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Rashba torque beyond the Boltzmann regime

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We study spin torques induced by Rashba spin-orbit coupling in two-dimensional ferromagnets under the good-metal condition $\epsilon_F\tau/\hbar \gg 1$ (ϵ_F the Fermi energy, τ the electron lifetime) by employing the Kubo formula. We find that, in the presence of spin-dependent disorder the Rashba torque changes greatly as the system evolves out of the weak disorder limit where \hbar/τ is much smaller than any intrinsic energy scale characterizing the multiband structure. The antidamping-like component of Rashba torque can be comparable to and larger than the field-like one out of the weak disorder limit. The semiclassical Boltzmann theory produces the same results as microscopic linear response calculations only in the weak disorder limit. Our analysis indicates that rich behaviors of various nonequilibrium phenomena beyond the Boltzmann theory may also be present even when $\epsilon_F\tau/\hbar \gg 1$ in multiband systems where ϵ_F is not the unique intrinsic energy scale.

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I. INTRODUCTION

In describing nonequilibrium phenomena of conduction electrons in solids, the widely-employed semiclassical Boltzmann theory [1] is valid only if the band structure is well-defined, i.e., the disorder broadening \hbar/τ (τ the electron lifetime) of bands is much smaller than any intrinsic energy scale characterizing the multiband structure [2–4]. This Boltzmann regime is often called the weak disorder limit [2–4] or weak scattering limit [5]. In simple systems where the Fermi energy ϵ_F is the unique intrinsic energy scale characterizing the conduction band such as the conventional parabolic band [8] and linear Dirac band [9], the Boltzmann regime is practically equivalent to the good-metal limit $\epsilon_F\tau/\hbar \gg 1$.

In multiband systems, some intrinsic energy scales characterizing the multiband structure of conduction bands may not be large compared to \hbar/τ , nonequilibrium phenomena then exhibit behaviors beyond the Boltzmann regime even in the good-metal limit $\epsilon_F\tau/\hbar \gg 1$. This is the possible case in systems with spin-orbit coupling, such as the Rashba spin-orbit coupling which widely exists in inversion-asymmetric structures [6]. In a Rashba system with both subbands partially occupied, the band splitting Δ_k due to the Rashba coupling (and exchange coupling in ferromagnetic Rashba systems) provides an intrinsic energy-scale [7]. Because the Rashba coupling is weak in many cases [6], the competition between Δ_k and \hbar/τ can lead to rich behaviors of nonequilibrium phenomena beyond the Boltzmann regime.

In the present paper we reveal such behaviors of Rashba torque [12–14] in the 2D ferromagnetic Rashba model with both subbands partially occupied. Under the good-metal condition $\epsilon_F\tau/\hbar \gg 1$ there are still two different limits [7, 10]: the weak disorder limit $\Delta_k \gg \hbar/\tau$ and the opposite limit $\Delta_k \ll \hbar/\tau$. This latter limit is often called the diffusive limit [11], because in this limit the spin relaxation time is much larger than the momentum relaxation time and thus when considering spin dynamics the motion of electron is diffusive.

The Rashba torque arises from the s-d coupling between the spin of Rashba electrons and the local magnetization: an applied electric field induces a nonequilibrium spin density of conduction electrons via the Rashba spin-orbit coupling, this spin density then exerts a torque on the local magnetization. The Rashba torque possesses two components: a field-like torque odd in the magnetization direction $\hat{\mathbf{M}}$ and an antidamping-like torque even in $\hat{\mathbf{M}}$ [12]. Although there have been plenty of researches on the Rashba torque in the weak disorder limit [3, 12, 13] or diffusive limit [14], some basic characters have not been revealed, such as the possibility that the antidamping-like component becomes larger than the field-like one and the evolution of Rashba torque from the weak disorder limit to the diffusive limit. Besides, most papers on the Rashba torque employing the Boltzmann theory or other phenomenological treatment [17–20] did not point out which limit (weak disorder limit or diffusive limit) their theories work in.

We consider the 2D Rashba model with a perpendicular magnetization [13] in order to avoid the complexity induced by the in-plane anisotropy. This anisotropy is important for the angular dependence of Rashba torque [19], but is not the interest here. By employing the Kubo formula under the non-crossing approximation, we find that both the field-like and antidamping-like Rashba torques change considerably from the weak disorder limit to the diffusive limit, provided that the disorder is not completely spin-independent. Especially, the antidamping-like torque, which is much smaller than the field-like one in the weak disorder limit, becomes comparable to and even larger than the field-like one out of the weak disorder limit.

