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Phys. Rev. B **95**, 245119 — Published 16 June 2017

DOI: [10.1103/PhysRevB.95.245119](https://doi.org/10.1103/PhysRevB.95.245119)

High-field magnetization and magnetic phase diagram of α -Cu₂V₂O₇

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High-field magnetization of the spin-1/2 antiferromagnet α -Cu₂V₂O₇ was measured in pulsed magnetic fields of up to 56 T in order to study its magnetic phase diagram. When the field was applied along the easy axis (the a -axis), two distinct transitions were observed at $H_{c1} = 6.5$ T and $H_{c2} = 18.0$ T. The former is a spin-flop transition typical for a collinear antiferromagnet and the latter is believed to be a spin-flip transition of canted moments. The canted moments, which are induced by the Dzyaloshinskii-Moriya interactions, anti-align for $H_{c1} < H < H_{c2}$ due to the anisotropic exchange interaction that favors the antiferromagnetic arrangement along the a -axis. Above H_{c2} , the Zeeman energy of the applied field overcomes the antiferromagnetic anisotropic interaction and the canted moments are aligned along the field direction. Density functional theory was employed to compute the exchange interactions, which were used as inputs for quantum Monte Carlo calculations and then further refined by fitting to the magnetic susceptibility data. Contrary to our previous report in Phys. Rev. B **92**, 024423, the dominant exchange interaction is between the third nearest-neighbor spins, which form zigzag spin-chains that are coupled with one another through an intertwining network of the nonnegligible nearest and second nearest-neighbor interactions. In addition, elastic neutron scattering under the applied magnetic fields of up to 10 T reveals the incommensurate helical spin structure in the spin-flop state.

PACS numbers: 71.20.-b, 75.30.Gw, 71.70.Gm, 75.50.Ee, 25.40.Dn

I. INTRODUCTION

A spin-flop transition in collinear antiferromagnetic systems can be observed when a magnetic field is applied parallel to the easy axis of the antiferromagnet. The strength of the applied magnetic field that forces the spins to flop depends on exchange interactions in the systems. The spin-flop transition, if present, causes the spins to reorient themselves perpendicular to the applied magnetic field in order to compromise the exchange-interaction energy with the Zeeman energy. This phenomenon was predicted eighty years ago¹ and has been observed in several compounds²⁻⁵. Generally, the spin-flop transition can be observed as a single transition with a sudden increase of magnetization M at a critical field H_c as well as the change of magnetic susceptibility defined by the slope of the $M - H$ curve below and above H_c . However, there are a few cases in which two successive magnetic phase transitions are observed, for example, in the quasi-one-dimensional BaCu₂Si₂O₇ system⁶⁻⁸, of which the underlying mechanism is still unresolved. In this article, we report on the two-stage spin reorientation in α -Cu₂V₂O₇ using high-field magnetization measurements on single crystal samples. Despite a single spin-flop transition being observed in its cousin phase

β -Cu₂V₂O₇⁹ or other antiferromagnetic systems, we instead found two successive jumps in the magnetization of α -Cu₂V₂O₇ similar to those observed in BaCu₂Si₂O₇.

α -Cu₂V₂O₇ crystallizes in the orthorhombic system ($Fdd2$) with $a = 20.645(2)$ Å, $b = 8.383(7)$ Å, and $c = 6.442(1)$ Å^{10,11}. Below $T_N = 33.4$ K, the system undergoes a paramagnetic to antiferromagnetic transition. In the ordered state, $S = 1/2$ Cu²⁺ spins align antiparallel along the crystallographic a -axis with their nearest-neighbors^{12,13}. The magnetization and powder neutron scattering studies suggest small spin canting along the c -axis^{12,13} as a result of the antisymmetric Dzyaloshinskii-Moriya (DM) interaction. The exchange interactions in α -Cu₂V₂O₇ are, to date, still open to debate. Our previous analysis using quantum Monte Carlo (QMC) simulation¹² showed two possible models with different values of the nearest-neighbor interaction J_1 and second nearest-neighbor interaction J_2 that can be equally used to describe the broad maximum observed in the magnetic susceptibility data. On the other hand, density functional theory (DFT) calculations by Sannigrahi *et al.*¹⁴ revealed the dominant third nearest-neighbor antiferromagnetic interaction J_3 (see Fig. 1 for the diagram). The latest study on a powder sample using inelastic neutron scattering also supports the leading J_3 model¹⁵. Both DFT

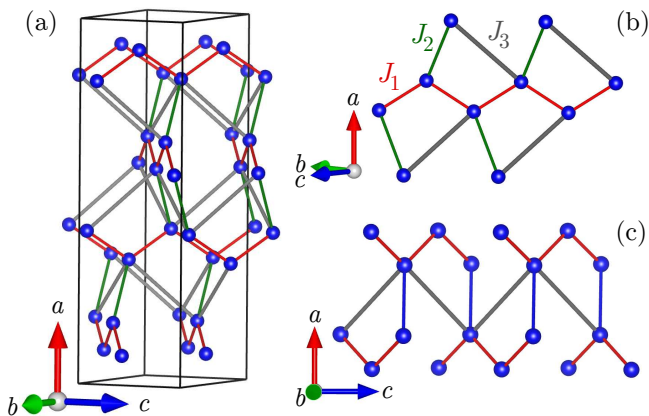


FIG. 1. (Color online) Diagrams showing the network of Cu^{2+} ions in $\alpha\text{-Cu}_2\text{V}_2\text{O}_7$. (a) The nearest-, second-nearest, and third-nearest neighbor interactions, J_1 , J_2 , and J_3 , are represented by red, green, and grey lines, respectively. (b) The nearest-neighbor interaction J_1 forms zigzag chains which run along the $[011]$ and $[01\bar{1}]$ directions. (c) The third nearest-neighbor interaction forms zigzag chain along the c -axis.

and powder inelastic neutron scattering studies qualitatively suggest that the antiferromagnetic third nearest neighbor interaction J_3 forming zigzag chains along the c -axis [Fig. 1(c)] via a complex Cu–O–V–O–Cu pathway (through the VO_4 tetrahedra) is non-negligible and possibly the strongest exchange interactions. In addition, the interconnection between electricity and magnetism in $\alpha\text{-Cu}_2\text{V}_2\text{O}_7$ has been studied to reveal its magnetoelectric properties^{13,14}, which might find useful applications. This variety of interesting phenomena and inconclusive understanding of the nature of the exchange interactions in $\alpha\text{-Cu}_2\text{V}_2\text{O}_7$ have led us to this more detailed investigation of the magnetic properties of the system.

This paper presents a study of the magnetic properties of single-crystal $\alpha\text{-Cu}_2\text{V}_2\text{O}_7$. The experimental details are described in Sec. II. In Sec. III A, we discuss the magnetization measurements at low field. In Sec. III B, the DFT calculation and QMC simulation are discussed and compared to the low-field magnetic susceptibility data. In Sec. III C, we investigate the magnetic phase transitions using high-field magnetization and present the magnetic phase diagram of this system. Elastic neutron scattering measurements under applied magnetic fields of up to 10 T are discussed in Sec. III D followed by the conclusion in Sec. IV

II. EXPERIMENT

The single crystals of $\alpha\text{-Cu}_2\text{V}_2\text{O}_7$ studied in this paper were grown by the vertical Bridgman technique. The detailed method of crystal growth and characterization are described elsewhere¹². The crystals with dimensions of about $4 \times 4 \times 4 \text{ mm}^3$ were aligned using a four-circle X-

ray diffractometer with $\text{MoK}\alpha$ radiation and cut perpendicular to the crystallographic b - and c -axes (the a -axis is the naturally cleaved facet). Magnetic properties at low fields (up to 7 T) were studied using a superconducting quantum interference device (MPMS-XL, Quantum Design) down to the base temperature of 1.8 K. Magnetization as a function of field and temperature was measured when the magnetic field was applied parallel to each of the crystallographic axes. To study the magnetic properties at high fields, the nondestructive pulsed magnet at the International MegaGauss Science Laboratory, Institute for Solid State Physics (ISSP), University of Tokyo was used to generate pulsed magnetic fields of up to 56 T. Magnetization was measured by induction using a coaxial pick-up coil. The single-crystal sample was aligned so that the applied field was either parallel or perpendicular to the a -axis, and cooled to the base temperature of 1.4 K using a liquid ^4He cryostat.

