Time reversal of a wave packet with temporal modulation of gauge potential

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Phys. Rev. B **94**, 140303 — Published 26 October 2016

DOI: 10.1103/PhysRevB.94.140303

Time reversal of a wave-packet with temporal modulation of gauge potential

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Abstract

We show that time-reversal can be achieved with a spatially-uniform temporal modulation of the gauge potential. The method can be applied to the quantum wavefunction of electrons where there are natural gauge potentials, and to classical waves such as the acoustic and optical waves where effective gauge potentials have been demonstrated recently. This time-reversal process can provide a compensation of higher-order dispersion effects. Moreover, our approach can be used to time-reverse the propagation of a wavepacket in three dimensions, as well as a topologically-protected one-way edge state where the underlying Hamiltonian does not have time-reversal symmetry. Our work shows that modulation of gauge potentials provides a versatile approach for dynamically controlling wave propagation.

The capability to achieve a time-reversal operation for classical acoustic and electromagnetic waves[1–12] is of fundamental importance for controlling wave dynamics, and has found practical applications in communication, sensing and imaging[1,13,14]. For acoustic waves, time-reversal has been achieved by a process of recording and playing back using an array of transducers[3,5,6]. For optical waves, time-reversal can be achieved by using either four-wave mixing, or electro-optic modulations[2,7–10].

In this Letter we introduce an alternative mechanism for achieving a time reversal operation. We show that time reversal of a wavefunction in general can be achieved by a spatially-uniform temporal modulation of the gauge potential that couples to the wave. This capability is potentially important in two different contexts:

First of all, inspired by the importance of achieving time-reversal for classical waves, it is certainly of interest to achieve a time-reversal operation for the quantum wavefunction of electrons. Mathematically speaking, the time-reversal operator for quantum wavefunctions of electrons is similar to that for the classical waves. However, none of the available physical mechanisms for time-reversal of classical waves can be straightforwardly applied for electrons. The mechanism we describe here naturally applies to an electronic wave function since an electron, being a charged particle, naturally couples to an associated gauge potential.

Secondly, the particles that represent excitations of a quantized classical field, such as photons and phonons, are neutral particles, and have no natural gauge potentials associated with them. On the other hand, there have been significant recent developments in synthesizing an effective gauge potential for photons and phonons[15–26]. These developments have been largely motivated by seeking to achieve non-reciprocity and to realize topologically non-trivial photon and phonon states[27–34]. Here we show that these developments can be also important for time-reversal of light and sound waves. In addition, the use of gauge potential also leads to unusual capability for time reversal. For example, we show that this scheme can be used to reverses all orders of dispersion, and to achieve time-reversal of a wavefunction in full three dimensions. Finally, we show that the time-reversal operation can be carried out in systems without time-reversal symmetry, by demonstrating the time-reversal of topologically protected one-way edge states.

We first provide a mathematical definition for a time reversal operation. We consider an

arbitrary wave function $\Psi_1(\vec{r},t)$, which evolves following the equation

$$H_1\Psi_1(\vec{r},t) = i\frac{\partial}{\partial t}\Psi_1(\vec{r},t), \tag{1}$$

at $t<0$. Here Ψ_1 can be either scalars, spinors or vectors. We note that the Schrödinger equation for an electron and the Maxwell's equation for electromagnetic waves can both be written in the form of Eq. (1)[35,36]. For notation simplicity, we refer to Eq. (1) as the Schrödinger equation for a particle, and H_1 as the Hamiltonian, with no further distinction made between electrons or photons for the rest of the paper. For electrons we set $e=\hbar=1$ throughout the paper. Hence the gauge potential has the same unit as the wavevector.

Our purpose is to apply a transformation of the Hamiltonian from H_1 to H_2 around the time $t=0$, such that at $t>0$, the wavefunction Ψ_2 whose evolution is governed by the Hamiltonian H_2, satisfies:

$$|\Psi_2(\vec{r},t)|^2 = |\Psi_1(\vec{r},-t)|^2. \tag{2}$$

Eq. (2) reverses the propagation of the probability density. For applications of time reversals for either photons or phonons, as discussed in Refs.[1–12], reversing the probability density propagation is sufficient. The two cases considered below as implementation of Eq. (2) both involves time-reversal for the underlying wave amplitude:

Case (1):

$$\Psi_2(\vec{r},t) = \Psi_1(\vec{r},-t). \tag{3}$$

Here for simplicity we ignore a global phase factor. More generally to satisfy Eq. (2) one can have $\Psi_2(\vec{r},t) = e^{\theta(\vec{r},t)}\Psi_1(\vec{r},-t)$, where $\theta(\vec{r},t)$ is an arbitrary real function. From Eq. (1), we therefore require

$$H_2(t)\Psi_2(\vec{r},t) = i\frac{\partial}{\partial t}\Psi_2(\vec{r},t) = i\frac{\partial}{\partial t}\Psi_1(\vec{r},-t) = -H_1(-t)\Psi_1(\vec{r},-t). \tag{4}$$

Hence, to achieve time reversal operation we can choose

$$H_2(t) = -H_1(-t). \tag{5}$$

Case (2):

$$\Psi_2(\vec{r},t) = \Psi_1^*(\vec{r},-t). \tag{6}$$

With a similar derivation as above, we can show that this results in the choice of

$$H_2(t) = H_1^*(-t). \tag{7}$$

In this letter, we focus on the operation in Eq. (5) and provide a brief discussion of Eq. (7) at the end of the paper. The objective of our work is to develop a physical mechanism for reversing the propagation of an arbitrary wave packet. Such a reversing process has been commonly referred to as "time-reversal" in the literature[1–12]. We emphasize that this mathematical description in Eq. (2) is in fact different from the time-reversal operator in standard quantum mechanics. The standard time-reversal operator in quantum mechanics was developed in order to elucidate the time-reversal symmetry property of a Hamiltonian, and as a result must be an anti-unitary operator[37,38]. On the other hand, our objective here rather is to develop a physical mechanism, to be implemented in an experiment, in order to reverse the propagation of a wave packet, which is described by a unitary time-evolution operator. Also, in the case of spinors, our time-reversal operation only reverses the wave propagation direction without affecting the spin degree of freedom, again different from the standard time-reversal operator in quantum mechanics.

Consider a wave packet with a center frequency ω_c, i.e.: $\Psi_1(\vec{r},t) = \tilde{\Psi}_1(\vec{r},t)e^{-i\omega_c t}$, the envelope $\tilde{\Psi}_1(\vec{r},t)$ satisfies a modified Schrödinger equation

$$(H_1 - \omega_c)\tilde{\Psi}_1(\vec{r},t) = i\frac{\partial}{\partial t}\tilde{\Psi}_1(\vec{r},t). \tag{8}$$

The time-reversal in Eq. (5) then corresponds to a sudden change of Hamiltonian

$$(H_1 - \omega_c) \rightarrow -(H_1 - \omega_c). \tag{9}$$

The operation of Eq. (9) has been implemented for electromagnetic waves in[7,10,39]. Here we show that such an operation can be straightforwardly achieved by modulating a gauge potential. Consider a wave with a dispersion relation $\omega(k)$, where ω and k are the frequency and the wavevector, respectively. The dispersion relation can be linearized around the frequency ω_c as:

$$\omega - \omega_c = v_g(k - k_c), \tag{10}$$

where k_c is the center wavevector of the wavepacket. On the other hand, suppose the system has time-reversal symmetry, and hence $\omega(k) = \omega(-k)$, then in the vicinity of the frequency ω_c the dispersion relation should also has a second branch centered around the wavevector $-k_c$, as represented by:

$$\omega - \omega_c = -v_g(k + k_c), \tag{11}$$

For a system described by the dispersion relation $\omega(k)$, applying a uniform gauge potential A results in a new system as described by a dispersion relation $\omega(k-A)$[17,40]. For the system above, by applying a change of the gauge potential:

$$A = 0 \rightarrow A = 2k_c. \tag{12}$$

The second branch of the dispersion relation as described above is then translated in the wavevector space to become:

$$\omega - \omega_c = -v_g(k - A + k_c) = -v_g(k - k_c). \tag{13}$$

Suppose we apply such a gauge potential modulation to the system, while the wavepacket is in the system. Since a constant gauge potential preserves the translational symmetry, the wavevectors of the wavepacket do not change, and yet the dispersion relation of the system around the wavevector k_c is now described by Eq. (13). Therefore we have indeed achieved the time-reversal operation as prescribed in Eq. (9).

We emphasize that Eq. (12) is not a gauge transformation, where one describes the same physical system with two different gauge potential choices. Rather, Eq. (12) describes a scenario where at $t = 0$ we physically change the gauge potential applied to the system from $A = 0$ to $A = 2k_c$. Experimentally, this corresponds to applying an electric field pulse at $t = 0$. Immediately before and after the application of such an electric field pulse, the wavefunction does not change, i.e. $\Psi(x, t = 0^-) = \Psi(x, t = 0^+)$. It is precisely the application of such an electric field pulse that performs a time-reversal operation.

As an illustration of the concept above, we consider a tight-binding Hamiltonian in one dimension (as shown in Fig. 1(a))

$$H_1 = g \sum_m \left(c_m^\dagger c_{m+1} e^{-i\phi_m(t)} + c_{m+1}^\dagger c_m e^{i\phi_m(t)} \right), \tag{14}$$

where g is the coupling strength, c_m ($c_m^\dagger$) is the annihilation (creation) operator at the m-th lattice site, and ϕ_m is the phase of the coupling between sites m and $m+1$. With such a tight binding model Ψ in Eq. (2) then becomes a scalar field. Such a tight-binding model, however, can be used to treat vector Maxwell's field in three dimensions by expanding the 3D Maxwell field on a basis of vector wavefunctions. The dynamics of the coefficients of such expansion, which are scalars, are then described by the tight-binding model[41]. The

vector potential is introduced in the system through $\phi_m(t)$ via the Peierls substitution as:[42]

$$\phi_m(t) = \int_m^{m+1} \vec{A}(\vec{r}, t) \cdot d\vec{l}, \tag{15}$$

where $\hat{l}$ is the unit vector along the direction from the site m to the site $m+1$.

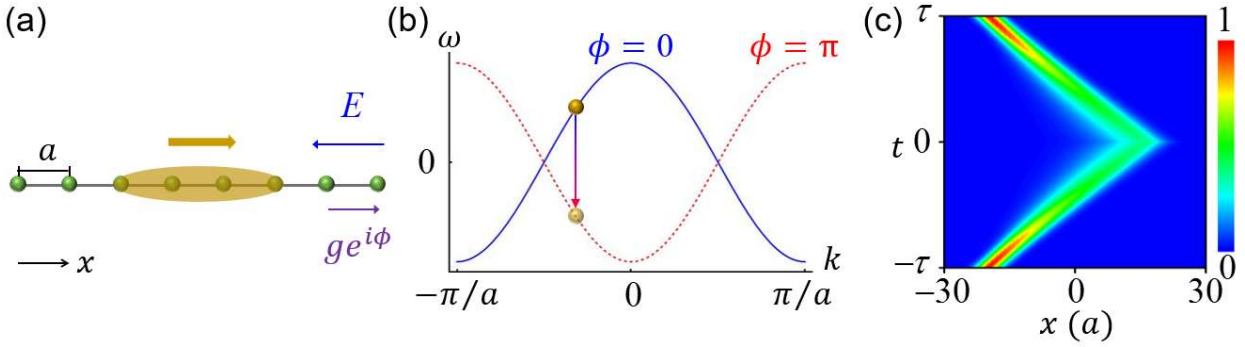

FIG. 1: (a) Schematic showing the time-reversal of a pulse (orange) in a tight-binding lattice by applying an electric field. (b) The band structure $\omega(k)$ for the tight-binding lattice with $\phi = 0$ (blue curve) and $\phi = \pi$ (red curve). (c) Propagation of a wave packet in the one-dimensional lattice. An electric field pulse is applied around the time $t = 0$. Colors show the intensity density of the wave packet.

We consider the case where $\phi_m = \phi$ is uniform throughout the system, hence a spatially uniform A. Eq. (14) is then described by a bandstructure:

$$\omega(k) = 2g\cos(k - \phi). \tag{16}$$

The bandstructures corresponding to $\phi = 0$ and $\phi = \pi$ are plotted in Fig. 1(b). With a shift of the gauge potential that changes from $\phi = 0$ to $\phi = \pi$, we have a time-reversal operation as defined by Eq. (5). Note that unlike the case discussed in Eq. (12) for a general dispersion relation, here the shift is sufficient to time-reverse any pulse in this system. Such a shift of gauge potential corresponds to the application of an electric field $E(t) \equiv -dA/dt$ that satisfies an area law:

$$\int E(t)dt = (2n+1)\pi/a, \tag{17}$$

where n is an arbitrary integer and a is the lattice constant. Any electric field profile satisfying Eq. (17) is capable for achieving time-reversal for arbitrary wavepacket in the system.

As a numerical demonstration, we perform the simulation for the dynamics of a wave packet

$$|\Psi(t)\rangle = \sum w_m(t) c_m^\dagger |0\rangle, \tag{18}$$

in the system as described by Hamiltonian of Eq. (14). Here w_m is the probability amplitude at the m-th site. The lattice includes 61 sites. The simulation is performed from $t = -\tau$ to $t = \tau$ ($\tau = 20\ g^{-1}$). The initial wave packet has the shape of

$$w_m(t = -\tau) = e^{-x_m^2/\Delta x^2 - ik_0 x_m}, \tag{19}$$

where $x_m = ma$, $\Delta x = 3.75a$ and $k_0 = 1/a$. To modulate the gauge potential, we apply an electric field pulse centered at $t = 0$ with the form

$$E(t) = \frac{\sqrt{\pi}}{a\Delta t} e^{-t^2/\Delta t^2}, \tag{20}$$

with $\Delta t = 0.06\ g^{-1}$. The pulse satisfies Eq. (17) with $n = 0$. The distribution of $|w_m(t)|^2$ is plotted in Fig. 1(c). The wavepacket initially moves to the right and expands in space as time progresses from $t = -\tau$. Upon the application of the gauge potential modulation, however, the pulse reverses its propagation direction and compresses, returning to its original shape at $t = \tau$.

For a general dispersion relation of $\omega(k)$, the time-reversal operation as prescribed in Eqs. (10)-(13) in fact reverses all odd-order dispersions, as can be seen by including higher-order terms in the Taylor expansion of Eqs. (10) and (11). This already represents an improvement over most of conventional time-reversal schemes[7–11], which only reverse the group velocity. Moreover, for the tight-binding model, the time-reversal scheme considered here results in an exact implementation of Eq. (5) and hence reverses dispersion of all orders. This is partially evident in Fig. 1(c), since the spread and compression of the wavepacket are controlled by the second-order Tayler expansion of the dispersion relation.

Most previous works on time-reversal in classical waves operates either on a one-dimensional wavepacket[7–12] or a two-dimensional wavefront[3–6]. On the other hand, the proposed scheme here enables time-reversal of a wavepacket in full three dimensions. As an illustration, considering a tight-binding Hamiltonian on a cubic lattice:

$$H = g \sum_{m,n,l} \left(c_{m,n,l}^\dagger c_{m+1,n,l} e^{-i\phi_x} + c_{m,n,l}^\dagger c_{m,n+1,l} e^{-i\phi_y} + c_{m,n,l}^\dagger c_{m,n,l+1} e^{-i\phi_z} + h.c. \right), \tag{21}$$

where (m, n, l) labels a lattice site along the x, y, z axes. Its band structure has the form:

$$\omega(k_x, k_y, k_z) = 2g\cos(k_x - \phi_x) + 2g\cos(k_y - \phi_y) + 2g\cos(k_z - \phi_z)\,. \tag{22}$$

By modulating the gauge potential such that the coupling phases along each axis vary according to $\phi_{x(y,z)} = 0 \to \pi$, we can achieve the time-reversal as indicated in Eq. (5). Such a simultaneous change of the coupling phases $\phi_{x(y,z)}$ along all three dimensions corresponds to an application of an electric field along the [111] direction for the cubic lattice. The component of the electric field in each direction satisfies the area law in Eq. (17).

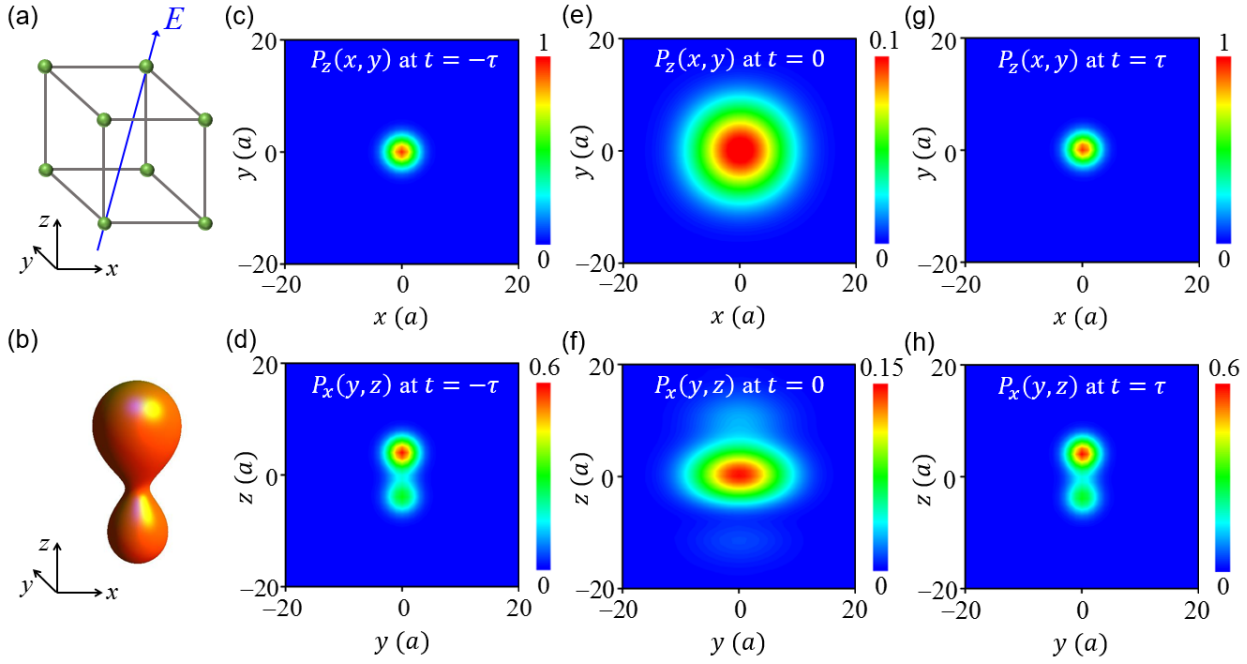

FIG. 2: (a) Three-dimensional lattice with an electric field pulse pointing at the [111] direction. (b) The initial wave packet distribution at $t = -\tau$. (c)-(h) Normalized projections of the calculated intensity distribution on the xy plane ($P_z(x, y, t) = \sum_l |w_{m,n,l}(t)|^2$) in (c),(e),(g) and on the yz plane ($P_x(y, z, t) = \sum_m |w_{m,n,l}(t)|^2$) in (d),(f),(h), at times $-\tau$, 0, and τ, respectively.

As a numerical demonstration, we simulate the Schrödinger equation on a $41a \times 41a \times 41a$ lattice. We apply an electric field pulse. Each of its components has the form in Eq. (20), with $\Delta t = 0.06\ g^{-1}$. At $t = -\tau = -10\ g^{-1}$, we initialize the wave packet with the amplitudes at the lattice site (m, n, l) having the form:

$$w_{m,n,l}(t = -\tau) = e^{-\left[x_m^2 + y_n^2 + (z_l - 4a)^2\right]/\Delta} + 0.7e^{-\left[x_m^2 + y_n^2 + (z_l + 4a)^2\right]/\Delta}, \tag{23}$$

with $x_m = ma$, $y_n = na$, $z_n = la$, $\Delta = 3.75a$, as shown in Fig. 2(b). At different times ($-\tau$, 0, and τ), we calculate the resulting wavepacket distribution, and plot its normalized projection of wave packet distribution on the xy plane (Figs. 2(c),(e),(g)) and on the yz plane (Figs. 2(d),(f),(h)), respectively. The normalized projections are defined as $P_z(x, y, t) = \sum_l |w_{m,n,l}(t)|^2$ and $P_x(y, z, t) = \sum_m |w_{m,n,l}(t)|^2$. With the application of the electric field pulse around $t = 0$, the three-dimensional wavepacket indeed returns to its original shape at $t = \tau$, providing a direct illustration of a three-dimensional time-reversal operation.

Up to this point, we have demonstrated time-reversal operation in a lattice that has time-reversal symmetry and full translational symmetry for all lattice vectors. Both symmetry constraints can in fact be relaxed. As long as the dispersion relation of the system has a symmetry of $\omega(k_x, k_y, k_z) = -\omega(k_x + \pi/a, k_y + \pi/a, k_z + \pi/a) + \text{const.}$, the use of gauge potential as described above will lead to a perfect time reversal. As a final illustration, we consider a two-dimensional lattice system subject to a constant magnetic field. We show that the same electric field pulse as prescribed above can be used to achieve a time-reversal for one-way edge states of the system.

We consider the Hamiltonian

$$H = g \sum_{m,n} \left(c^\dagger_{m,n} c_{m+1,n} e^{-i\phi_x} + c^\dagger_{m,n} c_{m,n+1} e^{-im\pi/2 - i\phi_y} + h.c. \right), \tag{24}$$

as schematically depicted in Fig. 3(a). In the second term of Eq. (24), the m-dependent phases correspond to a Landau gauge for a constant magnetic field of a strength $B = \pi/2a^2$. In the absence of the electric field pulse, (i.e. $\phi_x = \phi_y = 0$), the band structure of the system, for a stripe having a finite width of $80a$ in the y-direction, and infinite in the x-direction, is shown Fig. 3(b). The band structure consists of four groups of bands corresponding to the four magnetic sub-bands in the infinite bulk system[28]. The two center groups of bands touch at $\omega = 0$, and are separated from the two other groups of bands above and below in frequency by two topologically non-trivial band gaps. Within each gap there are a pair of topologically protected one-way edge states localized at either the top or the bottom edges of the stripe, as represented by the red and blue lines in Fig. 3(b). On the bottom edge, there is a right-going one-way edge state in the upper band gap in the wavevector range of $k_x > 0$, and a left-going state in the lower band gap with $k_x < 0$. Fig. 3(c) shows the same band structure, shifted along the k_x axis by π/a. In the shifted band structure, in the wavevector range of $k_x > 0$, there is now a left-going one-way edge. Therefore, for a

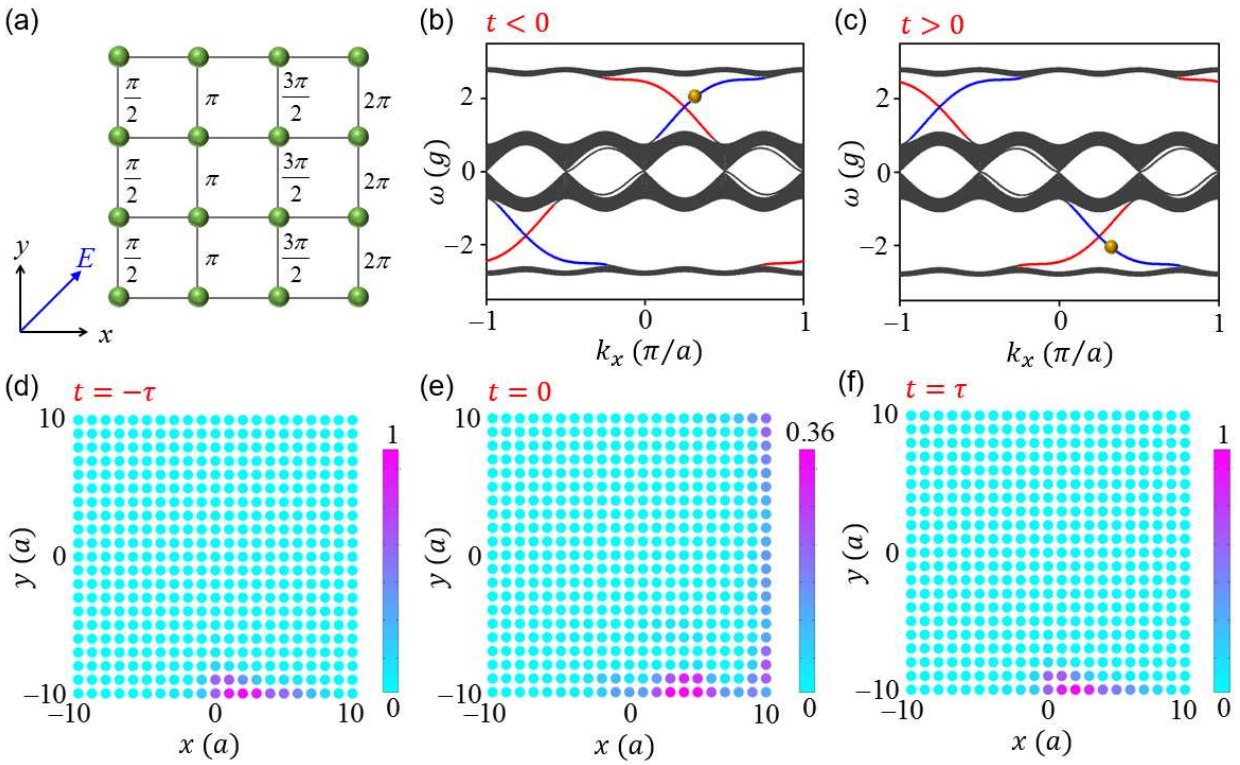

FIG. 3: (a) Two-dimensional lattice under a constant magnetic field, which is described by the Laudau gauge phase distribution. An electric field pulse pointing at the [11] direction is in addition applied. (b) and (c) The band structure at $t < 0$ and $t > 0$, respectively, for a stripe of tight-binding lattice shown in (a). The stripe is infinite in the x-direction and finite along the y-direction. Red (blue) curve shows the edge state on top (bottom) edge and grey curves show the bulk bands. A gold dot represents a wavepacket. (d)-(f) Normalized wave packet intensity distribution at times $-\tau$, 0, and τ, respectively.

right-going wavepacket consisting of one-way edge states at the bottom edge, as represented by the gold dot in Fig. 3(b), applying a spatially-uniform temporal modulation of the gauge potential in the form of a pulse of electric field in the x-direction as discussed in this paper will convert it to a left-going wavepacket as represented by the gold dot in Fig. 3(c), and hence achieving a time-reversal operation of one-way edge state.

As a numerical demonstration, we simulate the Hamiltonian of Eq. (24) on a $21a \times 21a$ lattice. The simulated system has edges along both the $x-$ and $y-$directions. Therefore we apply an electric field along the [11] direction, with both the $x-$ and $y-$ components of the field in the form of Eq. (20) with $\Delta t = 0.06\ g^{-1}$. The lattice is excited by a source pulse

located at the position $(0, -10)$, which has the expression

$$s(t) = e^{-(t-t_c)^2/\eta^2 - i\omega_s t}, \tag{25}$$

where $t_c = -25\ g^{-1}$, $\eta = 1.7\ g^{-1}$, and $\omega_s = 2g$. All frequency components of this pulse are located in the band gap[28]. The simulation is performed from $t = -1.5\tau$ to $t = \tau$ (with $\tau = 20\ g^{-1}$). We plot the normalized wave packet distributions at times $-\tau$, 0, and τ in Figs. 3(d)-(f). The initial wavepacket propagates counter-clockwise. Upon the application of the electric field pulse, it reverse its propagation direction. The wavepacket has the same shape at τ and $-\tau$, providing the evidence of the time-reversal operation.

We note that our approach here is reminiscent, but different, from the Thouless quantum pump, where an adiabatic variation of the magnetic field transfers between the one-way edge modes with the same energy on the two edges[43,44]. In this case, we achieve a transfer between one-way edge modes at the same edge with opposite chirality through the use of an additional electric field. These modes have different energies and are located in different energy gaps. The ideal form of variation is sudden rather than adiabatic. The observation of this effect can be achieved in photonic/acoustic topological-insulator systems straightforwardly. For electronic systems, the final state as discussed above after the application of the electric field pulse needs to be initially un-occupied in order for the time reversal to occur.

Up to now we have described a time-reversal operation based on Eq. (5). There are other possibilities of using gauge potential modulation to achieve time reversal. Assuming that at $t < 0$, a wave packet $\Psi_1(r, t)$ evolves in time as governed by a Hamiltonian with a gauge potential $\vec{A} = \vec{A}_1$, such that at $t = 0$, the wavefunction has the form $\Psi_1(\vec{r}, t = 0) = \rho(\vec{r})e^{i\theta(\vec{r})}$. We apply a sudden change of gauge potential at $t = 0$: $\vec{A} = \vec{A}_1 \rightarrow -\vec{A}_1 + 2\nabla\theta$. We can then obtain a time-reversal operation: at $t > 0$, the wavefunction evolves according to $\Psi_2(r, t) = \Psi_1^*(r, -t)e^{2i\theta(\vec{r})}$. This time evolution is equivalent to Eq. (7) after a gauge transformation. In this case, the required gauge potential is in general dependent upon the details of the wavepacket. In contrast, in the main results of the paper, which achieve a time-reversal operation based on Eq. (5), the same gauge potential change can be used to time-reverse an arbitrary wavepacket.

In summary, we propose a mechanism for achieving time reversal by applying a spatially-uniform temporal modulation of the gauge potential. Our proposal is applicable for both quantum wavefunctions of electrons, and for the classical electromagnetic or acoustic waves.

This time reversal operation has experimental feasibility for both electrons, by applying an electric field, and for photons and phonons, where the effective gauge fields have been proposed and realized in experiments in recent years[15–34]. Our works point to the important opportunities for dynamic control of wave propagation with the use of gauge potential.

Acknowledgments

We thank Prof. M. Fink for helpful discussion. This work is supported by U.S. Air Force Office of Scientific Research Grant No. FA9550-12-1-0488.

[1] A. Yariv, D. Fekete, and D. M. Pepper, Opt. Lett. **4**, 52 (1979).

[2] D. A. B. Miller, Opt. Lett. **5**, 300 (1980).

[3] M. Fink, IEEE Trans. Ultrason. Ferroelectr. Freq. Control **39**, 555 (1992).

[4] I. Freund, J. Opt. Soc. Am. A **9**, 456 (1992).

[5] M. Fink and C. Prada, Inverse Probl. **17**, R1 (2001).

[6] J. de Rosny and M. Fink, Phys. Rev. Lett. **89**, 124301 (2002).

[7] M. F. Yanik and S. Fan, Phys. Rev. Lett. **93**, 173903 (2004).

[8] J. B. Pendry, Science **322**, 71 (2008).

[9] A. V. Chumak, V. S. Tiberkevich, A. D. Karenowska, A. A. Serga, J. F. Gregg, A. N. Slavin, and B. Hillebrands, Nat. Commun. **1**, 141 (2010).

[10] Y. Sivan and J. B. Pendry, Phys. Rev. Lett. **106**, 193902 (2011).

[11] Y. Sivan and J. B. Pendry, Opt. Express **19**, 14502 (2011).

[12] C. Wang, R. Martini, and C. P. Search, Phys. Rev. A **86**, 063832 (2012).

[13] J. M. F. Moura and Y. Jin, IEEE Trans. Signal Process. **55**, 187 (2007).

[14] S. Denisov, S. Flach, and P. Hänggi, Phys. Rep. **538**, 77 (2014).

[15] M. Hafezi, E. A. Demler, M. D. Lukin, and J. M. Taylor, Nature Phys. **7**, 907 (2011).

[16] R. O. Umucalilar and I. Carusotto, Phys. Rev. A **84**, 043804 (2011).

[17] K. Fang, Z. Yu, and S. Fan, Phys. Rev. Lett. **108**, 153901 (2012).

[18] M. C. Rechstman, J. M. Zeuner, Y. Plotnik, Y. Lumer, D. Podolsky, F. Dreisow, S. Nolte, M. Segev, and A. Szameit, Nature **496**, 196 (2013).

[19] S. Longhi, Opt. Lett. **38**, 3570 (2013).

[20] E. Li, B. J. Eggleton, K. Fang, and S. Fan, Nat. Commun. **5**, 3225 (2014).

[21] F. Liu and J. Li, Phys. Rev. Lett. **114**, 103902 (2015).

[22] M. Schmidt, S. Kessler, V. Peano, O. Painter, and F. Marquardt, Optica, **2**, 635 (2015).

[23] L. Yuan and S. Fan, Phys. Rev. Lett. **114**, 243901 (2015).

[24] M. Xiao, W. -J. Chen, W. -Y. He, and C. T. Chan, Nat. Phys. **11**, 920 (2015).

[25] L. Yuan, S. Xu, and S. Fan, Opt. Lett. **40**, 5140 (2015).

[26] L. Yuan, Y. Shi, and S. Fan, Opt. Lett. **41**, 741 (2016).

[27] Z. Wang, Y. Chong, J. D. Joannopoulos, and M. Soljačić, Nature **461**, 772 (2009).

[28] K. Fang, Z. Yu, and S. Fan, Nat. Photonics **6**, 782 (2012).

[29] M. Hafezi, S. Mittal, J. Fan, A. Migdall, and J. M. Taylor, Nat. Photonics **7**, 1001 (2013).

[30] A. B. Khanikaev, S. H. Mousavi, W. -K. Tse, M. Kargarian, A. H. MacDonald, and G. Shvets, Nature Mater. **12**, 233 (2013).

[31] W. -J. Chen, S. -J. Jiang, X. -D. Chen, B. Zhu, L. Zhou, J. -W. Dong, and C. T. Chan, Nat. Commun. **5**, 5782 (2014).

[32] R. Fleury, D. L. Sounas, C. F. Sieck, M. R. Haberman, and A. Alù, Science **343**, 516 (2015).

[33] A. P. Slobozhanyuk, A. B. Khanikaev, D. S. Filonov, D. A. Smirnova, and A. E. Miroshnichenko, and Y. S. Kivshar, Sci. Rep. **6**, 22270 (2016).

[34] T. Ozawa, H. M. Price, N. Goldman, O. Zilberberg, and I. Carusotto, Phys. Rev. A **93**, 043827 (2016).

[35] J. D. Joannopoulos, S. G. Johnson, J. N. Winn, and R. D. Meade, Photonic Crystals: Molding the Flow of Light, 2nd ed. (Princeton University Press, Princeton, NJ, 2008).

[36] J. N. Winn, S. Fan, J. D. Joannopoulos, and E. P. Ippen, Phys. Rev. B **59**, 1551 (1999).

[37] J. J. