Interplay of magnon and electron currents in magnetic heterostructure

Yihong Cheng, Kai Chen, and Shufeng Zhang

Phys. Rev. B **96**, 024449 — Published 31 July 2017

DOI: 10.1103/PhysRevB.96.024449

Interplay of magnon and electron currents in magnetic heterostructure

Yihong Cheng, Kai Chen and Shufeng Zhang*

Department of Physics, University of Arizona, Tucson, Arizona 85721, USA

Abstract

In magnetic materials, both electrons and magnons are capable of carrying angular momentum currents. An external electric field can efficiently drive a charge and spin current of electrons, but it is unable to directly produce a charge-less magnon current. The generation of the magnon current is conventionally achieved via thermal gradients or the electron spin injection from interfaces. Here we investigate the magnon current induced by the momentum and angular momentum transfer from conduction electrons in magnetic layered systems. By using the generic exchange interaction between electrons and magnons, we derive the coupled diffusion equations for electron spins and magnons and we find, a) the ratio between the magnon current and the electric charge current is substantial at room temperature for conventional conducting ferromagnets, b) the spin diffusion length of electrons is significantly modified by the presence of the non-equilibrium magnon density, and c) the giant magnetoresistance of the magnetic multilayers for the current perpendicular to the plane of layers is reduced compared to the prior theory without taking into account the magnon current.

I. INTRODUCTION

One of the most important issues in spintronics is to identify and manipulate spin currents. Both electron spins and magnons have intrinsic angular momenta, and the translational flow of these particles (or quasiparticles) lead to a spin current or angular momentum current. Most of studies on the spin current has been focused on the electron spin current due to direct connections between the electron spin current and measurable physical phenomena. For example, the giant magnetoresistance (GMR)[1,2] and the spin transfer torque (STT)[3–5] are proportional to the spin current or spin polarization of conduction electrons in the magnetic multilayers and at the interfaces. By experimentally measuring GMR and STT for a given structure, the quantitative values of electron spin current can be obtained. The magnon current, on the other hand, is much harder to experimentally quantify. Since magnon is charge-less and it does not couple with the external electrical field, both generation and detection of magnon accumulation and magnon current are experimentally challenging.

At present, the magnon current has been generated by two methods. The spin Seebeck effect[6–9] utilizes a temperature gradient in a ferromagnetic material such that the thermal magnon density is non-uniform and magnons would diffuse from high to low temperature region, leading to a diffusive magnon current. While the thermal gradient could in principle works for all ferromagnetic materials, it is usually less efficient for magnetic metals due to practical difficulties in maintaining a large thermal gradient with a well-defined heat flow direction[10]. Another method is to utilize spin injection from the neighboring layer. For example, in a bilayer consisting of a heavy metal and a magnetic insulator, e.g., Pt/YIG bilayer, an in-plane electron charge current in the Pt layer could induce a magnon current in YIG layer via the conversion of the electron spin current to the magnon current[11–14]. In this case, the electron spin current and the magnon current are located in different spatial regions (electron spin current in Pt and magnon current in YIG, respectively), and the interaction between electron spin and magnon occurs only at the interface.

We shall first clarify that the magnon current we study here is the translational *flow* of quasi-particle magnons. The direction of the angular momentum of the magnon current is always *parallel* to the magnetization (or order parameter) because the magnon is defined as a quantum state without a transverse component relative to the quantization axis (magnetization direction). The spin-wave spin-current[15], sometimes referred as spin supercurrent[16], considers spatial and temporal dependence of the classical magnetization which is described by Landau-Lifshitz-Gilbert equation for transverse magnetization dynamics. A clear distinction of these magnon currents was discussed in Ref.[17].

In this paper, we theoretically formulate the spin-magnon transport by explicitly taking into account the exchange coupling between the electron spin and magnon in magnetic metals. The intrinsic strong exchange coupling in itinerant ferromagnets inevitably leads to substantial magnon current. Even for a uniformly magnetized conducting ferromagnet, an electric charge current is always accompanied with a magnon current at finite temperature. Up until now, the study of electric and spin transport has mostly neglected the role of the magnon current. Experimentally, several measured phenomena such as the spin accumulation and spin transfer directly depend on the total angular momentum current in which the magnon current is a part of. Other phenomena, such as the GMR and spin Hall effect, are sensitive to the electron spin current and seemingly independent of the magnon current. However, we will show that the presence of the magnon current could indirectly modify key parameters for electron spin transport. Thus, a schematic investigation on the correlation between spin current and magnon current would lead to a better understanding of angular momentum transport in general. At present, theoretical studies on the coupling and conversion between magnon and electron spin currents are mostly focused on the interface[11,18–22].

The paper is organized as follows. In Sec. II, we derive the coupled electron and magnon diffusion equations by using the generalized Boltzmann equations for electrons and magnons. In Sec. III, we apply the diffusion equations to magnetic multilayers. We conclude the paper in Sec. IV.

II. DERIVATION OF MAGNON AND ELECTRON SPIN DIFFUSION EQUATIONS

We start with a magnetic metal in which the exchange interaction between conduction electron spin $\boldsymbol{\sigma}$ and localized magnetic moment $\mathbf{S}_i$ at site i takes an isotropic form $V_{int} = -J_{sd}\sum_i \boldsymbol{\sigma}\cdot\mathbf{S}_i$, where J_{sd} is given by the exchange integral. In the spin-wave approximation, V_{int} is written as[23,24]

$$V_{int} = -\left(J_{sd}\sqrt{\frac{S}{2N}}\right)\sum_{\mathbf{kq}}\left(a^\dagger_{\mathbf{q}}c^\dagger_{\mathbf{k}\uparrow}c_{\mathbf{k+q}\downarrow} + a_{\mathbf{q}}c^\dagger_{\mathbf{k+q}\downarrow}c_{\mathbf{k}\uparrow}\right), \tag{1}$$

where N is the number of atomic sites, S is the spin per site, $c_{\mathbf{k}}\left(c_{\mathbf{k}}^{\dagger}\right)$ and $a_{\mathbf{q}}\left(a_{\mathbf{q}}^{\dagger}\right)$ represent the annihilation(creation) operators for the electron and the magnon. The linearized Boltzmann equation for the electrons in a layered structure, in which the spatial dependence is only one-dimensional, is[25,26]