The DFT calculations were performed using the QUANTUM ESPRESSO simulation package¹⁶. All calculations were done within the generalized gradient approximation in the form of Perdew, Burke and Ernzerhof (PBE)¹⁷ for the exchange and correlation potentials with the Hubbard U correction (GGA+ U) in order to explicitly take into account the correlated effect of the 3d electrons of Cu^{2+} ions. We adopted the values of the on-site Coulomb and exchange interaction parameters $U = 7.0 \text{ eV}$ and $J = 0.5 \text{ eV}$ according to similar compounds^{18,19}. To cross-check the choice of the Coulomb parameters, we calculated the electronic structure e.g. a band gap for several values of U and evaluated the exchange coupling for $U = 6.0, 7.0, \text{ and } 8.0 \text{ eV}$. The effect of core electrons was modeled through the use of ultrasoft pseudopotentials with the planewave cutoff of 80 Ry. The Gaussian broadening technique was used and meshes of $2 \times 4 \times 4$ and $4 \times 6 \times 6$ k -points were sampled for the Brillouin-zone integrations. All calculations were done with the experimental crystal structure whose lattice parameters are $a = 20.6786 \text{ \AA}$, $b = 8.4052 \text{ \AA}$ and $c = 6.4462 \text{ \AA}$.¹² The internal lattice coordinates from the experimental measurements were also used in the calculations. The crystal structure of $\alpha\text{-Cu}_2\text{V}_2\text{O}_7$ belongs to the $Fdd2$ space-group thus yielding the 88-atom unit cell. To address the consistency of the structural data, we performed the structural relaxation; the discrepancy of the atomic coordinates is less than 0.2 \AA and the forces do not exceed 0.001 Ry/a.u. This small distortion in the atomic coordinates weakly affects the electronic structure and the exchange coupling. The obtained exchange parameters were then used to construct a spin network for the QMC simulation with LOOP algorithm²⁰ using the simulation package ALPS²¹ to calculate the magnetic susceptibility for comparison with the experimental data.

Finally the spin-flop state was investigated microscopically using elastic neutron scattering at the SPINS instrument, NIST Center for Neutron Research (NCNR), USA. The single crystal of mass 1.39 g was aligned so that the bc -plane was in the scattering plane. The fixed final

154 neutron energy of 5 meV was utilized with the horizontal
 155 collimations of open – 80' – sample – 80' – detector. The
 156 vertical magnetic field between 0 to 10 T was applied
 157 along the crystallographic a -axis to investigate the spin-
 158 flop transition and the magnetic structure of α -Cu₂V₂O₇
 159 in the spin-flop state.

160 III. RESULTS AND DISCUSSION

161 A. Low-field magnetization

162 In our previous work,¹² the magnetization as a function
 163 of magnetic field $M(H)$ on single-crystal α -Cu₂V₂O₇
 164 was measured with the applied magnetic fields of up to 7
 165 T along two orthogonal directions, i.e., $H \parallel a$ and $H \perp a$.
 166 The results showed magnetic anisotropy between the a -
 167 axis and bc -plane. Weak ferromagnetism, which suggests
 168 canted moments as a result of the DM interaction, was
 169 observed in the ordered state for $H \perp a$. A later study
 170 on this system by Lee *et al.*¹³ revealed, from the mag-
 171 netization measurements along all three crystallographic
 172 axes, that the spins are only canted along the c -axis and
 173 the canting angle varies from 2° to 7° depending on the
 174 applied magnetic field from 0 to 9 T. As a result, the rel-
 175 evant DM vectors between the pairs of nearest-neighbors
 176 ($\mathbf{D}_{ij} \cdot \mathbf{S}_i \times \mathbf{S}_j$) can only point along the b -axis given the
 177 collinear spin structure along the a -axis. In this work, we
 178 performed a detailed investigation of the magnetization
 179 as a function of field as well as magnetic susceptibility as
 180 a function of temperature on the aligned single crystals
 181 when the magnetic field was applied along all three crys-
 182 tallographic axes. The samples studied in this work are
 183 from the same batch as those reported in our previous
 184 study¹².

185 Figure 2 shows the magnetization as a function of field
 186 between –1 T and 1 T for the applied field along each
 187 of the crystallographic axes at 1.8 K. These results con-
 188 firm that the weak ferromagnetism exists only for the
 189 field along the c -axis, where the spontaneous magnetiza-
 190 tion is clearly observed, in agreement with the work by
 191 Lee *et al.*¹³. The remnant magnetization as the field ap-
 192 proaches zero $M(0)$ is determined from the linear fit for
 193 $H > 0.1$ T. The interpolation gives $M(0) = 0.082(1)\mu_B$,
 194 from which the canting angle η can be calculated using
 195 $\eta = \sin^{-1} \frac{M(0)}{g\mu_B S}$ yielding $\eta = 4.7(1)^\circ$. Note that the value
 196 of $M(0)$ in our previous report¹² was not precisely deter-
 197 mined since the magnetic field was applied perpendicular
 198 to the a -axis but not precisely along the c -axis. The dif-
 199 ferent values of $M(0)$ suggest that the applied field in
 200 Ref. 12 was $\sim 30^\circ$ away from the c -axis.

201 The magnetization along the a - and b -axis, on the other
 202 hand, show a linear relation through zero field implying
 203 that the spin component along those axes are antiparallel
 204 resulting in zero net spontaneous magnetization, which
 205 is also consistent with the magnetic structure reported
 206 earlier^{12,13}. Since the canting is along the c -axis and the
 207 spins anti-align along the a -axis, the relevant component

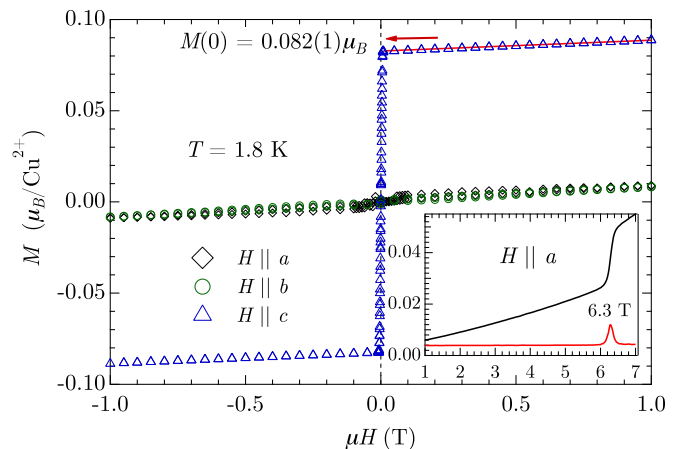


FIG. 2. (Color online) Magnetization as a function of field at 1.8 K near the zero field which is applied along the a -axis (black diamonds), b -axis (green circles) and c -axis (blue triangles). The solid line is a linear fit to the magnetization at $H > 0.1$ T and interpolated to $H \rightarrow 0$. Inset: the magnetization along the a -axis up to the field of 7 T shows the magnetic phase transition at 6.3 T indicated by a peak in dM/dH and denoted by the red line.

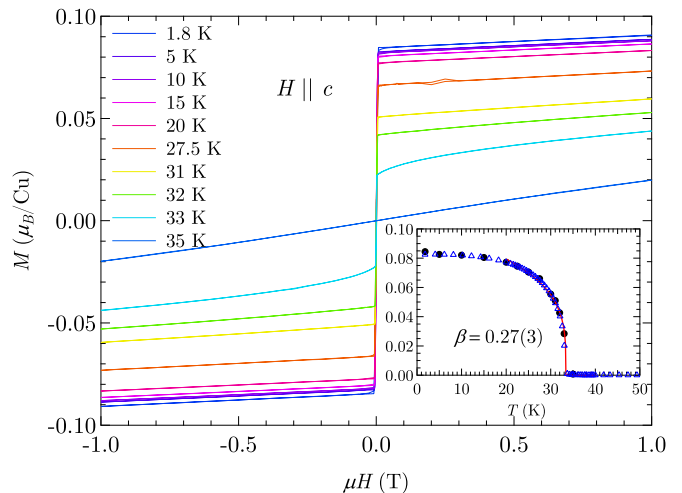


FIG. 3. (Color online) Magnetization as a function of applied field with $H \parallel c$ at different temperatures from 1.8 K to 35 K (only selected temperatures are shown). Inset shows the power-law fit to the magnetization at zero field $M(0)$ (black circles). Error bars are smaller than the plot symbol. The blue triangle in the inset is the magnetization as a function of temperature when the field of $H = 100$ Oe is applied along the c -axis.

