-2\pi|\nu|)$. For $2\pi|\nu| < 1/2$ the curves grow monotonically, which suggests that noise enhances the adiabaticity. For $2\pi|\nu| > 1/2$, i.e. when the transition is adiabatic in the absence of noise, the curves exhibit minima, suggesting that a weak noise suppresses the adiabaticity, while the strong noise, with magnitude exceeding the gap, enhances it.

Appendix A: Time evolution of the level population in the limit of small survival probability

In general, the level populations, $P_\downarrow(t)$ and $P_\uparrow(t)$ exhibit strong oscillations in the domain $t \sim J/v$, where the LZ transition takes place. These oscillations originate from the interference of the terms $\propto \exp(i\Phi(t))$ and $\propto \exp(-i\Phi(t))$, Eq. (16). The reason why we were able to find the noise-dependent correction analytically is that, for small bare survival probability, these oscillations are suppressed. We established this fact upon analysis of the asymptotes of the parabolic cylinder functions in the domain $t \sim J/v$. It is instructive to trace how the result Eq. (15)

$$P_\downarrow(t) - P_\uparrow(t) = \frac{\frac{vt}{2}}{(J^2 + \frac{v^2t^2}{4})^{1/2}} \tag{A1}$$

emerges from the alternative description based on the spin dynamics. In the literature, the effect of noise on the LZ transition is studied within this description.

The difference $P_\downarrow(t) - P_\uparrow(t) = S_z(t)$ can be viewed as spin polarization, while the system Eq. (2) describes the evolution of the $\uparrow$ and $\downarrow$ spin amplitudes in the effective magnetic field, $\boldsymbol{B}$, with components $B_z(t) = \frac{vt}{2}$ and $B_x = J$. Three equations of motion for the spin projections following from $\frac{d\boldsymbol{S}}{dt} = \boldsymbol{B}\times\boldsymbol{S}$ can be reduced to a single integral-differential equation for $S_z(t)$

$$\frac{\partial S_z(t)}{\partial t} = -\int\limits_{-\infty}^{t} dt' \cos\left(\int\limits_{t'}^{t} dt'' B_z(t'')\right) B_x(t)B_x(t')S_z(t'). \tag{A2}$$

The crucial simplification, which allows to solve this equation in the limit $|\nu| \gg 1$ is that, for relevant times $t \sim J/v$, the argument of cosine $\int\limits_{t'}^{t} dt'' B_z(t'') = \frac{v}{4}(t^2 - t'^2)$

is big. For $B_x(t) = B_x(t') = J$, Eq. (A2) takes the form

$$\frac{\partial S_z(t)}{\partial t} = -J^2 \int_{-\infty}^{t} dt' \cos\left[\frac{v}{4}(t^2 - t'^2)\right] S_z(t'). \tag{A3}$$

Strong oscillations of cosine suggest that the major contribution to the integral comes from $(t-t') \ll t$. To make use of this condition, we perform the integration by parts in the right-hand side

$$\frac{\partial S_z(t)}{\partial t} = -\frac{2J^2}{v} \int_{-\infty}^{t} dt' \sin\left[\frac{v}{4}(t-t')(t+t')\right] \frac{\partial\left(\frac{S_z(t')}{t'}\right)}{\partial t'}. \tag{A4}$$

Next, we set $t + t' = 2t$ in the argument of sine and set $t = t'$ in the derivative. This yields

$$\frac{\partial S_z(t)}{\partial t} = -\frac{2J^2}{v} \frac{\partial\left(\frac{S_z(t)}{t}\right)}{\partial t} \int_{-\infty}^{t} dt' \sin\left[\frac{v}{2}(t-t')t\right]. \tag{A5}$$

Now the integration over t' can be carried out leading to

$$\frac{\partial S_z(t)}{\partial t} = -\frac{4J^2}{v^2 t} \frac{\partial\left(\frac{S_z(t)}{t}\right)}{\partial t} = -\frac{4J^2}{v^2 t}\left[\frac{1}{t}\frac{\partial S_z(t)}{\partial t} - \frac{S_z(t)}{t^2}\right]. \tag{A6}$$

The first order differential equation Eq. (A6) can be easily solved. With initial condition $S_z(-\infty) = -1$, the result reads

$$S_z(t) = \frac{\frac{vt}{2}}{\left(J^2 + \frac{v^2 t^2}{4}\right)^{1/2}} = \frac{B_z}{\left(B_x^2 + B_z^2\right)^{1/2}}, \tag{A7}$$

i.e. the polarization is equal to cosine of the angle between magnetic field and the z-axis. Using Eq. (A7), the projection $S_y(t)$ can be calculated from the equation $\frac{dS_z}{dt} = B_x S_y$ and turns out to be

$$S_y(t) = \frac{J\frac{v}{2}}{\left(J^2 + \frac{v^2 t^2}{4}\right)^{3/2}} = \frac{B_x \frac{\partial B_z}{\partial t}}{\left(B_x^2 + B_z^2\right)^{3/2}}. \tag{A8}$$

Subsequently, the projection $S_x(t)$ calculated from $\frac{dS_x}{dt} = -B_z S_y$ acquires the form

$$S_x(t) = \frac{J}{\left(J^2 + \frac{v^2 t^2}{4}\right)^{1/2}} = \frac{B_x}{\left(B_x^2 + B_z^2\right)^{1/2}}. \tag{A9}$$

From the expressions Eqs. (A7)-(A9), we can estimate the accuracy of the approximations made. These expressions are valid if $S_y \ll 1$. Indeed, it follows from (A7), (A9) that $S_z^2 + S_x^2 = 1$. On the other hand, it follows from Eq. (A8) that the maximal value of S_y is $\frac{v}{J^2} = \nu^{-1} \ll 1$. Thus, the results Eqs. (A7)-(A9) are valid with accuracy ν^{-1}. Uncertainty $\sim \nu^{-1}$ is much bigger than the inaccuracy of the result $S_z(\infty) = 1$, which follows from Eq. (A7). Inaccuracy of this result is $\exp(-2\pi\nu)$, i.e. it is exponentially small.

Numerical results for the spin projections in the limit $\nu \gg 1$ are presented in Ref. 20. They seem to be in good agreement with analytical expressions Eqs. (A7)-(A9).

Acknowledgements

Illuminating discussion with V. L. Pokrovsky is gratefully acknowkedged. The work was supported by the Department of Energy, Office of Basic Energy Sciences, Grant No. DE- FG02-06ER46313.

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