$$v_{k_z}\frac{\partial f_\sigma(z,\mathbf{k})}{\partial z}-eE(z)\,v_{k_z}\frac{\partial f^0}{\partial\varepsilon_{\mathbf{k}}}=-\frac{f_\sigma(z,\mathbf{k})-\overline{f_\sigma}(z)}{\tau_\sigma}-\frac{f_\sigma(z,\mathbf{k})-\overline{f_{\sigma'}}(z)}{\tau_{\uparrow\downarrow}}+\left[\frac{\partial f_\sigma(z,\mathbf{k})}{\partial t}\right]_{sd} \quad (2)$$

where $f_\sigma(z,\mathbf{k})$ is the electron distribution function for spin $\sigma=\uparrow,\downarrow$ (or $\pm$), z is the coordination normal to the plane of the layers, τ_σ and $\tau_{\uparrow\downarrow}$ represent spin conserving and spin-flip scattering relaxation times, the overbar on the distribution function is the average over the momentum $\mathbf{k}$, and the last term is due to the exchange interaction V_{int}[27–29],

$$\begin{aligned}
\left[\frac{\partial f_\uparrow(z,\mathbf{k})}{\partial t}\right]_{sd} &= J_{sd}^2\frac{\pi S}{N\hbar}\sum_{\mathbf{q}}\beta\left[1-f^0(\varepsilon_{\mathbf{k+q}})\right]f^0(\varepsilon_{\mathbf{k}})\,N_m^0\left(\varepsilon_{\mathbf{q}}^m\right)\delta\left(\varepsilon_{\mathbf{k}}+\varepsilon_{\mathbf{q}}^m-\varepsilon_{\mathbf{k+q}}\right)\\
&\quad\times\left\{\left[\delta\mu_\downarrow(z)-\delta\mu_\uparrow(z)-\delta\mu_m(z)\right]+\left[g_\downarrow(z,\mathbf{k}+\mathbf{q})-g_\uparrow(z,\mathbf{k})-g_m(z,\mathbf{q})\right]\right\}\\
\left[\frac{\partial f_\downarrow(z,\mathbf{k})}{\partial t}\right]_{sd} &= J_{sd}^2\frac{\pi S}{N\hbar}\sum_{\mathbf{q}}\beta\left[1-f^0(\varepsilon_{\mathbf{k-q}})\right]f^0(\varepsilon_{\mathbf{k}})\left[N_m^0\left(\varepsilon_{\mathbf{q}}^m\right)+1\right]\delta\left(\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k-q}}-\varepsilon_{\mathbf{q}}^m\right)\\
&\quad\times\left\{\left[\delta\mu_\uparrow(z)-\delta\mu_\downarrow(z)+\delta\mu_m(z)\right]+\left[g_\uparrow(z,\mathbf{k}-\mathbf{q})-g_\downarrow(z,\mathbf{k})+g_m(z,\mathbf{q})\right]\right\}
\end{aligned}$$

where $\beta=(k_BT)^{-1}$ is the inverse of the temperature, f^0 and N_m^0 are the equilibrium distribution functions of the electron and the magnon, $\varepsilon_{\mathbf{k}}$ and $\varepsilon_{\mathbf{q}}^m$ are the dispersion relations for the electron and for the magnon, and we have separated the distribution functions into the sum of the equilibrium and non-equilibrium parts

$$\begin{aligned}
f_\sigma(z,\mathbf{k}) &= f^0(\mathbf{k})-\frac{\partial f^0(\mathbf{k})}{\partial\varepsilon_{\mathbf{k}}}\left[\delta\mu_\sigma(z)+g_\sigma(z,\mathbf{k})\right]\\
N_m(z,\mathbf{q}) &= N_m^0(\mathbf{q})-\frac{\partial N_m^0(\mathbf{q})}{\partial\varepsilon_{\mathbf{q}}^m}\left[\delta\mu_m(z)+g_m(z,\mathbf{q})\right]
\end{aligned}$$

in which non-equilibrium parts are further separated into the isotropic term $\delta\mu_{\sigma,m}(z)$ and anisotropic term $g_\sigma(z,\mathbf{k})[g_m(z,\mathbf{q})]$ with respect to the momentum, i.e., $\int d\mathbf{k}g_\sigma(z,\mathbf{k})=\int d\mathbf{q}g_m(z,\mathbf{q})=0$. We should point out that in the conventional Boltzmann equation for electrons, the spin-magnon interaction is phenomenologically included as a part of relaxation times (τ_s and $\tau_{\uparrow\downarrow}$)[30]. The explicit spin-magnon scattering in Eq. (2) would allow us to fully address both spin and magnon currents.

Similar, the Boltzmann equation for the magnon is

$$v_{q_z}\frac{\partial N_m(z,\mathbf{q})}{\partial z}=-\frac{N_m(z,\mathbf{q})-\overline{N_m}(z)}{\tau_m}-\frac{N_m(z,\mathbf{q})-N_m^0}{\tau_{th}}+\left[\frac{\partial N_m(z,\mathbf{q})}{\partial t}\right]_{sd} \quad (3)$$

where τ_m and τ_{th} represent the magnon number conserving and non-conserving relaxation times[31], respectively, and the last term is the momentum transfer between electron and magnon,

$$\left[\frac{\partial N_m(z,\mathbf{q})}{\partial t}\right]_{sd} = J_{sd}^2 \frac{\pi S}{N\hbar} \sum_{\mathbf{k}} \beta \left[1 - f^0\left(\varepsilon_{\mathbf{k+q}}\right)\right] f^0\left(\varepsilon_{\mathbf{k}}\right) N_m^0\left(\varepsilon_{\mathbf{q}}^m\right) \delta\left(\varepsilon_{\mathbf{k+q}} - \varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{q}}^m\right)$$
$$\times \left\{\left[\delta\mu_\downarrow(z) - \delta\mu_\uparrow(z) - \delta\mu_m(z)\right] + \left[g_\downarrow(z,\mathbf{k}+\mathbf{q}) - g_\uparrow(z,\mathbf{k}) - g_m(z,\mathbf{q})\right]\right\}$$

Comparing Eq. (2) and (3), we notice that there is no drift term in Eq. (3); this is because the magnon has no charge and the electric field does not drive the magnon motion. If no thermal gradient is applied, the magnon current would come from the momentum transfer from the electron, i.e., the last term in Eq. (3) becomes a source term for the magnon current.

