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Quantum perfect crossed Andreev reflection in top-gated quantum anomalous Hall insulator-superconductor junctions

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We investigate the quantum tunneling and Andreev reflection in a top-gated quantum anomalous Hall insulator proximity-coupled with a superconductor junction. A quantized perfect crossed Andreev reflection with its coefficient being integer 1 is obtained and all other scattering processes (the normal reflection, normal tunneling and local Andreev reflection) are completely suppressed, when the topological superconductor phase with Chern number \( N = 1 \) is realized. This perfect crossed Andreev reflection originates from the tunneling of the chiral Majorana edge states, and the phase of tunneling amplitude only being 0 and \( \pi \) plays a decisive role. Furthermore, because of the chiral characteristic of the Majorana edge states, the perfect crossed Andreev reflection is robust against the disorder and can work in a wide range of system parameters.

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I. INTRODUCTION

With the expectation on the utilization of entanglement effects in quantum communication and computation, quantum entanglement has been an extremely active area attracting many researchers1–3. One of the most key issues is searching for the methods of creating entangled particles. A Cooper pair in the superconductor is a pair of electrons binding together in a certain manner, which are both spin and momentum entangled. Thus superconductor is deemed as natural sources for generating nonlocal Einstein-Podolsky-Rosen electron pairs4,5. A Cooper pair can be spatially separated with the help of the crossed Andreev reflection (CAR)6–10, a process of converting an incoming electron from one terminal into an outgoing hole at the other terminal. These spatially-separated entangled electrons are the key building blocks for its promising application in quantum communication and quantum computing11–17. Therefore, many Cooper-pair splitters by coupling a superconductor with quantum dots9,18–21, carbon nanotubes22,23, Luttinger liquid wires24, graphene25–27, etc.,28 have been theoretically put forward and experimentally implemented in part.

However, besides the CAR, there also exists local Andreev reflection (LAR)29, where the outgoing hole returns back to the same terminal as the incoming electron. Since the incoming electrons and outgoing holes in the LAR reside in the same terminals, the LAR is usually stronger than CAR. Moreover, CAR is often completely masked by another nonlocal process known as quantum tunneling, which does not involve Cooper pairs and is therefore a parasitic process8,30. In the experiment, impurities and disorders exist inevitably, which will cause the normal reflection and weaken the CAR. Therefore, the coefficient of the CAR is usually very small. In order to obtain a perfect CAR with its coefficient being 1, it is necessary to propose a system where the LAR, normal tunneling and the normal reflection are all completely suppressed and then all incident electrons are converted into holes in the other terminal.

Quantum anomalous Hall insulator (QAHI) is a special kind of material where quantum Hall effects can be realized in the absence of an external magnetic field, and the unidirectionality of the chiral edge states promises the absence of backscattering31–35. QAHI in proximity to an s-wave superconductor can induce a topological superconductor (TSC)36, which supports topologically protected chiral Majorana edge states37,38. In the past proposed system based on the QAHI proximity-coupled with
a superconductor junction\(^3\), it has been found that the LAR could be completely suppressed and CAR could be considerably improved. But the normal tunneling process exists, and the CAR is not perfect still.

In this paper, we investigate normal tunneling, LAR and CAR in top-gated QAHI proximity-coupled with a superconductor, as shown in Fig.1(a), in which the top-gated voltage can moderate the transition of TSC phase with Chern number from \( N = 2 \) to \( N = 1 \). We find that in the TSC phase with \( N = 1 \), a quantized perfect CAR occurs in which the CAR coefficient shows a plateau with its value being integer 1, and all other scattering processes, e.g. LAR, normal tunneling and normal reflection, are completely suppressed. This perfect CAR originates from the tunneling of the chiral Majorana edge states, and the phase of tunneling amplitude only being 0 and \( \pi \) plays a decisive role. Because of the chiral characteristic of the Majorana edge states, the perfect CAR is robust against the disorder and can survive in a wide range of system parameters.

The rest of the paper is organized as follows. In Sec. II, we present the model Hamiltonian of the QAHI-TSC-QAHI junction and show the formulas of the tunneling coefficient, LAR coefficient and CAR coefficient. In Sec. III, we investigate the quantum perfect CAR effect. At last, the results are summarized in Sec. IV.

II. MODEL AND METHOD

For definiteness, we consider the simplest QAHI model Hamiltonian realized with low-energy states near the \( \Gamma \) point

\[
H_{\text{QAHI}}(p) = \begin{pmatrix}
 m + Bp^2 + \mu_L & A(p_x - ip_y) \\
 A(p_x + ip_y) & -m - Bp^2 + \mu_L
\end{pmatrix},
\]

(1)

where \( A, B \) and \( m \) are material parameters, and \( \mu_L \) is the potential energy of the QAHI leads. The basis vector is \((c^\dagger_\uparrow(p), c^\dagger_\downarrow(p))^T\), where \( c^\dagger_\uparrow(p) \) (\( c^\dagger_\downarrow(p) \)) is the operator annihilating an electron of momentum \( p \) and spin \( \uparrow \) (\( \downarrow \)). The sign of \( m/B \) determines the topological properties of the system, and the QAHI is obtained by setting \( m/B < 0 \). Since the tight-binding representation is used in our calculations, the Hamiltonian can be mapped onto a nearest neighbor tight-binding representation on a two dimensional square lattice,

\[
H = \sum_i [\psi_i^\dagger T_0 \psi_i + (\psi_i^\dagger T_x \psi_{i+\delta x} + \psi_i^\dagger T_y \psi_{i+\delta y}) + \text{H.c.}],
\]

(2)

where \( \psi_i = (c^\dagger_\uparrow, c^\dagger_\downarrow)^T \) and \( c^\dagger_\uparrow \) (\( c^\dagger_\downarrow \)) is the annihilation operator on site \( i \) with spin \( \uparrow \) (\( \downarrow \)). \( \sigma_0 \) and \( \sigma_{x,y,z} \) are the unit 2 \( \times \) 2 matrix and the Pauli matrix for spin, \( a \) is the lattice length, and \( \delta x (\delta y) \) is the unit vector along \( x (y) \) direction. In our calculations, we set \( A = B = 1 \), the lattice length \( a = 1 \) and \( h = 1 \).

In proximity to an s-wave superconductor, a finite pairing potential \( \Delta \) can be induced in QAHI. This gives us the Bogoliubov-de Gennes (BdG) Hamiltonian

\[
H_{\text{BdG}} = \frac{1}{2} \begin{pmatrix}
 H_{\text{QAHI}}(p) + \mu_S & i\Delta_g \\
-i\Delta_g^\dagger & -H_{\text{QAHI}}(-p) - \mu_S
\end{pmatrix},
\]

(3)

where \( \mu_S \) is potential energy varied by the top-gated voltage and the basis vector is \((c^\dagger(p), c^\dagger(-p), c^\dagger_\uparrow(-p), c^\dagger_\downarrow(p))^T\) for \( m < -\sqrt{\Delta^2 + \mu_S^2} \), the TSC phase with \( N = 2 \) is obtained, which owns two chiral Majorana modes\(^7\). Whereas the TSC phase with \( N = 1 \) is realized by setting \(-\sqrt{\Delta^2 + \mu_S^2} < m < \sqrt{\Delta^2 + \mu_S^2} \), which supports single Majorana edge state propagating at the edges of the sample. To directly picture this edge state, we calculate the energy dispersion of the TSC phase with \( N = 1 \) in Fig.2(a). One can see that a pair of chiral gapless edge state mode traverses across the bulk band gap. The corresponding wave functions \( \Psi_i \) of edge states are shown in Fig.2(b). One can see that wave functions \( \Psi_i \) are completely localized at the system boundary for \( \mu_S = 0.0 \) and \( \mu_S = 0.6 \). However wave functions \( \Psi_i \) oscillates and its localization length is considerably increased while \( |m| \ll \mu_S \). The reason is that the system at \( |m| \ll \mu_S \) is metal with high density of state for \( \Delta = 0 \). For \( m > \sqrt{\Delta^2 + \mu_S^2} \), the system is in normal superconductor phase of \( N = 0 \).