In the discussion part of this paper we also address other nonequilibrium phenomena such as the anomalous Hall effect [5], spin Hall effect [15] and Edelstein effect [16] in 2D Rashba systems. In all these cases, we show that the semiclassical Boltzmann theory is consistent with microscopic linear response calculations only in the weak disorder limit rather than in the whole regime

of the good-metal limit.

The present paper is organized as follows. General formulations are presented in Sec. II, whereas calculation results are given and analyzed in Sec. III. Section IV makes some discussions and concludes the paper.

II. FORMULATION

A. Model

The 2D ferromagnetic Rashba Hamiltonian is

$$\hat{H}_0 = \frac{\hbar^2 \mathbf{k}^2}{2m} + \alpha_R \hat{\sigma} \cdot (\mathbf{k} \times \hat{\mathbf{z}}) - J_{ex} \hat{\sigma} \cdot \hat{\mathbf{M}}, \quad (1)$$

where m is the effective mass of conduction electron, $\mathbf{k} = k(\cos \phi, \sin \phi)$ the 2D wavevector, $\hat{\sigma} = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$ are the Pauli matrices, α_R is the Rashba parameter, J_{ex} the exchange coupling, the direction of the local magnetization is chosen to be $\hat{\mathbf{M}} = \hat{\mathbf{z}}$ for the isotropic model. We only consider the case $\epsilon_F > J_{ex}$, i.e., both Rashba bands partially occupied (more accurately, we demand $\epsilon_F - J_{ex} \gg \hbar/\tau$). For any energy $\epsilon > J_{ex}$ there are two iso-energy rings corresponding to the two Rashba bands $\eta = \pm$: $k_\eta^2(\epsilon) = \frac{2m}{\hbar^2}(\epsilon - \eta \Delta_\eta(\epsilon))$ where $\epsilon_R = m(\frac{\alpha_R}{\hbar})^2$ and $\Delta_\eta(\epsilon) \equiv \Delta_{k_\eta(\epsilon)} = \sqrt{J_{ex}^2 + \alpha_R^2 k_\eta^2(\epsilon)} = \sqrt{\epsilon_R^2 + J_{ex}^2 + 2\epsilon_R \epsilon - \eta \epsilon_R}$. The density of states in η band is $D_\eta(\epsilon) = D_0 \frac{\Delta_\eta(\epsilon)}{\Delta_\eta(\epsilon) + \eta \epsilon_R}$ with $D_0 = \frac{m}{2\pi \hbar^2}$.

The short-range (pointlike) disorder can be classified according to the spin structure of scattering potential as [21]: class A $\hat{V} = V_A \hat{\sigma}_0$, class B $\hat{V} = V_B \hat{\sigma}_z$, class C $\hat{V} = V_c \hat{\sigma}_\pm / \sqrt{2}$. Here $\hat{\sigma}_\pm = \hat{\sigma}_x \pm i \hat{\sigma}_y$, $\hat{\sigma}_0$ is the identity matrix in spin space. It was shown that in the weak disorder limit the contribution from class A disorder to the anomalous Hall effect is quite different from that of classes B and C disorder, even with opposite sign [21]. While the contributions from class B and C disorder are similar [21]. Thus our calculation only takes into account class A and class B disorder. We will show that the spin structure of short-range disorder strongly affects the behavior of Rashba torque when the system evolves from the weak disorder limit to the diffusive limit under the good-metal condition. For this purpose it is sufficient to assume Gaussian disorder [13, 14, 22].

B. Kubo-Streda formalism

In the linear response analysis, the average value of an observable A (Hermitian operator \hat{A} , which can represent a vector, scalar, etc) in the presence of a dc uniform weak electric field \mathbf{E} and disorder is generally given by [23] $A = Tr \langle \hat{\rho}^0 \hat{A}^0 \rangle_c + Tr \langle (\delta^{\mathbf{E}} \hat{\rho}) \hat{A}^0 \rangle_c + Tr \langle \hat{\rho}^0 \delta^{\mathbf{E}} \hat{A} \rangle_c$, where $\hat{\rho} = \hat{\rho}^0 + \delta^{\mathbf{E}} \hat{\rho}$ is the total density matrix, $\langle \dots \rangle_c$ denotes the average over disorder configurations and Tr

the trace over relevant degrees of freedom. $\hat{\rho}^0$ and \hat{A}^0 are operators in equilibrium, $\delta^{\mathbf{E}} \hat{\rho}$ and $\delta^{\mathbf{E}} \hat{A}$ represent the out-of-equilibrium change of operators linear in \mathbf{E} . The last term of A is relevant usually in thermal related effects, such as the heat current response to an electric field [23]. Here we do not consider thermal related effects and focus on the case $\delta^{\mathbf{E}} \hat{A} = 0$. Then

$$\delta A = Tr \langle \hat{A}^0 (\delta^{\mathbf{E}} \hat{\rho}) \rangle_c, \quad (2)$$

with $\delta A = A - A_0$ and $A_0 \equiv Tr \langle \hat{\rho}^0 \hat{A}^0 \rangle_c$.