The solutions of the above Boltzmann equations in multilayered structures depend on many parameters including the band dispersions, momentum and spin relaxations for electrons and magnons, and also detailed boundary conditions at the interface. Thus, it would be less physical rewarding to numerically solve the equations. A more physically meaningful approach is to simplify Eqs. (2) and (3) to a set of macroscopic equations such that the experimentally measurable quantities can be directly compared with. In the absence of the electron-magnon coupling, such simplifications have led to a set of spin diffusion equations of electrons[32] which provide a powerful tool to analyze experimental data for the GMR of magnetic multilayers with the current perpendicular to the plane of the layers[33]. We should extend this approach by explicitly taking into account the coupling between spin and magnon.

Four macroscopic variables are: spin accumulation $\delta n_s(z)$, electron spin current $j_s(z)$, magnon accumulation $\delta n_m(z)$ and magnon current $j_m(z)$.

$$\delta n_s(z) \equiv (2\pi)^{-3} \int d\mathbf{k} \left[f_\uparrow(z,\mathbf{k}) - f_\downarrow(z,\mathbf{k})\right]$$
$$j_s(z) \equiv (2\pi)^{-3} \int d\mathbf{k} v_{k_z} \left[f_\uparrow(z,\mathbf{k}) - f_\downarrow(z,\mathbf{k})\right]$$
$$\delta n_m(z) \equiv (2\pi)^{-3} \int d\mathbf{q} \left[N_m(z,\mathbf{q}) - N_m^0(\mathbf{q})\right]$$
$$j_m(z) \equiv (2\pi)^{-3} \int d\mathbf{q} v_{q_z} N_m(z,\mathbf{q})$$

Note that we have used the spin/magnon accumulation in the unit of the particle number per volume, and spin/magnon current in the unit of the particle number density current.

To convert into the angular momentum current, charge current, or accumulation, one can simply multiply the resulting variables by $\hbar$, e, or μ_B. To obtain the macroscopic equations for these variables, a number of approximations are needed. We provide the detail of these approximations in Appendix A. The resulting spin-magnon diffusion equations and the extended Ohm's law are,

$$\frac{d^2}{dz^2}\begin{pmatrix}\delta n_s(z)\\ \delta n_m(z)\end{pmatrix} = \begin{pmatrix}\lambda_s^{-2} & \lambda_{sm}^{-2}\\ \lambda_{ms}^{-2} & \lambda_m^{-2}\end{pmatrix}\begin{pmatrix}\delta n_s(z)\\ \delta n_m(z)\end{pmatrix} \tag{4}$$

and

$$\begin{pmatrix}j_s(z)\\ j_m(z)\end{pmatrix} = j_e\begin{pmatrix}P_s\\ P_m\end{pmatrix} + \begin{pmatrix}-\sigma_s & \sigma_{ms}\\ \sigma_{sm} & -\sigma_m\end{pmatrix}\frac{d}{dz}\begin{pmatrix}\delta n_s(z)\\ \delta n_m(z)\end{pmatrix} \tag{5}$$

where all coefficients in the above equations are given in the Appendix A. We shall briefly discuss the physical meaning of these coefficients here. The two lengths λ_s and λ_m in Eq. (4) are spin and magnon diffusion lengths. As usual, λ_s are related to the geometry mean of the momentum and spin-flip relaxation times $\sqrt{\tau_e\tau_{\uparrow\downarrow}}$ ($\tau_e^{-1} = \tau_\uparrow^{-1} + \tau_\downarrow^{-1}$) and λ_m is similarly related to $\sqrt{\tau_{th}\tau_m}$. However, due to coupling between spins and magnons, these lengths are modified by the interactions, see Appendix A. The off-diagonal matrix elements λ_{sm} and λ_{ms} describe the conversions between the spin and magnon accumulations, and both λ_{sm} and λ_{ms} are inversely proportional to J_{sd}^2, as expected. The generalized Ohm's law, Eq. (5), describes the drift spin/magnon current (first term) by an applied charge current j_e ($\equiv j_\uparrow + j_\downarrow$) and the diffusive spin/magnon current (second term) due to spin-magnon accumulation in an inhomogeneous structure. Clearly, in the absence of spin-magnon coupling, $\sigma_{ms} = \sigma_{sm} = 0$ and $P_m = 0$ since the electric field cannot drive a magnon current, and $P_s = P_0 \equiv (\tau_\uparrow - \tau_\downarrow)/(\tau_\uparrow + \tau_\downarrow)$ is the spin polarization of the ferromagnet. With the spin-magnon coupling, both P_s and P_m depend on a number of scattering parameters, e.g. J_{sd}, τ_e, τ_m, see Appendix A in detail. Finally, the spin-magnon conductivity is a 2×2 matrix whose diagonal elements are the electron spin conductivity σ_s and the magnon conductivity σ_m, and the off-diagonal element σ_{ms} (σ_{sm}) is the inter-conductivity induced by the exchange coupling and is proportional to J_{sd}^2.

Equations (4) and (5) are our main results. The diffusion equation, Eq. (4), explicitly describes how the electron spin diffusion is affected by the magnon diffusion. When there is no coupling, i.e., $\lambda_{sm}^{-1} = \lambda_{ms}^{-1} = 0$, spin and magnon have their own diffusion lengths. When the coupling becomes strong, these two length scales become mutually dependent.

We will study the diffusion properties of the coupled system in layered structure in the later Sections. The extended Ohm's law, Eq. (5), indicates a non-zero drift term for the magnon current $P_m j_e \neq 0$. The origin comes from the momentum transfer rather than the angular momentum transfer from the spin to the magnon. To see this, we consider the case $P_0 = 0$ and we still have a finite magnon current since the first term in P_m would still survive. Then, the question is how a charge current j_e without spin polarization induce a magnon current? The answer becomes obvious when we consider the electron spin and magnon together: the magnon current induced by momentum transfer of the electron will in turn create a spin polarization of the electron and thus P_s is no longer zero, as shown in the Appendix A. The induced P_s, though it is small, is proportional to the electron momentum relaxation time τ_e, rather than the spin-flip relaxation times $\tau_{\uparrow\downarrow}$, further illustrating that the momentum transfer is responsible for creating a magnon current even for $P_0 = 0$.

III. APPLICATION TO THE MAGNETIC MULTILAYERS

A. Two-angular-momentum current model

The conventional two-current model in spintronics[34] refers to a charge-current carried by spin-up and spin-down conduction electrons. For a ferromagnetic conductor such as NiFe, the spin-up and spin-down electrons have a different density of states at the Fermi level and a different scattering rate (or relaxation time). One may define a conductivity for each of the spin channel (up and down) such that the Ohm's law is applied to each spin channels. Alternatively, one can introduce a charge current, which is the sum of the two spin channel currents, and a spin current, the difference of the two. Since the charge current is conserved, the conventional two current model in fact reduces to just one spin current model. In the present case, we are dealing with truly two-angular momentum current model: both electron spin and magnon currents are carrying angular moments and neither of them is conserved. Next, we shall estimate a relative magnitude of the spin and magnon current in a conventional magnetic metal.

We show in Fig. 1 the spin current and the magnon current relative to the applied charge current, $P_s = j_s/j_e$ and $P_m = j_m/j_e$ for several plausible parameters closely related to the transition magnetic metals such as Ni, Co, Fe, and their alloys. As the exchange coupling J_{sd}

increases, the magnon current increases, and in a large coupling limiting case, the magnon current becomes saturated. The saturated magnon current is limited by the magnon scattering rate: even if the transfer of the spin-magnon is efficient for a large J_{sd}, the steady-state magnon current would be a balance between the magnon momentum relaxation and the momentum transfer from the electron current. On the other hand, the electron spin current decays as the coupling increases due to increased spin angular momentum loss from the electron to the magnon. The spin and magnon currents are highly temperature dependent because the number of magnon carriers increase with the temperature and thus the transfer between the electron current and magnon current is more efficiency at high temperatures. From Fig. 1, we conclude that the magnon current is comparable to the spin current at the room temperature and consequently, the total angular momentum current must include the magnon current when one studies the angular momentum transfer in multilayered systems, which we will discuss next.

B. Spin and magnon accumulation in layered structure

In this subsection, we determine the spin/magnon accumulation for a hypothetical bilayer where two identical ferromagnetic layers are in contact at $z = 0$, with their magnetization antiparallel aligned. Experimentally, a thin nonmagnetic layer is needed to separate the magnetic coupling between two layers such that the antiparallel of the two layers can be achieved. This simple example would provide insights on the spatial and temperature dependence of the spin and magnon distributions. We should first solve the spin-magnon diffusion equations, Eq. (4), for each layer

$$\delta n_s(z) = A_L \exp\left(\frac{z}{\lambda_+}\right) + B_L \exp\left(\frac{z}{\lambda_-}\right) \tag{6}$$

for $z < 0$, and

$$\delta n_s(z) = A_R \exp\left(-\frac{z}{\lambda_+}\right) + B_R \exp\left(-\frac{z}{\lambda_-}\right) \tag{7}$$

for $z > 0$, where the two characteristic lengths are given by

$$\frac{1}{\lambda_\pm^2} = \frac{1}{2}\left(\frac{1}{\lambda_s^2} + \frac{1}{\lambda_m^2}\right) \pm \frac{1}{2}\sqrt{\left(\frac{1}{\lambda_s^2} - \frac{1}{\lambda_m^2}\right)^2 + \frac{4}{\lambda_{sm}^2\lambda_{ms}^2}} \tag{8}$$

and A_L, B_L, A_R and B_R are four constants of the integration to be determined by the boundary conditions. The magnon accumulation $\delta n_m(z)$ can be similarly obtained without

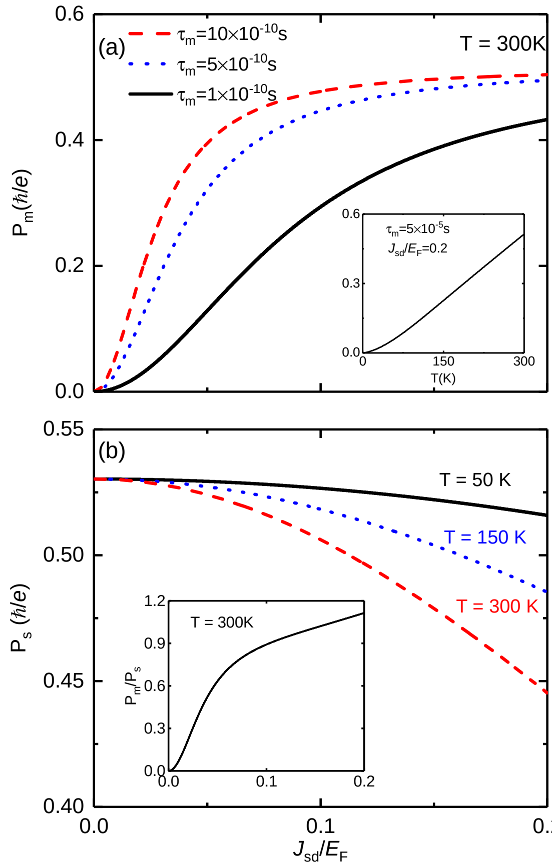

FIG. 1. (Color Online) (a) The ratio of the magnon current to the charge current and (b) The ratio of the spin current to the charge current, as a function of the exchange coupling magnitude for several different magnon momentum relaxation times and temperatures. The insert in (a) shows the temperature dependence at a fixed $J_{sd}/E_F = 0.2$, and the insert in (b) shows the ratio of the magnon current to the spin current at a fixed temperature $T = 300K$. Other parameters are $E_F = 5eV$, $T_C = 550K$, $\tau_e = 7 \times 10^{-16}s$.

a new constant of the integration since $\delta n_m(z)$ can be expressed by $\delta n_s(z)$ from Eq. (4), namely, $\delta n_m(z) = \lambda_{sm}^2[\delta n_s''(z) - \lambda_s^{-2}\delta n_s(z)]$. By using the extended Ohm's law, Eq. (5), the spin and magnon current can also be expressed in terms of these constants. For the perfect interface, the continuity of the spin and magnon currents and accumulations gives four boundary equations at $z = 0$, and thus we are able to completely determine these four constants (A_L, B_L, A_R and B_R) and therefore the position dependent spin and magnon accumulations and currents. The explicit solution could be found in Appendix B.

In Figure. 2, we show the spatial distribution of the spin and magnon accumulations and currents near the interface. When the coupling J_{sd}/E_F is small, the magnon accumulation and magnon currents are small, as expected. The magnon accumulation increases as J_{sd} increases since the source of the non-equilibrium is through the coupling. On the other hand, the electron spin accumulation decreases due to additional electron relaxations to the magnon. Interestingly, the magnon accumulation displays a non-monotonic behavior near the interface. The origin comes from the interplay between two length scales (λ_+ and λ_-) in the solution of the accumulation, see Eq. (6)-(8), and Appendix B. The supposition of the two exponential functions leads to a local extrema at the position between these two lengths. Similarly, the magnon current also displays a non-monotonic function. The maximum value could even exceed the value of the uniform layer without the interface.

C. Magnetoresistance

Magnetoresistance of magnetic multilayers for the current perpendicular(CPP) to the layers has been theoretically modeled using Valet-Fert's[32] spin diffusion equations for electrons at zero temperature where the electron-magnon spin-flip scattering is frozen out. The essential physics picture is that the spin accumulation creates an additional interface resistance. Since the spin accumulation depends on the magnetization configuration in magnetic multilayers, the resulting resistance varies with the relative magnetization of each layer. In the above example of the bilayer, there is no spin accumulation when the magnetizations of the two magnetic layers are parallel. The maximum spin accumulation is created for the antiparallel magnetization. Without taking into count the coupling between electrons and magnons, Valet-Fert's model immediately leads to a magnetoresistance ΔR, defined as the resistance difference between the antiparallel and parallel aligned magnetization,

$$\Delta R = P_0^2 \lambda_s \rho_F \tag{9}$$

where ρ_F is the resistivity of the magnetic layer. The above Valet-Fert spin diffusion theory successfully provides an essential method for analyzing the CPP magnetoresistance at low temperatures[33]. At higher temperatures, the magnons become important and the above simple expression breaks down. Indeed, the experimental findings on strong temperature dependence of the CPP GMR[35,36] have not been satisfactorily explained. With our formalism, the electron spin accumulation would be decreased due to electron-magnon angular momen-

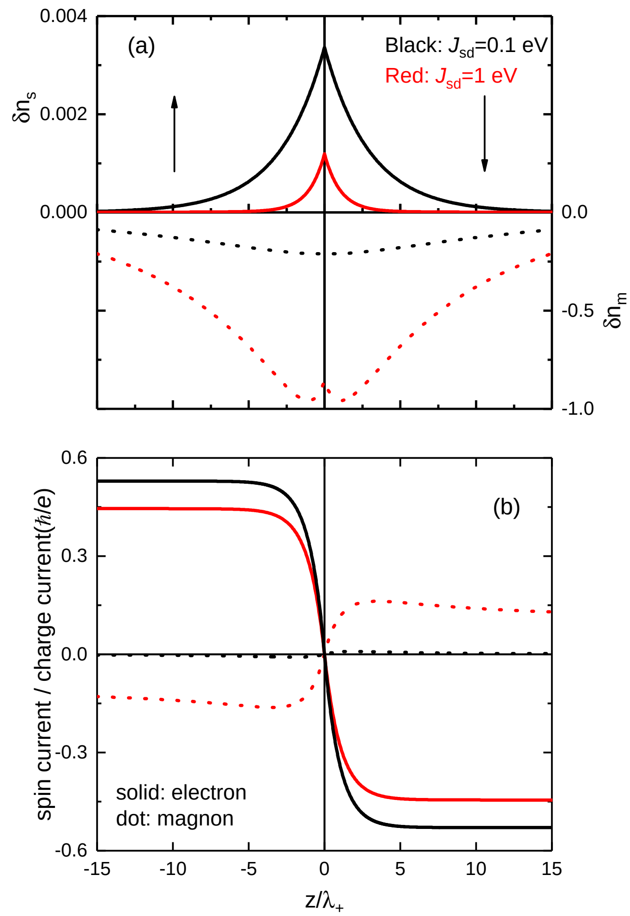

FIG. 2. (Color Online) (a) The position dependence of the spin and magnon accumulation. (b) the position dependence of the spin current and magnon current. The parameters are the same as used in Fig. 1.

tum transfer and thus provides a plausible explanation for the temperature dependent GMR. Unfortunately, the analytical expression of CPP GMR becomes rather cumbersome due to the multiple diffusion lengths as well as additional scattering channels involving magnons, see Appendix B. However, the essential physical picture remains intact: the magnetoresistance comes from the additional resistance generated by the spin accumulation which is strongly affected by the electron-magnon coupling at high temperatures. In Fig. 3, we show the magnetoresistance as function of the exchange coupling at several temperatures.

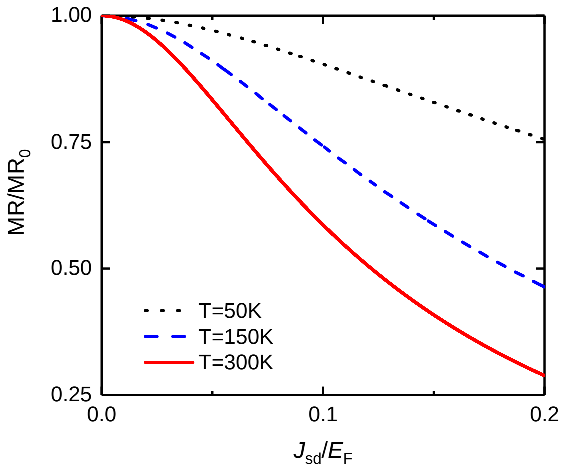

FIG. 3. (Color Online) Magnetoresistance of the magnetic bilayer, normalized to the classical value without the spin-magnon coupling, as a function of the exchange coupling for several different temperatures. We have used a temperature-independent relaxation time in order to single out the temperature dependent contribution from the magnon accumulation.