208 of the DM vector, which in our previous work was pro-
 209 posed to lie within the bc -plane, must be solely along
 210 the b -axis. Interestingly, when the field is applied along
 211 the a -axis, a magnetic phase transition appears at 6.3
 212 T as shown in the inset of Fig. 2. This magnetic phase
 213 transition, which is not observed when $H \parallel b$, is due to

214 the spin-flop transition and will be discussed in detail in
215 Section III C.

216 A series of $M(H)$ measurements at different tempera-
217 tures (Fig. 3) shows that the remnant magnetization
218 and hence the value of $M(0)$ decreases as tempera-
219 ture increases; $M(0)$ goes to zero at T_N (the inset of
220 Fig. 3). A fit of the measured temperature dependence
221 of $M(0)$ to the power-law $M(0, T) \propto (1 - T/T_N)^\beta$ for
222 $20 \text{ K} < T < 33.4 \text{ K}$ yields $\beta = 0.27(3)$. This value of
223 the critical exponent is quite close to that obtained from
224 the order parameter measurement of the magnetic Bragg
225 intensity using neutron scattering [$\beta = 0.21(1)$]¹². The
226 inset also shows the field-cooled magnetization, measured
227 at the low-field of 100 Oe along the c -axis, as a function
228 of temperature which, as expected, perfectly follows the
229 temperature dependence of $M(0)$.

230 The magnetic susceptibility measured at the applied
231 field of 1 T along the a - and c -axis are shown as a function
232 of temperature in Fig. 4. The data for $H \parallel b$ (Fig. 8)
233 will be discussed in Section III B. When the field is ap-
234 plied along the a -axis, there is a sharp Néel transition
235 at $T_N \approx 35 \text{ K}$ which, as shown in the inset of Fig. 4(a),
236 slightly decreases toward lower temperature when the ap-
237 plied field is increased (see Fig. 11 for the $H(T)$ phase
238 diagram). For $H \parallel c$, there is a spontaneous magnetiza-
239 tion below T_N due to the spin canting as described above.
240 The value of the remnant magnetization as $T \rightarrow 0$ along
241 the c -axis is much higher than that along the other two
242 axes. Above 50 K the magnetic susceptibility shows a
243 clear and smooth curve following the Curie-Weiss law up
244 to 300 K. It should be noted that the previously observed
245 broad peak in the magnetic susceptibility data for $H \perp a$
246 around $T = 50 \text{ K}$ can now be observed only in the $H \parallel b$
247 data [Fig. 8(a)]. This broad peak will be analyzed and
248 fitted in the next section.

249 B. Density functional theory calculation & 250 Quantum Monte Carlo simulation

251 In order to derive the exchange interactions between
252 the Cu–Cu couplings, we performed total energy calcula-
253 tions for 120 different magnetic structures including the
254 ferromagnetic, antiferromagnetic and other spin configura-
255 tions. The calculations show that structures with ferro-
256 magnetic and random spin structures are more energetic
257 than the antiferromagnetic structure. The energy of the
258 antiferromagnetic ordered state is about 3.3 meV per formu-
259 la unit cell lower than the others. Therefore, it is in
260 agreement with the known ground state of $\alpha\text{-Cu}_2\text{V}_2\text{O}_7$.
261 The total and atomic-resolved density of states (DOS)
262 of the ground state of $\alpha\text{-Cu}_2\text{V}_2\text{O}_7$ is shown in Fig. 5.
263 The Fermi level is at zero energy. The DOS of spin-up
264 and spin-down electrons are symmetric as expected for
265 an antiferromagnetic state. The band gap is estimated
266 to be about 1.8 eV, thus rendering the system an insu-
267 lator. The bottom of the conduction band comprises
268 the Cu 3d, V 3d and O 2p electrons, whereas the top of

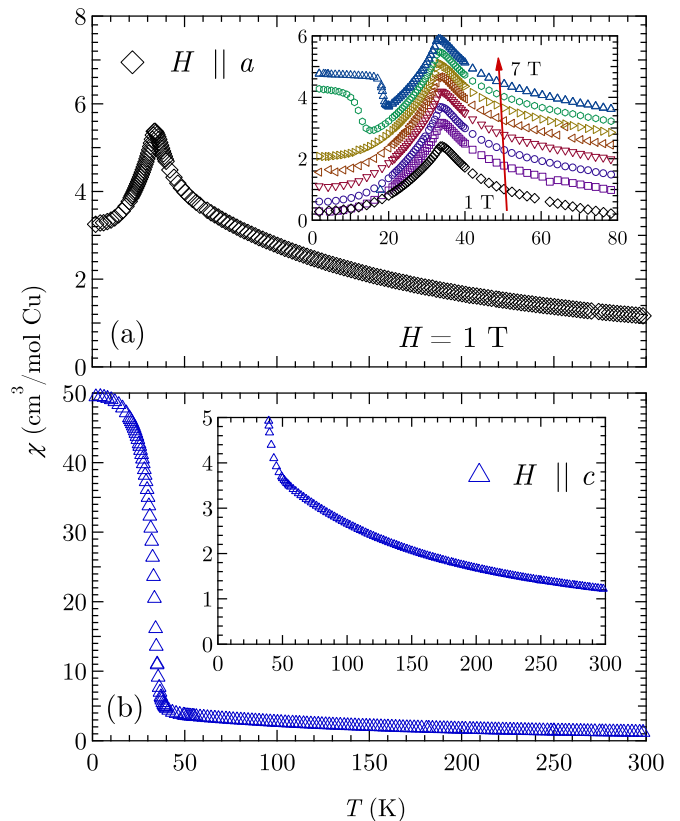


FIG. 4. (Color online) Temperature dependence of the mag-
netic susceptibility when a field of 1 T is applied (a) along
the a -axis and (b) c -axis. Inset in (a) shows the Néel transi-
tion at different applied field from 1 T (black diamonds) to
7 T (blue triangles) with y -offset. The inset in (b) shows a
clear and smooth decrease in the magnetic susceptibility as
the temperature increases following the Curie-Weiss law.

269 the valence band is primarily composed of the O 2p elec-
270 trons with some contributions from Cu 3d and V 3d. It
271 is evident that the O 2p orbitals hybridizes strongly with
272 the Cu 3d and V 3d orbitals in the valence band region.
273 To elucidate the electronic nature and chemical bonding
274 of the system, we plotted the orbital-resolved density of
275 states of the Cu 3d orbitals as depicted in Fig. 6. The
276 magnetic Cu^{2+} ions in $\alpha\text{-Cu}_2\text{V}_2\text{O}_7$ have been regarded
277 as having a distorted octahedral environment as a result
278 of the Jahn-Teller effect^{22,23}. The d^9 electronic confi-
279 guration of Cu^{2+} implies the splitting of the crystal field
280 into the $(t_{2g})^6$ and $(e_g)^3$ orbitals, which consist of the
281 xy , xz and yz orbitals and the $x^2 - y^2$ and $3z^2 - r^2$
282 orbitals, respectively. This implies that the lower lying t_{2g}
283 orbitals are fully filled, while the e_g orbital is partially
284 filled. Hence, the e_g orbitals would play a crucial role for
285 the hybridization with O 2p as evidenced by Fig. 6. Here
286 most of the states in the vicinity of the Fermi energy be-
287 long to the e_g contribution, i.e., $3z^2 - r^2$ and $x^2 - y^2$
288 with an especially large contribution from the $3z^2 - r^2$
289 orbitals near the Fermi energy indicating that these orbitals are

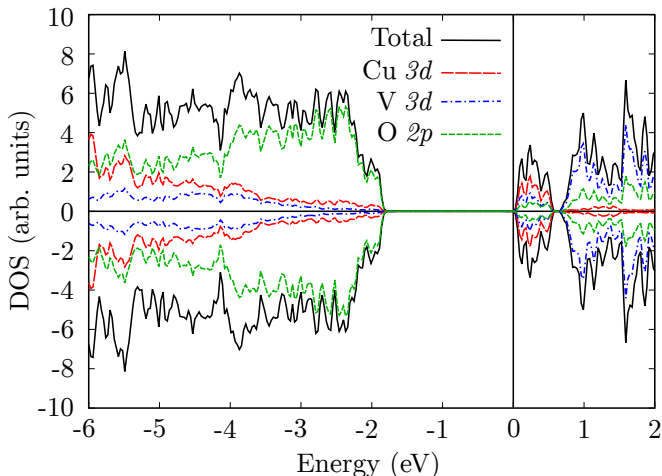


FIG. 5. (Color online) Total and atomic-resolved density of states per formula unit of the α - $\text{Cu}_2\text{V}_2\text{O}_7$ in the collinear antiferromagnetic state. The positive and negative DOS refer to the spin up and spin down contributions, respectively. The Fermi energy is set to zero.