The linear response (2) in the single-particle picture with only elastic electron-impurity scattering can be found by the Kubo-Streda formula [24, 25] for the correlation function between \hat{A} and the electric current operator $\hat{j} = e \hat{\mathbf{v}}$. For instance, at low-temperature limit the Kubo-Streda formula for the electric-field induced nonequilibrium spin density $\delta S_\alpha = \chi_{\alpha\beta} E_\beta$ reads [22, 25] $\chi_{\alpha\beta} = \chi_{\alpha\beta}^{I(a)} + \chi_{\alpha\beta}^{I(b)} + \chi_{\alpha\beta}^{II}$, where

$$\chi_{\alpha\beta}^{I(a)} = \frac{\hbar}{2\pi} Tr \langle \hat{S}_\alpha \hat{G}^R(\epsilon_F) \hat{j}_\beta \hat{G}^A(\epsilon_F) \rangle_c, \quad (3)$$

$$\chi_{\alpha\beta}^{I(b)} = -\frac{\hbar}{2\pi} \text{Re} Tr \langle \hat{S}_\alpha \hat{G}^R(\epsilon_F) \hat{j}_\beta \hat{G}^R(\epsilon_F) \rangle_c, \quad (4)$$

$$\chi_{\alpha\beta}^{II} = \frac{\hbar}{2\pi} \text{Re} \int d\eta f^0(\epsilon) \quad (5)$$

$$\times Tr \left\langle \hat{S}_\alpha \hat{G}^R(\epsilon) \hat{j}_\beta \frac{d\hat{G}^R(\epsilon)}{d\epsilon} - \hat{S}_\alpha \frac{d\hat{G}^R(\epsilon)}{d\epsilon} \hat{j}_\beta \hat{G}^R(\epsilon) \right\rangle_c.$$

Here $\alpha, \beta = x, y$, \hat{S}_α is the α -component of spin operator, $\hat{G}^{R/A}(\epsilon) = (\epsilon - \hat{H} \pm i0^+)^{-1}$ is the retarded/advanced Green's function operator with $\hat{H} = \hat{H}_0 + \hat{V}$, f^0 is the Fermi distribution function.

C. Formal expressions for Rashba torque

In the case of class A (B) disorder, the imaginary part of the retarded Born self-energy $\Sigma^R = \sum_{\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} G_{\mathbf{k}'}^{0,R} V_{\mathbf{k}'\mathbf{k}}$ is diagonal in spin space and inversely proportional to the electron lifetime $\tau = \left(\frac{2\pi}{\hbar} n_{im}^{A(B)} V_{A(B)}^2 D_0 \right)^{-1}$ with $n_{im}^{A(B)}$ the density of class A (B) disorder. The dressed retarded Green's function is then [5] $G_{\mathbf{k}}^R = \sum_i G_{i\mathbf{k}}^R \hat{\sigma}_i$ with $G_{0\mathbf{k}}^R = \frac{1}{2} \sum_\eta G_{\eta\mathbf{k}}^R$, $G_{x\mathbf{k}}^R = \frac{\sin \theta \sin \phi}{2} \sum_\eta \eta G_{\eta\mathbf{k}}^R$, $G_{y\mathbf{k}}^R = -\frac{\sin \theta \cos \phi}{2} \sum_\eta \eta G_{\eta\mathbf{k}}^R$, $G_{z\mathbf{k}}^R = -\frac{\cos \theta}{2} \sum_\eta \eta G_{\eta\mathbf{k}}^R$ and $G_{\eta\mathbf{k}}^R(\epsilon_F) = (\epsilon_F - \epsilon_k^\eta + \frac{i\hbar}{2\tau})^{-1}$. Here $\cos \theta = J_{ex}/\Delta_k$.

Under the good-metal condition, $\chi_{\alpha\beta}^{II}$ and $\chi_{\alpha\beta}^{I(b)}$ are approximated by their disorder-free parts [5, 26] which are zero in the considered case. Thus we get $\chi_{\alpha\beta} = \chi_{\alpha\beta}^{I(a)}$,

which is calculated by bubble with ladder vertex corrections [7, 11] under the non-crossing approximation: $\chi_{\alpha y} = \frac{\hbar e}{2\pi} \frac{\hbar}{2} \sum_{\mathbf{k}} \text{tr} [\sigma_{\alpha} G_{\mathbf{k}}^R(\epsilon_F) \Upsilon_y G_{\mathbf{k}}^A(\epsilon_F)]$, with Υ_y the dressed velocity vertex and tr the trace in spin space.

In this section we only consider the presence of class A or class B disorder alone. Assuming $\Upsilon_y = a \frac{\hbar k_y}{m} + b \sigma_x + c \sigma_y$ with a, b, c real numbers, we get

$$\begin{aligned} \chi_{yy} &= -\frac{\hbar e}{\pi} \frac{\hbar}{2} \frac{2\pi\tau D_0}{\hbar} (ibI_2 - cI_1), \\ \chi_{xy} &= \frac{\hbar e}{\pi} \frac{\hbar}{2} \frac{2\pi\tau D_0}{\hbar} (aI_3 + bI_1 + ciI_2), \end{aligned} \quad (6)$$

where $I_3 = -\alpha_R/\hbar$ and

$$\begin{aligned} I_1 &= \sum_{\eta} \left[\sin^2 \theta_{\eta} + \frac{1 + \cos^2 \theta_{\eta}}{1 + (2\Delta_{\eta}(\epsilon_F) \frac{\tau}{\hbar})^2} \right] \frac{D_{\eta}(\epsilon_F)}{4D_0}, \\ I_2 &= -i \frac{\tau}{\hbar} \sum_{\eta} \frac{J_{ex}}{1 + (2\Delta_{\eta}(\epsilon_F) \frac{\tau}{\hbar})^2} \frac{D_{\eta}(\epsilon_F)}{D_0}. \end{aligned} \quad (7)$$

Here $\cos \theta_{\eta} = \frac{J_{ex}}{\Delta_{\eta}(\epsilon_F)}$, $\sin \theta_{\eta} = \frac{\alpha_R k_{\eta}(\epsilon_F)}{\Delta_{\eta}(\epsilon_F)}$. We note that I_1 and I_2 depend on the parameter $2\Delta_{\eta}(\epsilon_F) \frac{\tau}{\hbar}$ which measures the competition between the intrinsic multiband splitting and the disorder-induced band broadening. In the weak disorder limit $2\Delta_{\eta}(\epsilon_F) \gg \hbar/\tau$, the multiband structure is well-defined, the picture of intraband and interband processes is clear [27] thus the semiclassical Boltzmann theory born in the band-eigenstate representation works well [3]. However, when $2\Delta_{\eta}(\epsilon_F) \lesssim \hbar/\tau$, the intrinsic multiband splitting of two Rashba bands is overwhelmed by the disorder broadening. This case, although still in the good-metal limit (the longitudinal electrical conductivity is still large due to $\epsilon_F \tau/\hbar \gg 1$), cannot be well treated by the Boltzmann theory.

For later convenience, here we work out

$$I_1 \simeq \frac{\epsilon_R \epsilon_F}{J_{ex}^2 + 2\epsilon_R \epsilon_F}, I_2 \simeq \frac{-i\hbar}{2\tau} \frac{J_{ex}}{J_{ex}^2 + 2\epsilon_R \epsilon_F} \quad (8)$$

in the weak disorder limit $\hbar/\tau \ll \Delta_{\eta}(\epsilon_F), \epsilon_F$, and

$$I_1 \simeq 1 - 2 \left(1 + \frac{J_{ex}^2}{\Delta_0^2} \right) \left(\frac{\bar{\Delta} \tau}{\hbar} \right)^2, I_2 \simeq -i \frac{2\tau}{\hbar} J_{ex} \quad (9)$$

in the diffusive limit $\Delta_{\eta}(\epsilon_F) \ll \hbar/\tau \ll \epsilon_F$. Here $\bar{\Delta} = \frac{1}{2} \sum_{\eta} \Delta_{\eta}(\epsilon_F)$ and the diffusive-limit condition also implies that $\epsilon_R, J_{ex}, \sqrt{\epsilon_R \epsilon_F} \ll \hbar/\tau \ll \epsilon_F$.

III. RESULTS

A. Class A disorder

In this case the dressed velocity vertex is given by $\Upsilon_y = v_y + \frac{\hbar}{2\pi\tau D_0} \sum_{\mathbf{k}'} G_{\mathbf{k}'}^R(\epsilon_F) \Upsilon_y G_{\mathbf{k}'}^A(\epsilon_F)$, yielding $a = 1$ and $b = c = 0$. Then

$$\chi_{yy} = 0, \chi_{xy} = -e\alpha_R D_0 \tau. \quad (10)$$

They remain the same forms from the weak disorder limit to the diffusive limit. This result is the same as that obtained in previous diagrammatic calculation [22] and consistent with the weak-disorder-limit result in a recently formulated semiclassical Boltzmann theory [3].

Besides, under the same approximations, the anomalous Hall effect vanishes $\sigma_{xy} = 0$ and the longitudinal conductivity takes the Drude form $\sigma_{yy} = n_e e^2 \tau/m$, where $n_e = 2D_0(\epsilon_F + \epsilon_R)$ is the carrier density. These two quantities also remain the same forms from the weak disorder limit to the diffusive limit. These results indicate that the scalar short-range disorder is very special in a Rashba system in the case of both bands partially occupied under the non-crossing approximation.

B. Class B disorder

In this case $\Upsilon_y = v_y + \frac{\hbar}{2\pi\tau D_0} \sum_{\mathbf{k}'} \sigma_z G_{\mathbf{k}'}^R \Upsilon_y G_{\mathbf{k}'}^A \sigma_z$ yields $a = 1$, $b = 2 \frac{\alpha_R}{\hbar} \frac{1+I_1}{(1+I_1)^2 + (iI_2)^2}$ and $c = \frac{iI_2}{1+I_1} b$. Then

$$\begin{aligned} \chi_{yy} &= -e\alpha_R D_0 \tau \frac{2iI_2}{(1+I_1)^2 + (iI_2)^2}, \\ \chi_{xy} &= -e\alpha_R D_0 \tau \frac{1 - I_1^2 - (iI_2)^2}{(1+I_1)^2 + (iI_2)^2}. \end{aligned} \quad (11)$$

Both depend on the competition between the intrinsic band-splitting and disorder broadening.

In the weak disorder limit, by Eq. (8), we get

$$\begin{aligned} \chi_{yy} &= -\hbar e\alpha_R D_0 \frac{J_{ex} (J_{ex}^2 + 2\epsilon_R \epsilon_F)}{(J_{ex}^2 + 3\epsilon_R \epsilon_F)^2}, \\ \chi_{xy} &= -e\alpha_R D_0 \tau \frac{J_{ex}^2 + \epsilon_R \epsilon_F}{J_{ex}^2 + 3\epsilon_R \epsilon_F}, \end{aligned} \quad (12)$$

which confirms the result obtained by a recent Boltzmann theory [3].

In the diffusive limit, by Eq. (9), we have

$$\chi_{xy} = -e\alpha_R D_0 \tau \left(\frac{\bar{\Delta} \tau}{\hbar} \right)^2, \chi_{yy} = -e\alpha_R D_0 \tau \frac{\tau J_{ex}}{\hbar}, \quad (13)$$

which strongly depend on the electron lifetime τ and are totally beyond the semiclassical Boltzmann theory. The ratio between the longitudinal and transverse nonequilibrium spin densities is very large (here we assume the exchange coupling is not too small $J_{ex} \gtrsim \sqrt{\epsilon_R \epsilon_F}$):

$$\frac{\chi_{yy}}{\chi_{xy}} = \frac{J_{ex}}{\bar{\Delta}} \frac{\hbar}{\Delta \tau} \gg 1. \quad (14)$$

The evolution of χ_{yy} and χ_{xy} as well as their ratio from the weak disorder limit to the diffusive limit is plotted in Fig. 1 where we set $\hbar/\tau = 1$ as the unit of energy and $\epsilon_F = 20$. In plotting Fig. 1 (and Fig. 2) we assume $\epsilon_R \ll \epsilon_F$ and thus can use the parameter $\bar{\Delta} \tau/\hbar$ to approximately control the evolution from the weak disorder limit $\bar{\Delta} \tau/\hbar \gg 1$ to the diffusive limit $\bar{\Delta} \tau/\hbar \ll 1$.

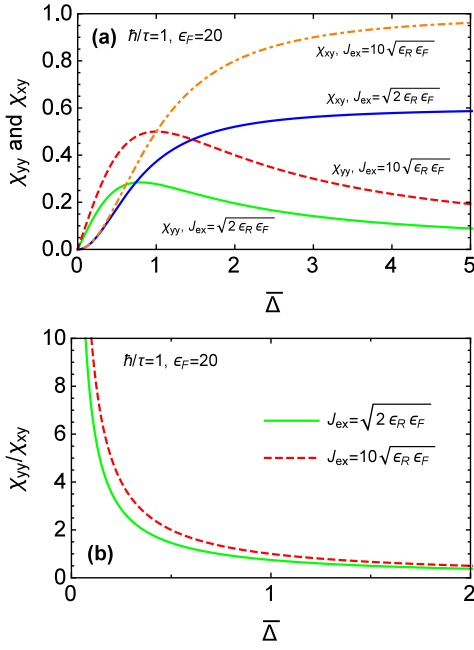


FIG. 1. The evolution of (a) χ_{yy} and χ_{xy} as well as (b) their ratio χ_{yy}/χ_{xy} from the weak disorder limit ($\bar{\Delta}\tau/\hbar \gg 1$) to the diffusive limit ($\bar{\Delta}\tau/\hbar \ll 1$) in the case of class B disorder, $\bar{\Delta} = \frac{1}{2} \sum_{\eta} \Delta_{\eta} (\epsilon_F)$. The ratio χ_{yy}/χ_{xy} represents the relative strength of antidamping-like and field-like components of the Rashba torque. Here and in Fig. 2 we assume $\epsilon_R \ll \epsilon_F$ and thus use the parameter $\bar{\Delta}\tau/\hbar$ to control the evolution from the weak disorder limit to the diffusive limit. We choose $\hbar/\tau = 1$, $\epsilon_F = 20$ in plotting the curves for the case of $J_{ex} = \sqrt{2\epsilon_R\epsilon_F}$ and $J_{ex} = 10\sqrt{\epsilon_R\epsilon_F}$. In (a) χ_{yy} and χ_{xy} are measured in the units of $-\epsilon\alpha_R D_0\tau$.

To simplify the analysis, we choose $J_{ex} = \sqrt{2\epsilon_R\epsilon_F}$ and $J_{ex} = 10\sqrt{\epsilon_R\epsilon_F}$ in plotting the curves. In Fig. 1(a) χ_{yy} and χ_{xy} are measured in the units of $-\epsilon\alpha_R D_0\tau$. Figure. 1 shows that both χ_{yy} (corresponds to an antidamping-like Rashba torque in the direction $\hat{\mathbf{M}} \times [(\hat{\mathbf{z}} \times \mathbf{E}) \times \hat{\mathbf{M}}]$) and χ_{xy} (corresponds to a field-like Rashba torque in the direction $\hat{\mathbf{M}} \times (\hat{\mathbf{z}} \times \mathbf{E})$) change greatly from the weak disorder limit to the diffusive limit, and χ_{yy}/χ_{xy} rapidly increases when the system evolves towards the diffusive limit. The non-monotonicity of χ_{yy} in Fig. 1(a) is just what can be expected from the two limiting values of χ_{yy} in Eqs. (12) and (13) in the case of $J_{ex} \propto \sqrt{\epsilon_R\epsilon_F}$.

For comparison, here we also present the values of electrical conductivities in the diffusive limit: $\sigma_{yy} = \frac{e^2}{\pi\hbar} \frac{\tau}{\hbar} (\epsilon_F + \epsilon_R)$, $\sigma_{xy} = \frac{e^2}{\pi\hbar} \frac{2\tau J_{ex}}{\hbar} \frac{\epsilon_R\tau}{\hbar}$, and $\sigma_{xy}/\sigma_{yy} \simeq \frac{2\tau J_{ex}}{\hbar} \frac{\epsilon_R}{\epsilon_F} \ll 1$. Unlike σ_{yy} whose leading contribution under the good-metal condition is always proportional to $\epsilon_F\tau/\hbar$ (not shown), χ_{xy} is not proportional to ϵ_F even in the weak disorder limit. Thus as the system evolves from the weak disorder limit to the diffusive limit, while σ_{yy} remains large, χ_{xy} may become much smaller and may not remain dominant over χ_{yy} . As a result, in complex multiband systems with multiple intrinsic energy scales under

the good-metal condition $\epsilon_F\tau/\hbar \gg 1$, the relaxation time approximation (or lifetime approximation) of the Boltzmann equation can be used as a first approximation for analyzing σ_{yy} even out of the Boltzmann regime (weak disorder limit), producing a result proportional to τ . But this is not the case for analyzing χ_{xy} which is of higher order in τ out of the Boltzmann regime unless in rare special cases such as that considered in Sec. III. A. This important difference between σ_{yy} and χ_{xy} explains why the Boltzmann analysis of the longitudinal charge (and thermoelectric) transport is usually qualitatively valid even out of the weak disorder limit in complex multiband systems [1], and also indicates that this kind of “extended” validity of the semiclassical Boltzmann theory may not be possible for other nonequilibrium phenomena which exhibit rich behaviors out of the weak disorder limit. This point will be further discussed in Sec. IV.