IV. DISCUSSION AND CONCLUSION

We have established the macroscopic diffusion equations for non-equilibrium electron spins and magnons of conducting ferromagnets. The essential conclusion is that the magnon current is always accompanied with the charge and spin electron current. The magnon current is the result of the momentum and angular momentum transfer from the conducting electrons. In magnetic multilayers, both spins and magnons are accumulated near the interfaces. As a result, the spin and magnon currents are spatially varying on length scales determined by multiple scattering mechanisms and by the coupling between spins and magnons. The spin-magnon diffusion equations can be broadly applied to various spintronics systems. Experimental determination of our predicted magnon accumulation and magnon currents are certainly challenging due to the lack of proper experimental tools that can directly couple to the magnon accumulation. Aside from the indirect methods such as converting the magnon current to the spin current or the spin transfer torques, directly measuring the magnon accumulation might be possible as well because the magnon accumulation is much larger than the electron spin accumulation. At present, the electron or hole spin accumulation is only measured in a semiconductor system[37]. For metallic magnets, the spin accumulation is too small even for the current density as high as $10^8 A/cm^2$. The magnon accumulation might offer a possible route for directly observing the accumulation of the magnetic moment.

This work was partially supported by National Science Foundation under Grant No. ECCS-1404542.

Appendix A: Spin-Magnon Diffusion Equations and the Extended Ohm's Law

In this Appendix, we specify approximations and calculations leading to Eqs. (4) and (5). As shown in [32], a key approximation to obtain a closed form for the macroscopic diffusion equations is to expand the non-equilibrium distribution functions into polynomial series P_n, namely, $g_\sigma(z,\mathbf{k}) = \sum_n g_\sigma^{(n)}(z,k)P_n(k_z/k)$ and $g_m(z,\mathbf{q}) = \sum_n g_m^{(n)}(z,q)P_n(q_z/q)$, and only keeping the lowest order in expansion($n=1$). In the same limiting cases, such approximations are justifiable[21,31]. By summing over $\mathbf{k}$ in Eq. (2) for each spin channel, and by summing over $\mathbf{q}$ in Eq. (3), we find, after tedious but straightforward calculations,

$$\frac{d}{dz}\begin{pmatrix} j_s(z) \\ j_m(z) \end{pmatrix} = -\begin{pmatrix} \tau_{11}^{-1} & \tau_{12}^{-1} \\ \tau_{21}^{-1} & \tau_{22}^{-1} \end{pmatrix}\begin{pmatrix} \delta n_s(z) \\ \delta n_m(z) \end{pmatrix} \tag{A1}$$

where the off-diagonal matrix elements of Eq. (A1) depend on the coupling between spin and magnon and density of states of both quasiparticles. At the temperature much lower than the Curie temperature, the expression reduces to

$$\tau_{12}^{-1} = \left(\frac{J_{sd}}{k_B T_F}\right)^2 \frac{k_B T}{\hbar}\left(\frac{T_C}{T}\right)^{1/2} S\alpha_1$$

and

$$\tau_{21}^{-1} = \left(\frac{J_{sd}}{k_B T_C}\right)^2 \frac{k_B T_C}{\hbar}\left(\frac{T}{T_F}\right)^{1/2} S\alpha_2$$

where α_1 and α_2 are numerical constants. The diagonal parts of the matrix elements are

$$\tau_{11}^{-1} = 2\left(\tau_{\uparrow\downarrow}^{-1} + \tau_{21}^{-1}\right), \qquad \tau_{22}^{-1} = \left(\tau_{th}^{-1} + \frac{1}{2}\tau_{12}^{-1}\right).$$

Next, we multiply v_{k_z} on both sides of Eq. (2) and v_{q_z} on both sides of Eq. (3) and sum over momentum, and we find

$$\frac{d}{dz}\begin{pmatrix} \delta n_s(z) \\ \delta n_m(z) \end{pmatrix} = \begin{pmatrix} \zeta_1 \\ \zeta_2 \end{pmatrix} j_e - \begin{pmatrix} \zeta_{11} & \zeta_{12} \\ \zeta_{21} & \zeta_{22} \end{pmatrix}\begin{pmatrix} j_s(z) \\ j_m(z) \end{pmatrix} \tag{A2}$$

where

$$\zeta_1 = \frac{3}{v_F^2}\left(P_0\tau_e^{-1} + \tau_{em}^{-1}\right), \qquad \zeta_2 = \frac{1}{2I_m}\tau_{em}^{-1}$$
$$\zeta_{11} = \frac{3}{v_F^2}\left(\tau_e^{-1} + \tau_{em}^{-1}\frac{T_F}{T}\right), \qquad \zeta_{22} = \frac{1}{I_m}\left(\tau_m^{-1} + \tau_{me}^{-1}\right)$$
$$\zeta_{12} = \frac{3}{v_F^2}\tau_{me}^{-1}, \qquad \zeta_{21} = \frac{1}{2I_m}\tau_{em}^{-1}\frac{T_C}{T_F}$$

where

$$\tau_{em}^{-1} = \frac{J_{sd}^2}{\hbar\left(k_B T_C\right)}\left(\frac{T}{T_F}\right)^{1/2} S\alpha_3$$
$$\tau_{me}^{-1} = \frac{J_{sd}^2}{\hbar\left(k_B T_F\right)}\left(\frac{T T_C}{T_F^2}\right)^{1/2} S\alpha_4$$

and v_F is Fermi velocity, the integral is defined as $I_m = \int d\mathbf{q}\left(-\partial N_m^0/\partial\varepsilon_{\mathbf{q}}^m\right) / \int d\mathbf{q}\left(-\partial N_m^0/\partial\varepsilon_{\mathbf{q}}^m\right) v_{q_z}^2$, α_3 and α_4 are other two numerical constants. From these four equations above we could easily arrive at the spin-magnon diffusion equations, Eq. (4) and the extended Ohm's law, Eq. (5) in the main text. The explicit form of the four length scales in Eq. (4) are