290 magnetically active. In contrast, the states of the t_{2g} or-
 291 bitals, i.e., xy , xz , and yz lie in the lower energy range
 292 of -7.5 eV to -4 eV.

293 We evaluated the exchange interaction through the
 294 isotropic Heisenberg model of spin interactions whose
 295 Hamiltonian is expressed as

$$\mathcal{H} = \sum_{ij} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j, \quad (1)$$

296 where J_{ij} denotes the coupling interaction between spins
 297 at the lattice sites i and j . We note that due to the
 298 complexity of the spin structures required to refine the
 299 anisotropic terms, the anisotropic interactions are ignored.
 300 However, this spin Hamiltonian should be sufficient to
 301 capture high-temperature susceptibility above the ordering
 302 temperature. For the complete description of the system,
 303 we will employ the spin Hamiltonian described in Eq. 2,
 304 which will be discussed later. To quantitatively extract
 305 the coupling constant, magnetic unit cells with different
 306 spin configurations are considered. Since the crystal
 307 structure of α - $\text{Cu}_2\text{V}_2\text{O}_7$ is known to have space group
 308 $Fdd2$, lower dimensional structures can be easily utilized
 309 to define the three dominant magnetic coupling constants,
 310 one intrachain interaction and two interchain interactions.
 311 In the bc -plane, Cu^{2+} cations form zigzag chains
 312 connected by two inequivalent O^{2+} ions. The coupling
 313 J_1 corresponds to the first nearest-neighbor Cu - Cu
 314 with the shortest intrachain bond of 3.138 Å. Another
 315 lower-dimensional structure linking all the 1-D chains
 316 in the crystal to form a network of the intertwining
 317 spin-chains defines the other two coupling constants,
 318 J_2 and J_3 . The coupling J_2 emerges from the
 319 two Cu^{2+} ions of different chains via the shorter
 320 3.982 Å bonds while J_3 relates to the longer bond
 of 5.264 Å

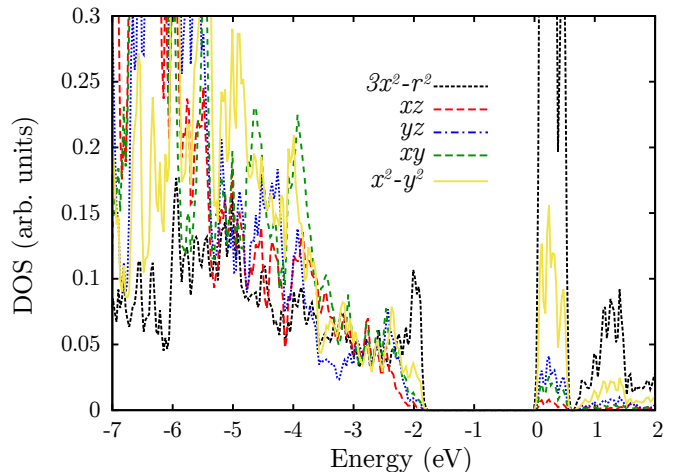


FIG. 6. (Color online) Projected density of states (spin only) of the five Cu 3d orbitals. The Fermi level is set to zero.

321 as depicted in Fig. 1. For each magnetic spin configu-
 322 ration, the pair energy of the parallel and antiparallel
 323 alignments corresponding to each of the coupling con-
 324 stants (E_{FM,J_i} and E_{AFM,J_i}) and the total energies
 325 are mapped to the Heisenberg model Hamiltonian. The
 326 coupling constants are then determined by least-square
 327 fitting. The calculated values of the exchange interactions
 328 are $J_1 = 3.02$ meV, $J_2 = 3.40$ meV and $J_3 = 6.12$ meV.

329 Figure 7 shows the isosurface of the valence electron
 330 density of α - $\text{Cu}_2\text{V}_2\text{O}_7$ for two different planes depict-
 331 ing the intrachain and interchain coupling between the
 332 magnetic Cu^{2+} ions. Here the intrachain Cu - Cu cou-
 333 pling can be observed through the charge density on the
 334 bc -plane as shown in Fig. 7(a). Strong covalency
 335 between Cu 3d and O 2p atomic orbitals is observed,
 336 underlying the J_1 coupling. In contrast, Figure 7(b)
 337 depicts two superexchange pathways corresponding to
 338 the two interchain interactions. The second nearest-
 339 neighbor interaction J_2 is attributed to the Cu - O - Cu
 340 pathway while the third nearest-neighbor interaction
 341 J_3 connects the two Cu atoms via the Cu - O - V - O - Cu
 342 pathway. It is clear that the charge distribution cross-
 343 ing the Cu - O - Cu pathway is finite but a more pro-
 344 nounced distribution can be observed along the Cu - O - V - O - Cu
 345 pathway. This result indicates that the strong exchange
 346 coupling J_3 is induced by the superexchange bridge
 347 by the V d^5 orbitals. This is reasonable since the
 348 Cu - O distances in the Cu - O - Cu pathway differ
 349 substantially (1.94 Å and 3.03 Å) while the Cu - O
 350 and V - O distances in the J_3 coupling are compar-
 351 able (ranging from 1.65 - 1.75 Å). These distances
 352 are short enough to accommodate the hybridization
 between the cation 3d and O 2p states.

353 The obtained values of the exchange interactions from
 354 the first-principles calculations were used to construct
 355 a spin network for the QMC simulation in order to
 356 describe the broad maximum and fit the measured mag-
 netic sus-

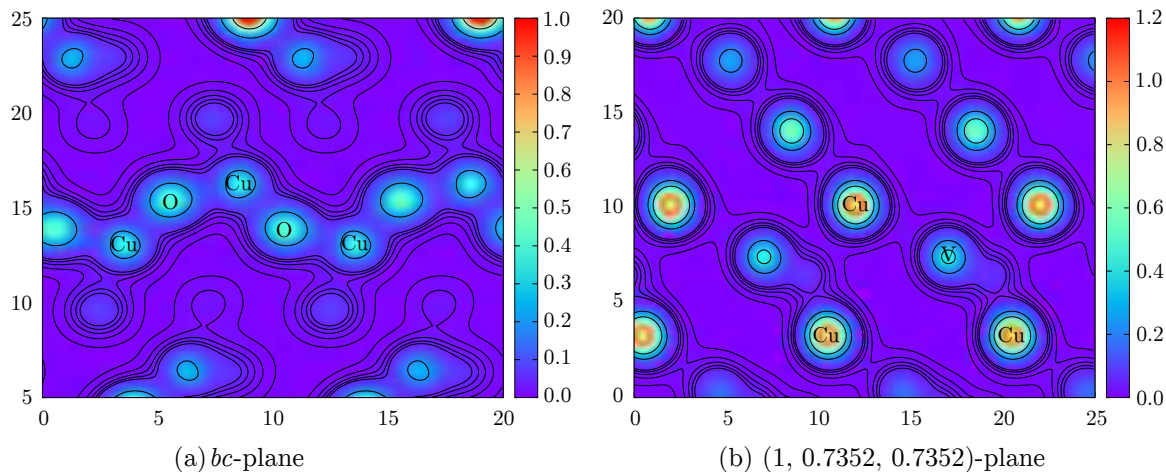


FIG. 7. (Color online) Isosurface of electron density at (a) the bc -plane indicating the Cu zigzag chain and (b) the $(1, 0.7352, 0.7352)$ plane facilitating the J_2 and J_3 superexchange pathways.

357 ceptibility for $H \parallel b$ [Fig. 8(a)]. For comparison, we used
 358 two different models; one is the $2J$ model in which we
 359 only consider the first and second nearest-neighbor in-
 360 teractions J_1 and J_2 , respectively, and the other is the
 361 $3J$ model that includes the third nearest-neighbor inter-
 362 action J_3 in the spin network (Fig. 1). The values of
 363 the exchange parameters for the $2J$ model were kept the
 364 same as those in our previous work¹², where the $J_1 : J_2$
 365 ratios of $1 : 0.45$ and $0.65 : 1$ were found to give the best
 366 fit to the experimental data for $H \perp a$. We note that the
 367 previous data is imprecise since the applied field was not
 368 perfectly aligned along the b -axis. However, it is clear
 369 from our new data shown in Figs. 4 and 8 that the broad
 370 peak at around 50 K only occurs when the magnetic field
 371 is applied along the b -axis. This broad peak is a result of
 372 short-range correlations and is related to the magnitude
 373 of the exchange couplings. To obtain a more accurate
 374 determination of the exchange interactions J_i , the mag-
 375 netic susceptibility calculated from the QMC simulations
 376 were refitted to the $H \parallel b$ data. The details of the QMC
 377 simulation and fitting are described elsewhere^{12,24}.

378 To re-examine our previous work, we first refitted the
 379 $2J$ model with the same $J_1 : J_2$ ratios of $1 : 0.45$ and
 380 $0.65 : 1$, the results of which are represented in Fig. 8(a)
 381 by the green and blue lines, respectively. The discrep-
 382 ancy between the experiment and calculations especially
 383 around the broad peak shown in the residue plot of
 384 Fig. 8(b) suggests that the $2J$ model falls short of cap-
 385 turing the accurate spin correlations. In the inset of
 386 Fig. 8(a), the maximum position of the broad peak is
 387 higher than those obtained from the calculations using
 388 the $2J$ model, which implies that the actual average value
 389 of the J_i must be higher than our previous estimation.
 390 We then compare the data to the QMC simulation with
 391 the $3J$ model by using the values of J_i obtained directly
 392 from the DFT calculations to construct the spin network.
 393 However, as shown by the red dashed line in Fig. 8(a),

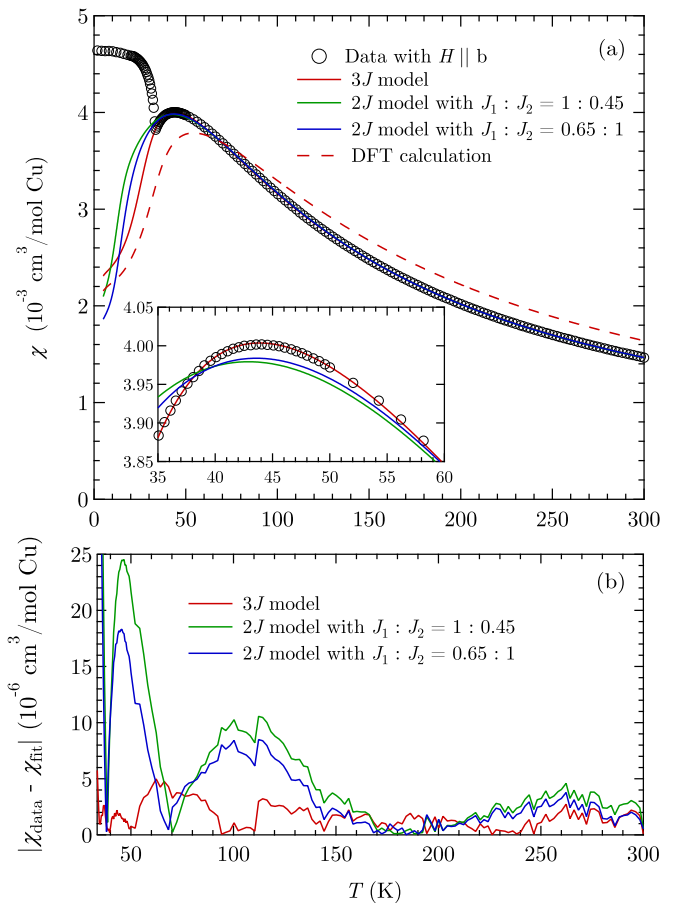


FIG. 8. (Color online) Magnetic susceptibility as a function of temperature with $H \parallel b$. (a) The broad peak at around 50 K is compared to the QMC simulations with $2J$ (green and blue line) and $3J$ (red solid line) models. The red dashed line is a direct result from the DFT calculation. (b) The discrepancy between the calculation and data for $2J$ and $3J$ models.

394 the results do not fit the experimental data very well.
 395 The discrepancy is most likely due to extra terms in the
 396 spin Hamiltonian, representing the anisotropic exchange
 397 and antisymmetric DM interactions^{25,26}. A more precise
 398 spin Hamiltonian which includes all relevant interactions
 399 as well as the Zeeman energy can be described by

$$\mathcal{H} = \sum_{i,j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j + \sum_{k,l} G_{kl} (S_k^x S_l^x - S_k^y S_l^y - S_k^z S_l^z) + \sum_{k,l} \mathbf{D}_{kl} \cdot (\mathbf{S}_k \times \mathbf{S}_l) - g_e \mu_B \sum_i \mathbf{S}_i \cdot \mathbf{B}, \quad (2)$$

400 where the summation $\sum_{i,j} (\sum_{k,l})$ is taken over the near-
 401 est, second-nearest, and third-nearest neighbours (near-
 402 est neighbours). The nearest-neighbor anisotropic ex-
 403 change interaction G_{kl} is denoted by G_1 and the DM
 404 vector $\mathbf{D}_{kl} = (D_{1a}, D_{1b}, 0)$, where D_{1a} and D_{1b} represent
 405 the a - and b -component, respectively. The c -component
 406 of the DM vector, even if present, cannot be determined
 407 by the magnetization or spin-wave data^{12,26}. As previ-
 408 ously mentioned, to first approximation these extra terms
 409 beyond the dominating isotropic exchange interactions
 410 are not fitted to the result of the DFT total energy cal-
 411 culations nor included in the QMC calculations, which
 412 are used to fit the susceptibility above T_N , due to the
 413 extremely complex degrees of freedom. The obtained ex-
 414 change parameters are therefore slightly overestimated
 415 when compared with the values obtained from the spin-
 416 wave data as shown in Table I. The anisotropic exchange
 417 interaction G_1 results in the collinear spin structure along
 418 the a -axis while the a -component of the DM vector D_{1a}
 419 favors the helical spin structure in the bc -plane. The
 420 competitive nature of these incompatible interactions
 421 gives rise to the nonreciprocal magnons and the low-field
 422 collinear structure but high-field helical structure²⁶. On
 423 the other hand, the b -component of the DM vector D_{1b}
 424 gives rise to the canted moments observed at low-field
 425 for $H \parallel c$ ¹² and at high-field for $H \parallel a$, which will be
 426 discussed below.

427 In order to obtain a better estimate of the exchange in-
 428 teractions J_i based on the $3J$ model, we slightly adjusted
 429 the values of exchange interactions obtained from the
 430 DFT calculations by converting them into a fraction with
 431 respect to J_1 ; this model is called the modified $3J$ model.
 432 As a result, the $J_1 : J_2 : J_3$ ratio is fixed at $1 : 1.12 : 2.03$.
 433 The spin network corresponding to the three values of the
 434 exchange parameters were then used for the QMC simu-
 435 lation, and the calculated magnetic susceptibility was
 436 again fitted to the experimental data [red solid line in
 437 Fig. 8(a)] yielding $J_1 = 2.45(1)$ meV, which differs by
 438 about 20% from the unnormalized DFT value. The fitted
 439 value of the Landé g -factor is $2.35(1)$, which is suffi-
 440 ciently close to the value of $2.44(3)$ obtained from the
 441 Curie-Weiss fit at high temperature ($T > 100$ K). The
 442 modified $3J$ model fits the experimental data much bet-
 443 ter than the $2J$ model especially around the broad peak
 444 as shown in the inset of Fig. 8(a) and in the residue plot
 445 in Fig. 8(b). The obtained fitted parameters are sum-
 446 marized in Table I. In contrast to our previous report¹²,

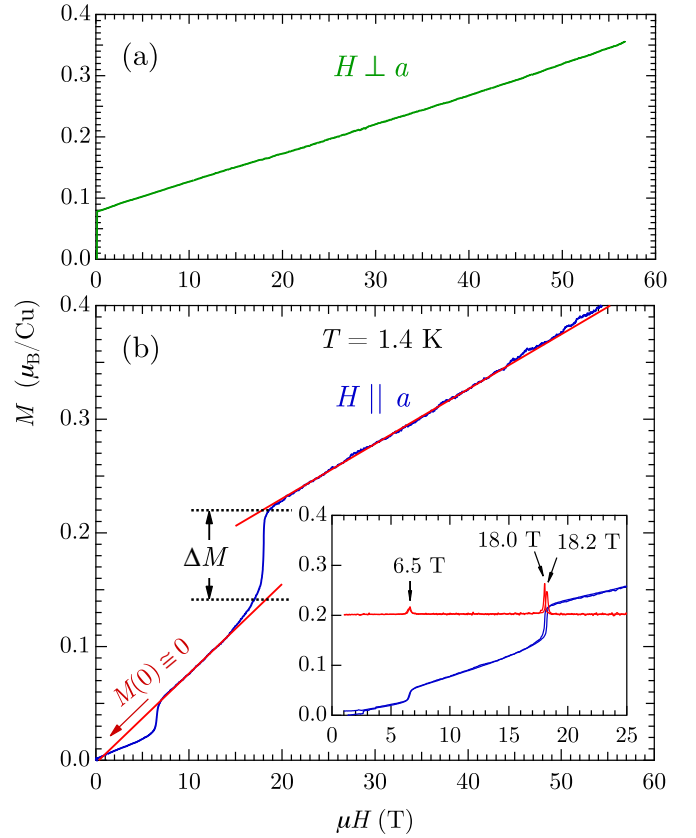


FIG. 9. (Color online) Magnetization as a function of mag-
 netic field when the field is applied parallel and perpen-
 dicular to the crystallographic a -axis at 1.4 K. (a) The magne-
 tization when the field is applied perpendicular to the crystal-
 lographic a -axis. The main panel in (b) shows all the data
 up to 56 T for $H \parallel a$. The red lines are the linear fit to
 the data at $8 \text{ T} < H < 14 \text{ T}$ yielding $M(0) \rightarrow 0$ T, and
 at $H > 20 \text{ T}$ for the calculation of ΔM as described in
 the text. The inset shows the transition field at $H_{c1} = 6.5 \text{ T}$
 and $H_{c2} = 18.2 \text{ T}$ (18.0 T) upon the increasing (decreasing)
 field defined by dM/dH in the red curve. A small amount of
 hysteresis can be observed at H_{c2} .

447 our new analysis on the broad peak at 50 K of the $H \parallel b$
 448 data indicates that the third nearest-neighbour J_3 is in
 449 fact the strongest interaction, which is consistent with
 450 the previous work^{14,15}. Using the combined DFT and
 451 QMC calculations, we were able to determine the magni-
 452 tudes of the exchange interactions more accurately than
 453 before. Furthermore, it should be noted that our DFT
 454 calculations indicate that J_1 , J_2 , and J_3 are all antifer-
 455 romagnetic, which is in disagreement with the work by
 456 Sannigrahi *et al.* where J_2 is ferromagnetic¹⁴.

C. High-field magnetization

457
 458 The high-field magnetization of single-crystal α -
 459 $\text{Cu}_2\text{V}_2\text{O}_7$ was measured in the pulsed magnetic field ap-

TABLE I. Parameters obtained from the fit of magnetic susceptibility with $H \parallel b$ using different lattice models, which are compared with the values obtained from fitting the spin-waves data²⁶.

	Modified 3J model	2J model (refitted) ¹²	Spin-waves ²⁶
J_1 (meV)	2.45(1)	5.79(1)	4.10(1)
J_2 (meV)	2.77	2.61	6.31
J_3 (meV)	4.97	–	–
G_1 (meV)	–	–	–
D_{1a} (meV)	–	–	–
D_{1b} (meV)	0.41(1)	0.814(1)	0.576(1)
g -factor	2.35(1)	2.24(1)	2.25(1)

460 plied along two orthogonal directions, i.e., $H \parallel a$ and
 461 $H \perp a$. The results at 1.4 K are shown in Fig. 9. When
 462 the field is applied perpendicular to the a -axis [Fig. 9(a)],
 463 the magnetization abruptly increases to about $0.08\mu_B$
 464 near zero field, which is consistent with that observed
 465 from the MPMS measurement with $H \parallel c$. From the
 466 value of $M(0)$, it can be inferred that the c -axis of the
 467 crystal was closely aligned parallel to the applied field.
 468 The magnetization was found to linearly increase with
 469 the field up to 56 T without saturation or further appear-
 470 ance of a phase transition. On the other hand, when the
 471 field was applied along the a -axis [Fig. 9(b)], we observed
 472 two magnetic phase transitions, indicated by the peaks
 473 in dM/dH , the first transition at $H_{c1} = 6.5$ T, which
 474 was already observed in the MPMS measurement (inset
 475 of Fig. 2), and the second at $H_{c2} = 18.2$ T (18.0 T) upon
 476 increasing (decreasing) field. In the ordered state, as pre-
 477 viously stated, the $S = 1/2$ Cu^{2+} spins align antiparallel
 478 with their nearest and next-nearest neighbors, and the
 479 majority of the spin component is along the crystallographic
 480 a -axis with small field-induced canting along the
 481 c -axis. When the applied magnetic field along the a -axis
 482 is between H_{c1} and H_{c2} ($6.5 \text{ T} < H < 18 \text{ T}$), the compe-
 483 tition between the exchange energy and Zeeman energy
 484 forces the spins to minimize the total energy by flopping
 485 altogether into the bc -plane making the spin direction
 486 perpendicular to the applied magnetic field. Due to the
 487 presence of the a -component of the DM vector, the in-
 488 plane spin components form a helical structure with the
 489 helical axis along the a -axis. This helical structure is con-
 490 firmed by the neutron scattering data, which will be dis-
 491 cussed in Part III D. The remnant magnetization at zero
 492 field $M(0)$ in the spin-flop state also approaches zero as
 493 shown by the linear fit in Fig. 9(b). In addition, as shown
 494 in the inset of Fig. 4(a), the magnetic susceptibility shows
 495 only a small upturn through the spin-flop transition be-
 496 low $T \simeq 20$ K where the magnetic susceptibility stays
 497 constant at about $0.004\mu_B$ as the temperature decreases
 498 toward 1.8 K. The small value of the remnant magne-
 499 tization at the base temperature suggests that after the
 500 transition into the spin-flop state, the small canted mo-
 501 ments along the a -axis resulting from the b -component
 502 of the DM interaction remain anti-aligned as depicted in
 503 the spin diagram in Region II of Fig. 11, which is consis-

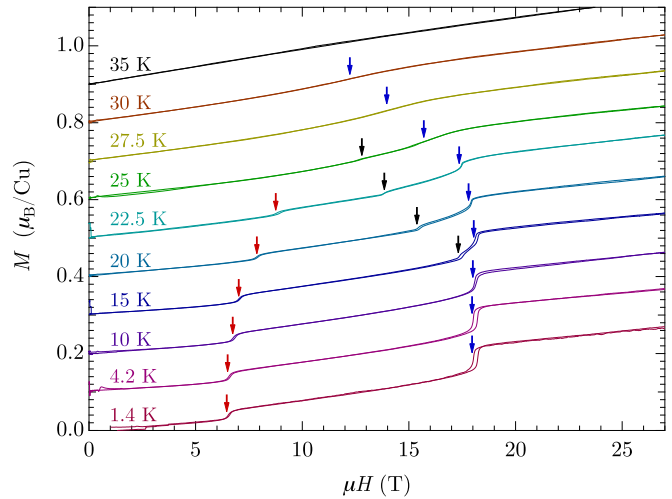


FIG. 10. (Color online) Magnetization at different temperatures from 1.4 K to 35 K. The stack is due to the offset for visualization. The transition field $H_{c1}(T)$ (red arrows) and $H_{c2}(T)$ (blue arrows) denotes the spin-flop and spin-flip transitions, respectively. The third transition denoted by $H_{c3}(T)$ (black arrows) appears between $T = 15$ K and 25 K.