C. Competition between classes A and B

In the presence of both class A and class B impurities, we assume $\langle V_A V_B \rangle_c = 0$ following Ref. 21 and expect that interference effects between class A and B scattering do not qualitatively alter the result in this subsection. Here we only give the main results, calculation details are present in Appendix A.

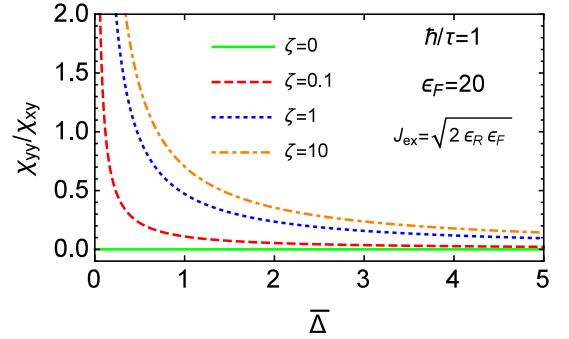


FIG. 2. χ_{yy}/χ_{xy} for fixed values of ζ in the presence of both class A and B disorder from the weak disorder limit to the diffusive limit. Here $\hbar/\tau = 1$, $\epsilon_F = 20$ and $J_{ex} = \sqrt{2\epsilon_R\epsilon_F}$.

Due to $\sum_{\eta} \frac{D_{\eta}}{\Delta_{\eta}} = 0$, the electron lifetime is still simple $\tau = \tau_A / (1 + \zeta) = (\tau_A^{-1} + \tau_B^{-1})^{-1}$, where $1/\tau_{A(B)} = 2\pi n_{im}^{A(B)} V_{A(B)}^2 D_0/\hbar$, and $\zeta = \tau_A/\tau_B$ represents the relative weight of the two scattering classes. The ζ -dependence of χ_{yy}/χ_{xy} from the weak disorder limit to the diffusive limit is shown in Fig. 2. As ζ increases from zero, the curve of χ_{yy} is shifted upward from the class A dominated regime due to the increasing contribution from class B scattering.

According to the results in this section, even under the good-metal condition $\epsilon_F\tau/\hbar \gg 1$ the Rashba torque exhibits rich behaviors, and the spin structure of disorder strongly affects the behavior of Rashba torque in the

whole good-metal regime.

IV. DISCUSSION AND CONCLUSION

We start this discussion section by pointing out that some previous papers that did not emphasize which limit (diffusive limit or weak disorder limit) they work in are in fact within the weak disorder limit. References 17–19 that employed the semiclassical Boltzmann theory should be considered in the weak disorder limit according to the analysis in the present paper. Reference 20 assumed the exchange coupling is the largest energy scale affecting the conduction electron spin dynamics in metallic Rashba ferromagnets, thus $2\Delta_\eta(\epsilon_F) \simeq 2J_{ex} \gg \hbar/\tau$ and this paper also works in the weak disorder limit.

We note that rich behaviors from the weak disorder limit to the diffusive limit can be found in various nonequilibrium phenomena in 2D Rashba systems with both subbands partially occupied provided that the scattering is not completely spin-independent. One can thus consider the spin Hall and anomalous Hall effects in the case of magnetic impurities [28, 29], as well as the spin Hall and Edelstein effects in the presence of spin-orbit scattering off scalar pointlike impurities [10, 30]. In all these cases, one can verify that the semiclassical Boltzmann theory only applies in the weak disorder limit.

For example, in the presence of spin-orbit scattering off scalar disorder, the semiclassical Boltzmann calculation [31] yields zero spin Hall conductivity in the first order of Rashba parameter, same as microscopic linear response calculations in the weak disorder limit [10]. Whereas in the diffusive limit $\bar{\Delta}\tau/\hbar \ll 1$ nonzero spin Hall effect due to the Elliot-Yafet spin relaxation caused by spin-orbit scattering [30] cannot be reproduced in the Boltzmann theory. Also, applying the semiclassical Boltzmann framework proposed in Ref. 3 to the spin Hall effect in the presence of isotropic pointlike magnetic impurities just yields the weak-disorder-limit value $\frac{\bar{e}}{7\pi}$ of the spin Hall conductivity obtained in Kubo diagrammatic calculations (Eq. (21) in Ref. 28), while the spin Hall conductivity vanishes as $(\bar{\Delta}\tau/\hbar)^2$ in the diffusive limit.

Some final remarks are in order.

First, the diagrammatic calculation of the anomalous Hall effect in model (1) in the weak disorder limit has been improved by going beyond the non-crossing approximation [32]. How this additional contribution changes from the weak disorder limit to the diffusive limit is left for future work. Quantitative correction from this contribution to our results about the Rashba torque is expected, but the qualitative aspect of the results in the present paper is not expected to change.

Second, if one considers finite-range or long-range disorder, other fine details besides the spin structure of disorder potentials should also be carefully treated.

Third, in the ferromagnetic Rashba model with a perpendicular magnetization where $J_{ex} < \epsilon_R$, there is a “window” [33] around the avoided band-anticrossing

point. The height of this window also provides an intrinsic energy scale characterizing the conduction band. Rich behaviors of the anomalous Hall effect beyond the Boltzmann regime when the Fermi energy is located in this window have been deeply investigated [33]. One can verify that, in the weak disorder limit (represented by $2J_{ex} \gg \hbar/\tau$ in this case), the semiclassical Boltzmann theory [2] produces the same results as microscopic linear response theories [5, 33]. However, rich behaviors of the anomalous Hall effect out of the weak disorder limit (called superclean case in Ref. 33) obtained in the Keldysh approach are beyond the scope of the Boltzmann theory. The band-anticrossing regime is relevant in the case of strong Rashba coupling that is possible in heavy-elements-related inversion-asymmetric structures. The behavior of Rashba torque in this energy regime is also left for future work.

In summary, we have studied the Rashba torque in 2D Rashba ferromagnets under the good-metal condition $\epsilon_F\tau/\hbar \gg 1$ by employing the Kubo-Streda formalism in the non-crossing approximation. It was shown that the widely-used semiclassical Boltzmann theory produces the same results as the Kubo formula only in the weak disorder limit. As the system evolves from the weak disorder limit to the diffusive limit, both the antidamping-like and field-like components of Rashba torque remain sensitive to the spin structure of disorder. The magnitude of the antidamping-like component can be comparable to and larger than the field-like one out of the weak disorder limit provided that the short-range disorder is not completely spin-independent. We expect these findings are helpful also in understanding spin-orbit torques in 2D anti-ferromagnetic Rashba model [34].

The rich behaviors of nonequilibrium phenomena, like those in Rashba systems, can also be expected in other multiband systems where the Fermi energy is not the unique intrinsic energy scale characterizing the band structure of conduction bands.

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Appendix A: Calculation details in the presence of both class A and B disorder

The dressed velocity vertex is given by $\Upsilon_y = v_y + n_{im}^A V_A^2 \sum_{\mathbf{k}'} [G_{\mathbf{k}'}^R \Upsilon_y G_{\mathbf{k}'}^A + \zeta \sigma_z G_{\mathbf{k}'}^R \Upsilon_y G_{\mathbf{k}'}^A \sigma_z]$. Then $a = 1$, $b = \frac{\alpha_R}{\hbar} + (1 - \zeta) bI_1' + (1 - \zeta) icI_2' + (1 - \zeta) I_3'$ and $c = (1 - \zeta) cI_1' - (1 - \zeta) ibI_2'$, where $I_i' = \frac{1}{1+\zeta} I_i$, $i = 1, 2, 3$. Here I_i take the forms $I_3 = \frac{\hbar}{2\pi\tau D_0} \sum_{\mathbf{k}} \frac{\hbar k_y}{m} 2 \text{Re} G_0^R G_x^A$

and $I_{1(2)} = \frac{\hbar}{2\pi\tau D_0} \sum_{\mathbf{k}} \left(G_0^R G_{0(z)}^A - G_z^R G_{z(0)}^A \right)$. They are given by Eq. (7) with $\tau^{-1} = (1 + \zeta) \tau_A^{-1}$. Thus

$$c = -\frac{\frac{1-\zeta}{1+\zeta} ibI_2}{1 - \frac{1-\zeta}{1+\zeta} I_1} = \frac{\alpha_R}{\hbar} \frac{-\frac{1-\zeta}{1+\zeta} \frac{2\zeta}{1+\zeta} iI_2}{\left(1 - \frac{1-\zeta}{1+\zeta} I_1\right)^2 + \left(\frac{1-\zeta}{1+\zeta}\right)^2 (iI_2)^2},$$

then $\chi_{yy} = \hbar e D_0 \tau c \frac{1+\zeta}{1-\zeta}$ and

$$\frac{\chi_{yy}}{\chi_{xy}} = \frac{2\zeta}{1 + \zeta} \frac{iI_2}{\left(1 - \frac{1-\zeta}{1+\zeta} I_1\right) (1 - I_1) + \frac{1-\zeta}{1+\zeta} (iI_2)^2}.$$

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