$$\lambda_s^{-2} = \zeta_{11}\tau_{11}^{-1}, \qquad \lambda_m^{-2} = \zeta_{22}\tau_{22}^{-1}$$
$$\lambda_{sm}^{-2} = \zeta_{11}\tau_{12}^{-1} + \zeta_{12}\tau_{22}^{-1}$$
$$\lambda_{ms}^{-2} = \zeta_{21}2\tau_{11}^{-1} + \zeta_{22}\tau_{21}^{-1}$$

and the coefficients in extended Ohm's law are

$$P_s = \frac{P_0}{1+\chi_s} + \frac{1}{\left(1+\chi_s\right)\left(1+\chi_m\right)}\frac{\tau_e}{\tau_{em}}$$
$$P_m = \frac{1}{2}\frac{1}{\left(1+\chi_m\right)}\frac{\tau_m}{\tau_{em}}\left[1 - \frac{P_0}{\left(1+\chi_s\right)}\frac{T_C}{T_F}\right]$$
$$\sigma_s = \frac{\sigma_s^0}{\left(1+\chi_s\right)}, \qquad \sigma_m = \frac{\sigma_m^0}{\left(1+\chi_m\right)}$$
$$\sigma_{ms} = \frac{2}{\left(1+\chi_s\right)\left(1+\chi_m\right)}\frac{\tau_e}{\tau_{me}}\sigma_m^0$$
$$\sigma_{sm} = \frac{1}{\left(1+\chi_s\right)\left(1+\chi_m\right)}\frac{\tau_m}{\tau_{em}}\frac{T_F}{T_C}\frac{\sigma_s^0}{2}$$

where $\chi_s = \frac{\tau_e}{\tau_{em}}\frac{T_F}{T}$ and $\chi_m = \frac{\tau_m}{\tau_{me}}$ are unitless quantities which characterize the relative strength of spin-magnon coupling in momentum scattering relaxation time(both proportional to J_{sd}^2 and also temperature dependent), $\sigma_s^0 = 2\tau_e v_F^2/3$ and $\sigma_m^0 = \tau_m I_m$ are electron spin conductivity and the magnon conductivity without considering coupling.

Appendix B: Characteristic Lengths and Boundary Conditions

The decay length associated with the diffusion equation, Eq. (4), can be readily obtained by taking the electron spin and magnon accumulations in the form of $e^{\pm\frac{z}{\lambda}}$ such that λ satisfies the eigenvalue equations,

$$\lambda^{-2}\begin{pmatrix}\delta n_s(z)\\ \delta n_m(z)\end{pmatrix} = \begin{pmatrix}\lambda_s^{-2} & \lambda_{sm}^{-2}\\ \lambda_{ms}^{-2} & \lambda_m^{-2}\end{pmatrix}\begin{pmatrix}\delta n_s(z)\\ \delta n_m(z)\end{pmatrix}$$

the non-zero solutions for δn_s and δn_m lead to two new characteristic lengths in Eq. (8). If the coupling(J_{sd}) is 0, $\lambda_\pm$ reduces to the original spin diffusion length $\lambda_s^0 = \sqrt{v_F^2\tau_e\tau_{\uparrow\downarrow}/3}$ and magnon diffusion length $\lambda_m^0 = \sqrt{I_m\tau_m\tau_{th}}$.

In a ferromagnetic bilayers with magnetization antiparallel aligned, we take the perfect interface condition as

$$\begin{aligned}\delta n_m\left(0^-\right) &= \delta n_m\left(0^+\right)\\ \delta n_s\left(0^-\right) &= \delta n_s\left(0^+\right)\\ j_s\left(0^-\right) &= j_s\left(0^+\right)\\ j_m\left(0^-\right) &= j_m\left(0^+\right)\end{aligned} \tag{B1}$$

and the general solution of accumulation functions are such

$$\begin{aligned}\delta n_s(z) &= A_L\exp\left(\frac{z}{\lambda_+}\right) + B_L\exp\left(\frac{z}{\lambda_-}\right)\\ \delta n_m(z) &= \alpha A_L\exp\left(\frac{z}{\lambda_+}\right) + \beta B_L\exp\left(\frac{z}{\lambda_-}\right)\end{aligned}$$

for $z < 0$ and

$$\begin{aligned}\delta n_s(z) &= A_R\exp\left(-\frac{z}{\lambda_+}\right) + B_R\exp\left(-\frac{z}{\lambda_-}\right)\\ \delta n_m(z) &= \alpha A_R\exp\left(-\frac{z}{\lambda_+}\right) + \beta B_R\exp\left(-\frac{z}{\lambda_-}\right)\end{aligned}$$

for $z > 0$, where $\alpha = \lambda_{sm}^2\left(\lambda_+^{-2} - \lambda_s^{-2}\right)$, $\beta = \lambda_{sm}^2\left(\lambda_-^{-2} - \lambda_s^{-2}\right)$.

The coefficients could be obtained by matching the boundary conditions

$$A_L = A_R = \lambda_+ j_e\frac{P_s(\sigma_{sm} - \beta\sigma_m) + P_m(\sigma_s - \beta\sigma_{ms})}{(\alpha-\beta)\sigma_s\sigma_m} \tag{B2}$$

$$B_L = B_R = -\lambda_- j_e\frac{P_m(\sigma_s - \alpha\sigma_{ms}) + P_s(\sigma_{sm} - \alpha\sigma_m)}{(\alpha-\beta)\sigma_s\sigma_m} \tag{B3}$$

then the magnon accumulation, spin current and magnon current would be further determined.