504 tent with the antiferromagnetic anisotropic exchange in-
 505 teraction in the a -component²⁶. The spin-flop transition
 506 was in fact also observed in its cousin phase $\beta\text{-Cu}_2\text{V}_2\text{O}_7$
 507 where the easy axis is along the c -axis⁹. However, the
 508 magnetization data up to 5 T only showed a single spin-
 509 flop transition for $H \parallel c$ at around 1.5 T in contrast to
 510 the two transitions in the a -phase.

511 When the applied magnetic field reaches 18 T, we ob-
 512 served a second magnetic phase transition with a small
 513 hysteresis [inset of Fig. 9(b)]. This second phase transi-
 514 tion at $H_{c2} = 18$ T is a result of the Zeeman energy that
 515 overcomes the antiferromagnetic anisotropic exchange in-
 516 teractions making the a -axis component of the canted
 517 moments that previously anti-align below H_{c2} align along
 518 the applied field giving rise to a non-zero $M(0)$. The
 519 change of magnetization ΔM at the antiferromagnetic-
 520 to-ferromagnetic transition at H_{c2} is considerably larger
 521 than that at the spin-flop transition at H_{c1} . In order
 522 to estimate the canting angle along the a -axis in the

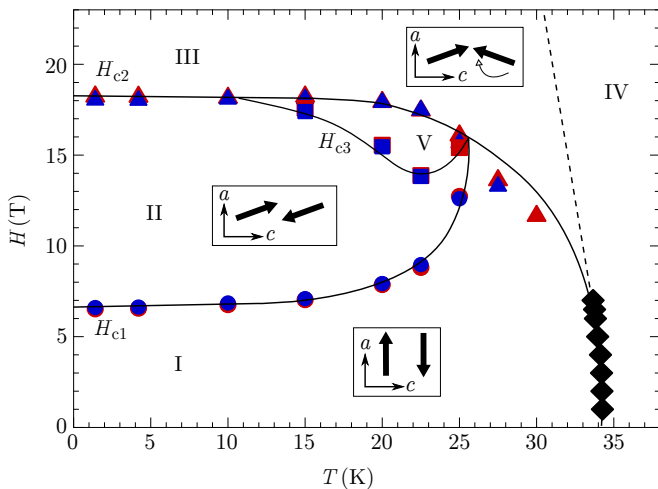


FIG. 11. (Color online) Magnetic phase diagram of α - $\text{Cu}_2\text{V}_2\text{O}_7$. Solid and dashed lines serve as guides to the eye. The solid lines at $H_{c1}(T)$ and $H_{c2}(T)$ represent the spin-flop and spin-flip transition, respectively, whereas $H_{c3}(T)$ represents the intermediate spin reorientation which occurs between $T = 15$ K and 25 K. Red (blue) symbols indicate the magnetic phase transition upon increasing (decreasing) field. The black diamond is the H_{c2} obtained from the Lorentzian fit to the peak at the transition temperature of the data in the inset of Fig. 4(a). The dashed line represents the cross-over between Region III and Region IV.

523 $H > 18$ T regime, a linear fit to the magnetization was
 524 performed to acquire the value of ΔM at H_{c2} , i.e., the
 525 change of magnetization where the second phase transi-
 526 tion occurs relative to the value in the spin-flop state
 527 as depicted in Fig. 9(b). The obtained high-field ΔM
 528 along the a -axis at 1.4 K is $0.081(1)\mu_B$, which is con-
 529 sistent with the value of $0.082(1)\mu_B$ obtained from the
 530 $H \parallel c$ data implying the same order of spin canting and
 531 a similar underlying mechanism. The value of $\Delta M =$
 532 $0.081(1)\mu_B$ yields a canting angle of $4.65(6)^\circ$ along the
 533 a -axis.

534 To further explore the magnetic phase transition for
 535 $H \parallel a$, the magnetization was measured at higher tem-
 536 peratures up to 35 K, i.e., above T_N . A series of data
 537 points collected from 1.4 K to 35 K is shown in Fig. 10.
 538 The phase transition denoted by H_{c1} and H_{c2} for the first
 539 and second jumps in magnetization are indicated by the
 540 red and blue arrows, respectively. The position of H_{c1}
 541 (H_{c2}) was found to increase (decrease) as the tempera-
 542 ture increases toward T_N . The resulting critical fields as
 543 a function of temperature $H_c(T)$ are presented as a mag-
 544 netic phase diagram in Fig. 11. In addition, we observed
 545 the unexpected third anomaly at H_{c3} as indicated by the
 546 black arrows in Fig. 10, which starts to appear at $T =$
 547 15 K and seems to merge with H_{c2} at around $T = 25$ K.
 548 Similar behavior was also observed in the kagome lat-
 549 tice antiferromagnet $\text{KFe}_3(\text{OH})_6(\text{SO}_4)_2$ where the spins
 550 on the alternating planes rotate 180° forcing the previous

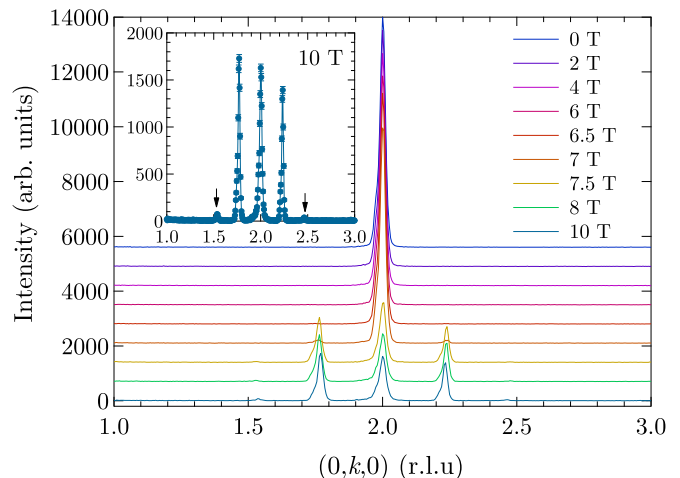


FIG. 12. (Color online) Elastic neutron scattering with applied magnetic fields from 0 to 10 T along K at $T = 2.5$ K. The inset shows harmonic peaks at $H = 10$ T, indicated by the arrows, that occurs at $(0, 2 \pm 2\delta, 0)$ with $\delta = 0.23(1)$.

551 oppositely canted moments between the alternating lay-
 552 ers to ferromagnetically align along the applied field²⁷.
 553 However, it is not clear from the available data whether
 554 the same mechanism occurs in α - $\text{Cu}_2\text{V}_2\text{O}_7$. We believe
 555 that there are two possible explanations for the presence
 556 of the intermediate transition at H_{c3} ; one is the spin-
 557 rotation and the other is the spin-flip. In the former case,
 558 the applied magnetic field must simultaneously overcome
 559 both the isotropic and anisotropic interactions. On the
 560 other hand, in the latter case, it takes considerably lower
 561 energy to flip the spins along the applied magnetic field in
 562 order to overcome only the antiferromagnetic anisotropic
 563 interaction, which is much weaker than the exchange in-
 564 teractions. Given that $H_{c2} = 18$ T (~ 1 meV) at H_{c2} , it is
 565 most probable that the magnetic phase transition at H_{c2}
 566 is due to the spin-flip and the anomaly at H_{c3} is a result of
 567 the competition between the applied magnetic field and
 568 the anisotropic exchange interaction with the presence of
 569 thermal fluctuations. The dashed line in Fig. 11 repre-
 570 sents the cross-over between the ordered stated in Region
 571 III and the paramagnetic state in Region IV, which has
 572 not been resolved. In order to verify the spin-flop state in
 573 Region II, in-field neutron scattering, which will be pre-
 574 sented in the next section, is necessary. However, even
 575 using the strongest magnet currently available for neutron
 576 scattering, we still cannot reach the second phase
 577 transition at H_{c2} , making it impossible to provide fur-
 578 ther evidence for the proposed spin-flip state in Region
 579 III.

580 D. Neutron scattering

581 In order to microscopically investigate the spin-flop
 582 state in Region II, elastic neutron scattering was per-

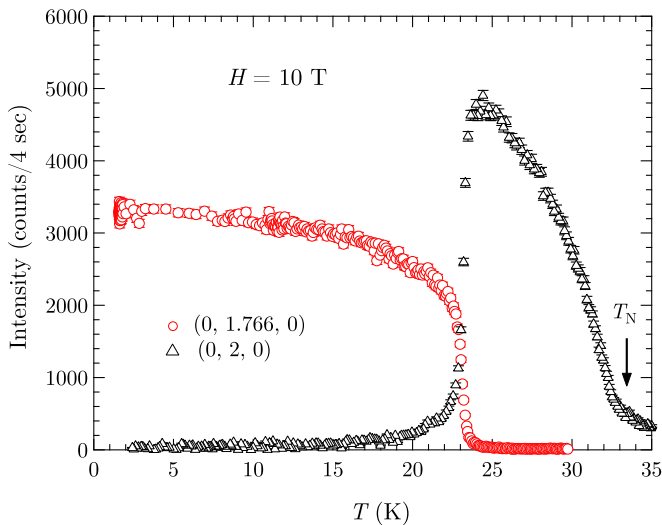


FIG. 13. (Color online) Order parameter scans as a function of temperature at $H = 10$ T of the magnetic $(0, 2, 0)$ (black triangles), and the incommensurate $(0, 1.766, 0)$ (red circle) reflections. The intensity at $(0, 2, 0)$ is background subtracted and divided by two. The Néel temperature $T_N = 33.4$ K is indicated by the black arrow.

583 formed on the single crystal with the applied magnetic
 584 fields of up to 10 T. The vertical field is applied along
 585 the a -axis with the bc -plane in the neutron scattering
 586 plane. The field dependence of the magnetic Bragg intensi-
 587 ty was measured around $\mathbf{Q} = (0, 2, 0)$. At zero field, the
 588 spins align antiparallel along the crystallographic a -axis
 589 resulting in the only observable $(0, 2, 0)$ magnetic Bragg
 590 reflection. As the applied magnetic field is increased,
 591 the intensity of $(0, 2, 0)$ decreases as shown in Fig. 12.
 592 On the other hand, we observed two extra Bragg peaks
 593 at $(0, 2 \pm \delta, 0)$ where $\delta = 0.23(1)$ for $H > 6$ T, which
 594 coincides with the first jump in the high-field magnetiza-
 595 tion data at H_{c1} . In addition, two much smaller Bragg
 596 peaks were observed at $\delta = 0.46(1)$, which can be in-
 597 terpreted as the second harmonic reflections (the arrows
 598 in the inset of Fig. 12), indicative of the incommensu-
 599 rate magnetic Bragg peaks. In contrast to a conven-
 600 tional spin-flop state, where the spins remains collinear,
 601 in the spin-flop state of α - $\text{Cu}_2\text{V}_2\text{O}_7$, the spins form a heli-
 602 cal structure. The shift of the magnetic Bragg intensity
 603 from the zone center to the incommensurate wavevectors
 604 is consistent with the transition from the collinear spin
 605 structure for $H < H_{c1}$, where the spins antiferromagnet-
 606 ically align along the a -axis, to the helical spin structure
 607 for $H > H_{c1}$, where the spins lie within the bc -plane
 608 with the helical axis along the a -axis. The helical struc-
 609 ture results from the a -component of the DM vector²⁶.
 610 Furthermore, in the spin-flop state ($H_{c1} < H < H_{c2}$),
 611 we did not observe a shift of the incommensurate peaks
 612 as a function of magnetic field (Fig. 12), indicative of no
 613 change in the modulation of the helical spin structure
 614 within the spin-flop state or at least up to the field of 10

615 T. Therefore, the modulation is determined by the DM
 616 interaction and not the applied field.

617 The magnetic scattering intensity as a function of tem-
 618 perature was measured at $(0, 2, 0)$ and $(0, 1.766, 0)$ to rep-
 619 resent the order parameters in the collinear state and
 620 spin-flop state, respectively. At 10 T, as temperature
 621 decreases from above T_N , the intensity of the $(0, 2, 0)$
 622 magnetic Bragg reflection monotonically increases be-
 623 fore abruptly decreasing to zero at the same temperature
 624 (~ 23 K) as the onset of the scattering intensity at the in-
 625 commensurate $(0, 1.766, 0)$ reflection as shown in Fig. 13.
 626 We note that the $(0, 2, 0)$ intensity is background sub-
 627 tracted and then divided by two, assuming that, at the
 628 transition from the spin-flop state to the collinear state,
 629 the two incommensurate peaks merge to form $(0, 2, 0)$
 630 and their intensities combine. However, it is clear that
 631 the maximum intensity at $(0, 2, 0)$ after the normaliza-
 632 tion is still higher than that at $(0, 1.766, 0)$. Qualitatively,
 633 this result confirms the fact that in the spin-flop state,
 634 the majority of the spin component lies in the bc -plane,
 635 i.e., the neutron scattering plane, hence resulting in a
 636 lower incommensurate magnetic intensity due to the ge-
 637 ometric factor of the scattering intensity²⁸. We note that
 638 with the current neutron diffraction data of the spin-flop
 639 state, we are unable to determine precisely the helical
 640 spin structure. However, having already studied the spin
 641 dynamics of the nonreciprocal magnons in this system²⁶,
 642 we expect that the ordered spins in the spin-flop state ar-
 643 range in the pattern closely similar to the spin structure
 644 shown in Fig. S9 in Supplementary Materials of Ref. 26,
 645 assuming that the closing of the spin gap of the nonre-
 646 ciprocal magnons gives rise to the helical spin structure
 647 at approximately the same wavevector.

648 IV. CONCLUSION

649 We have studied the magnetic properties of single-
 650 crystal α - $\text{Cu}_2\text{V}_2\text{O}_7$ by means of low-field and high-field
 651 magnetization measurements, as well as elastic neutron
 652 scattering. The combined DFT and QMC calculations
 653 confirm that the third nearest-neighbor interaction J_3
 654 is the strongest exchange coupling, in agreement with
 655 the previous studies, and refine the values of the spin
 656 Hamiltonian parameters. The high-field magnetization
 657 measurements for $H \parallel a$ reveal two consecutive magnetic
 658 phase transitions at H_{c1} and H_{c2} . The first transition
 659 at H_{c1} is due to the typical spin-flop transition similar
 660 to that observed in its cousin phase β - $\text{Cu}_2\text{V}_2\text{O}_7$. In
 661 the spin-flop state, the spins form the helical structure
 662 within the bc -plane with anti-aligned canted moments
 663 along the a -axis. As with the previously reported
 664 canted moments along the c -axis, the a -axis canted
 665 moments are a result of the DM interaction along the
 666 b -axis. The anti-alignment of the canted moments is
 667 a result of the antiferromagnetic anisotropic exchange
 668 interaction. Neutron scattering experiments reveal that
 669 for $H_{c1} < H < H_{c2}$, the incommensurate magnetic

670 Bragg reflections emerge suggesting the modulation of
671 the helical magnetic structure with the majority of the
672 spin component lying within the bc -plane. The second
673 transition at H_{c2} is believed to be the spin-flip transition
674 where the previously anti-aligned canted moments
675 become aligned with the applied magnetic field as the
676 Zeeman energy overcomes the anisotropic exchange
677 energy. The magnetic phase diagram was drawn from
678 the high-field magnetization data showing the presence
679 of the intermediate phase, which might be related to the
thermal effects, between the spin-flop and spin-flip states.

681 Acknowledgements

682 We acknowledge the support of the National Insti-
683 tute of Standards and Technology, U.S. Department of

686 Commerce, in providing the neutron research facilities
687 used in this work. The identification of any commer-
688 cial product or trade name does not imply endorsement
689 or recommendation by the National Institute of Stan-
690 dards and Technology. The authors would like to thank
691 Dr. Daisuke Okuyama, Dr. Fengjie Ma, and M. A. Allen
692 for fruitful discussions. S.Z. and M.S. acknowledge sup-
693 port from the US NSF (Grant no. DMR-1409510). Work
694 at Mahidol University was supported in part by the Thai-
695 land Research Fund Grant Number RSA5880037 and the
696 Thailand Centre of Excellence in Physics. Work at IM-
697 RAM was partly supported by a Grant-In-Aid for Scien-
698 tific Research (24224009) from the Japan Society for the
699 Promotion of Science (JSPS), and by the Research Pro-
700 gram “Dynamic Alliance for Open Innovation Bridging
701 Human, Environment and Materials”.

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