We now turn to analyze the scattering processes when an incident electron with the energy \( E \) flows from the left QAHI terminal into the central TSC region. By using the non-equilibrium Green’s function technique\(^3\), we can obtain the normal tunneling coefficient \( T \), LAR coefficient \( T_{\text{LAR}} \) and CAR coefficient \( T_{\text{CAR}} \):

\[
T(E) = \text{Tr}[\Gamma^R_{ee} \Gamma^\dagger_{eh} \Gamma^L_{ee} \Gamma^\dagger_{he}],
\]

(4)

\[
T_{\text{LAR}}(E) = \text{Tr}[\Gamma^R_{ee} \Gamma^\dagger_{eh} \Gamma^L_{eh} \Gamma^\dagger_{he}],
\]

(5)

\[
T_{\text{CAR}}(E) = \text{Tr}[\Gamma^R_{ee} \Gamma^\dagger_{eh} \Gamma^L_{eh} \Gamma^\dagger_{he}],
\]

(6)
The reason behind this observation is that there exist the tunneling between Majorana edge modes $\gamma_1$ and $\gamma_3$ at certain system lengths. Thus the outgoing Majorana state $\sqrt{2}(b_1 + b_1^\dagger)$ in the left terminal can be written as
\begin{equation}
(\sqrt{2}/2)(b_1 + b_1^\dagger) = r\gamma_1 + te^{i\varphi}\gamma_3
\end{equation}
where $te^{i\varphi}$ is the tunneling amplitude between $\gamma_1$ to $\gamma_3$ satisfying $r^2 + t^2 = 1$. Notice that the phase $\varphi$ can only take the value 0 or $\pi$ because the Majorana Fermion is self-Hermitian particle. Similarly, the other three outgoing Majorana edge states are:
\begin{equation}
(\sqrt{2}/2)(b_1 - b_1^\dagger) = \gamma_4,
\end{equation}
\begin{equation}
(\sqrt{2}/2)(b_2 + b_2^\dagger) = r\gamma_3 + te^{i\varphi}\gamma_1,
\end{equation}
\begin{equation}
(\sqrt{2}/2)(b_2 - b_2^\dagger) = \gamma_2.
\end{equation}
This gives a scattering matrix:
\begin{equation}
\begin{pmatrix}
\frac{b_1}{b_1^\dagger} \\
\frac{b_2}{b_2^\dagger}
\end{pmatrix} = \frac{1}{2} \begin{pmatrix}
 r & r & t_+ & t_- \\
 r & r & t_- & t_+ \\
 t_+ & t_- & r & r \\
 t_- & t_+ & r & r
\end{pmatrix} \begin{pmatrix}
a_1 \\
a_1^\dagger \\
a_2 \\
a_2^\dagger
\end{pmatrix},
\end{equation}
where $t_{\pm} = te^{i\varphi} \pm 1$. So the normal reflection coefficient $R = \frac{t_-}{t_+}$, LAR coefficient $T_{LAR} = \frac{t_-}{t_+}$, normal tunneling coefficient $T = |t_+|^2/4$ and the CAR coefficient $T_{CAR} = |t_-|^2/4$.

The results of Fig. 3 can be further illustrated as follow. For $\mu_S \sim |m|$, the Majorana edge state is localized...
oscillates and its localization length is considerably in-
creased for \( \mu_S \gg |m| \) [see Fig.2(b)]. Thus for a short 
TSC region, it is inevitable that \( \gamma_3 \) could arrive 
at the right terminal by tunneling itself to \( \gamma_3 \) and 
the tunneling amplitude \( t \) can almost be 1. In this case, 
the normal reflection and LAR are completely suppressed, 
and the normal tunneling or CAR coefficient is 1, 
which can be clearly seen in Fig.3. To be specific, in the Fig.1(b) 
we show the propagating route of the carriers. When an 
electron \( a_1 \) incoming from the left terminal spreads to the 
interface between QAHI and TSC, it will separate into 
two Majorana Fermions \( \gamma_1 \) and \( \gamma_2 \). Here \( \gamma_2 \) directly 
propagates to the right terminal, and \( \gamma_1 \) propagates along 
the interface between QAHI and TSC. If the quantum 
tunneling does not take place between the Majorana edge 
states \( \gamma_1 \) and \( \gamma_3 \), \( \gamma_1 \) will be reflected back to the left 
terminal once it arrives at the upper edge of the sample. 
In fact, for a short central TSC region, it is inevitable 
that \( \gamma_1 \) could arrive at the right terminal by tunneling 
it to \( \gamma_3 \). Therefore the outgoing states in the right 
terminal are \( \gamma_2 \) and \( e^{i\varphi}\gamma_1 \). Note that here \( \varphi \) can only be 0 
or \( \pi \) due to the Majorana Fermion being self-Hermitian 
particle. For \( \varphi = 0 \), normal tunneling coefficient \( T \) 
is 1 and the CAR coefficient \( T_{\text{CAR}} \) is 0. By contrast, for 
\( \varphi = \pi \), normal tunneling coefficient \( T \) is 0 and the CAR 
coefficient \( T_{\text{CAR}} \) is 1, i.e. the perfect CAR occurs and all 
other processes (T, LAR and R) are totally suppressed. 
Here we would like to emphasize that \( \varphi \) can only be 0 or 
\( \pi \) is a decisive factor for the appearance of the plateaus 
of \( T_{\text{CAR}} \) and \( T \).

In addition, from Eq.(11), we can see that the normal 
reflection and LAR coefficients are always equal 
(\( R = T_{\text{LAR}} = \frac{1-e^{i\varphi}}{4} \)) regardless of the system parameters 
and \( te^{i\varphi} \), which is also shown in the results of numerical 
calculations in Fig. 4. One can see that the normal 
reflection and LAR coefficients are exactly the same, 
although both \( R \) and \( T_{\text{LAR}} \) change complicatedly with the 
increase of \( \mu_S \). This is completely consistent with the 
Eq.(11) and clearly indicates why the physical picture for 
the quantum perfect CAR is reasonable. It is noteworthy 
that the TSC phase is \( N = 2 \) whenever the \( \mu_S \) satisfies 
\( m < -\sqrt{\Delta^2 + \mu_S^2} \). In this case, the normal 
tunneling coefficient \( T \) always is 1, so \( R \) and \( T_{\text{LAR}} \) remain 
still equal to 0.

Next we investigate how the perfect CAR is affected 
by the systemic parameters. Fig.3(a-c) show the normal 
tunneling coefficient \( T \), LAR coefficient \( T_{\text{LAR}} \) and CAR 
coefficient \( T_{\text{CAR}} \) for the different width \( N \). It can be clearly seen that the CAR plateau with \( T_{\text{CAR}} = 1 \) can 
well keep with the change of width except for the very 
narrow case. While the width is very narrow, there is 
the coupling between the upper and lower Majorana edge 
states, which can reduce \( T_{\text{CAR}} \). In addition, the perfect 
CAR can well survive with the change of the parameter 
\( m \) [see Fig.3(d)].

Fig.5 shows the normal tunneling coefficient \( T \), CAR 
coefficient \( T_{\text{CAR}} \) and LAR coefficient \( T_{\text{LAR}} \) versus the 
potential energy \( \mu_S \) for the different on-site energy \( \mu_L \) 
of the QAHI leads. The CAR plateaus with \( T_{\text{CAR}} = 1 \) 
can well hold at the small \( \mu_L \) as shown in Fig.5(a-b). For a large \( \mu_L \), the Fermi level is close the bulk 
states, which could trigger LAR and normal reflection 
and weaken normal tunneling and CAR. In this case, \( T \) 
and CAR show peaks with the values less than 1 [see 
Fig.5(c) with \( \mu_L = 0.4 \)]. Finally for a very large \( \mu_L \), the
Fermi level is in the bulk states of the QAHI, leading to an anomalous large LAR coefficient and the vanishing CAR as the general normal lead-superconductor system, see Fig.5(d)].

Furthermore, Fig.6 shows $T$, $T_{CAR}$ and $T_{LAR}$ for the different length $L$ of the central TSC region. One can see that the plateaus of $T$ and $T_{CAR}$ are broken with the increase of distance between the two terminals. Even in this case the larger $\mu_S$ is, the more robust the plateaus is. For a long length $L$, the tunneling probability $t$ between the Majorana edge states $\gamma_1$ and $\gamma_3$ is not equal to 1, thus both $T$ and $T_{CAR}$ are not also quantized. As the increasing $L$ reaches a limit value, $t$ tends towards zero and four coefficients tend towards $T = R = T_{LAR} = T_{CAR} = 1$.

In Fig. 7 we plot $T$, $T_{LAR}$ and $T_{CAR}$ as functions of the superconductor pairing potential $\Delta$ at the different potential energy $\mu_S$. For the small $\mu_S$, the normal tunneling shows a quantized plateau with value of integer 1 and the other three coefficients ($R$, $T_{LAR}$ and $T_{CAR}$) are zero [see Fig.7(a)]. The reason is that the central region is in the TSC phase of $\mathcal{N} = 2$ when the condition $m < -\sqrt{\Delta^2 + \mu_S^2}$ is satisfied. Whereas for $m^2 < \Delta^2 + \mu_S^2$, the central region is in the TSC phase of $\mathcal{N} = 1$. Now the perfect CAR with $T_{CAR} = 1$ can occur and all other scattering processes are totally suppressed [see Fig.7(b) and 7(d)]. Thus the perfect CAR can exist in a wide range of $\Delta$. And the larger $\mu_S$ is, the wider the plateau value of CAR coefficient is. In addition, the quantized normal tunneling can also occur in $m^2 < \Delta^2 + \mu_S^2$ [see Fig.7(c)], because of the alternate appearance of $T_{CAR} = 1$ and $T = 1$ with the change of $\mu_S$.

Next we consider the inevitable impurity scattering in the real samples. In Fig.8 we consider the Anderson disorder only existing in the central TSC region. One can see that the perfect CAR plateaus are robust against the disorder and the plateaus can be well kept while the disorder strength is $W \lesssim 2$. In fact, the perfect CAR originates from the Majorana edge states. As soon as the ribbon is wide enough, the coupling between the upper and lower edge states is suppressed, then the perfect CAR always occurs. On the other hand, when the disorder becomes stronger (eg. $W = 3.0$), the Majorana edge state as well TSC phase are destroyed because of the impurity scattering, then the quantized CAR plateau is destroyed and the CAR coefficient shows the peaks.

Up to now, we have demonstrated that a quantized perfect CAR occurs in the QAHI-$\mathcal{N} = 1$ TSC-QAHI system, in which two electrons from a Cooper pair at TSC are split and certainly go to two different leads. At last, we study the current in each terminal. By solving the transmission coefficients, the current $I_{L/R}$ from the left/right QAHI terminal can be obtained straightforwardly\cite{41,42}:

$$I_i = \frac{e}{h} \int \frac{dE}{2\pi} \left\{ (f_{i+} - f_S) T_{iS} + (f_{i+} - f_{i-}) T_{LAR} ight\} + \langle f_{i+} - f_{i-}\rangle T_{CAR} + \langle f_{i+} - f_{i+}\rangle T, \quad (12)$$

where $i = L/R$ corresponds the left/right terminal, $i = R$ for $i = L$ whereas $i = L$ for $i = R$, $f_{i \pm} (E) = 1/\{e^{(E + V_i)/k_B T} + 1\}$ and $f_S (E) = 1/\{e^{E/k_B T} + 1\}$ are Fermi distribution with the bias $V_i$ and temperature $T$. Here the bias of TSC terminal has been set to zero. While $|E| < \Delta$, the tunneling coefficient $T_{iS}$ from the QAHI terminal to TSC is zero. By setting the right QAHI terminal as a voltage probes with $V_R = 0$, we have $V_R = -V_L$ and $V_{R/L}$ tends infinite while the quantized perfect CAR occurs. On the other hand, we have $V_R = V_L$ when $T = 1$. Thus with the change of $\mu_S$, the $T_{CAR}$ and $T$ alternately are 1 (see Figs.3, 5 and 8), resulting in the alternately bias of the right terminal with value of $V_L$ and $-V_L$. \
FIG. 8: (Color online) $T$, $T_{\text{LAR}}$, and $T_{\text{CAR}}$ as a function of $\mu_S$, with the disorder strengths being $W = 0.5$ (a), 1.0 (b), 2.0 (c) and 3.0 (d). Other parameters are the same as those in Fig.3(b). Here the curves are averaged over 1000 random configurations.

IV. CONCLUSIONS

In summary, a quantized perfect crossed Andreev reflection is found in the QAHI-TSC-QAHI system when TSC phase of $N = 1$ is realized. The coefficient of the crossed Andreev reflection shows the plateaus with the value being integer 1, and all other scattering processes, the normal reflection, normal tunneling and local Andreev reflection, are completely suppressed. The quantized perfect crossed Andreev reflection originates from the chiral Majorana edge states and the tunneling between them, and it is robust against the disorder and can work in wide range of system parameters.

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