From the Boltzmann equations we could also get an expression for electron current j_e as a total derivative with respect to electron accumulation $\delta n_e(z)$ (which proportional to the summation of electrochemical potentials of two spins), spin accumulation $\delta n_s(z)$ and magnon accumulation $\delta n_m(z)$

$$j_e = \frac{d}{dz}\left[\sigma_e \delta n_e(z) + \sigma_{se}\delta n_s(z) + \sigma_{me}\delta n_m(z)\right] \tag{B4}$$

By using the continuity requirement of charge current across the interface, we find the magnetoresistance would be

$$\begin{aligned}\delta R &= \frac{\sigma_{se}}{\sigma_e}(A_L + B_L) + \frac{\sigma_{me}}{\sigma_e}(\alpha A_L + \beta B_L) \\ &\approx \frac{P_0}{1+\chi_s}(A_L + B_L) + \frac{1}{1+\chi_m}(\alpha A_L + \beta B_L)\end{aligned} \tag{B5}$$

the first term means the magnetoresistance would be largely reduced because both spin polarization and accumulation at interface would be smaller than the case without coupling, and the second term is high order effect coming from the magnon accumulation would in turn convert back into electron spin accumulation.

* zhangshu@email.arizona.edu

[1] M. N. Baibich, J. M. Broto, A. Fert, F. N. Van Dau, F. Petroff, P. Etienne, G. Creuzet, A. Friederich, and J. Chazelas, Phys. Rev. Lett. **61**, 2472 (1988).

[2] G. Binasch, P. Grünberg, F. Saurenbach, and W. Zinn, Phys. Rev. B **39**, 4828 (1989).

[3] J. C. Slonczewski, Journal of Magnetism and Magnetic Materials **159**, L1 (1996).

[4] L. Berger, Phys. Rev. B **54**, 9353 (1996).

[5] D. C. Ralph and M. D. Stiles, Journal of Magnetism and Magnetic Materials **320**, 1190 (2008).

[6] K.-i. Uchida, H. Adachi, T. Ota, H. Nakayama, S. Maekawa, and E. Saitoh, Appl. Phys. Lett. **97**, 172505 (2010).

[7] S. Y. Huang, W. G. Wang, S. F. Lee, J. Kwo, and C. L. Chien, Phys. Rev. Lett. **107**, 216604 (2011).

[8] M. Weiler, M. Althammer, F. D. Czeschka, H. Huebl, M. S. Wagner, M. Opel, I.-M. Imort,

G. Reiss, A. Thomas, R. Gross, and S. T. B. Goennenwein, Phys. Rev. Lett. **108**, 106602 (2012).

[9] K. Uchida, T. Nonaka, T. Kikkawa, Y. Kajiwara, and E. Saitoh, Phys. Rev. B **87**, 104412 (2013).

[10] S. Y. Huang, X. Fan, D. Qu, Y. P. Chen, W. G. Wang, J. Wu, T. Y. Chen, J. Q. Xiao, and C. L. Chien, Phys. Rev. Lett. **109**, 107204 (2012).

[11] S. Takahashi, E. Saitoh, and S. Maekawa, J. Phys.: Conf. Ser. **200**, 062030 (2010).

[12] L. J. Cornelissen, J. Liu, R. A. Duine, J. B. Youssef, and B. J. van Wees, Nat Phys **11**, 1022 (2015).

[13] J. Li, Y. Xu, M. Aldosary, C. Tang, Z. Lin, S. Zhang, R. Lake, and J. Shi, Nat Commun **7**, 10858 (2016).

[14] H. Wu, C. H. Wan, X. Zhang, Z. H. Yuan, Q. T. Zhang, J. Y. Qin, H. X. Wei, X. F. Han, and S. Zhang, Phys. Rev. B **93**, 060403 (2016).

[15] Y. Kajiwara, S. Takahashi, S. Maekawa, and E. Saitoh, IEEE Transactions on Magnetics **47**, 1591 (2011).

[16] E. B. Sonin, Advances in Physics **59**, 181 (2010).

[17] A. Rückriegel and P. Kopietz, Phys. Rev. B **95**, 104436 (2017).

[18] J. Xiao, G. E. W. Bauer, K.-c. Uchida, E. Saitoh, and S. Maekawa, Phys. Rev. B **81**, 214418 (2010).

[19] S. S.-L. Zhang and S. Zhang, Phys. Rev. B **86**, 214424 (2012).

[20] S. M. Rezende, R. L. Rodríguez-Suárez, and A. Azevedo, Phys. Rev. B **93**, 054412 (2016).

[21] S. M. Rezende, R. L. Rodríguez-Suárez, R. O. Cunha, A. R. Rodrigues, F. L. A. Machado, G. A. Fonseca Guerra, J. C. Lopez Ortiz, and A. Azevedo, Phys. Rev. B **89**, 014416 (2014).

[22] K. Chen, W. Lin, C. L. Chien, and S. Zhang, Phys. Rev. B **94**, 054413 (2016).

[23] T. Kasuya, Prog. Theor. Phys. **16**, 58 (1956).

[24] L. C. Davis and S. H. Liu, Phys. Rev. **163**, 503 (1967).

[25] A. Fert, J. Phys. C: Solid State Phys. **2**, 1784 (1969).

[26] J. Barnas, O. Baksalary, and Y. Bruynseraede, J. Phys.: Condens. Matter **7**, 6437 (1995).

[27] T. Kasuya, Prog. Theor. Phys. **22**, 227 (1959).

[28] I. Mannari, Prog. Theor. Phys. **22**, 335 (1959).

[29] D. A. Goodings, Phys. Rev. **132**, 542 (1963).

[30] D. L. Mills, A. Fert, and I. A. Campbell, Phys. Rev. B **4**, 196 (1971).

[31] S. S.-L. Zhang and S. Zhang, Phys. Rev. Lett. **109**, 096603 (2012).

[32] T. Valet and A. Fert, Phys. Rev. B **48**, 7099 (1993).

[33] J. Bass, in *Handbook of Spin Transport and Magnetism*, edited by E. Tsymbal and I. Zutic (CRC Press, 2012) pp. 69–93.

[34] N. F. Mott, Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences **153**, 699 (1936).

[35] M. A. M. Gijs, S. K. J. Lenczowski, and J. B. Giesbers, Phys. Rev. Lett. **70**, 3343 (1993).

[36] K. Liu, K. Nagodawithana, P. C. Searson, and C. L. Chien, Phys. Rev. B **51**, 7381 (1995).

[37] Y. K. Kato, R. C. Myers, A. C. Gossard, and D. D. Awschalom, Science **306**, 1910 (2004).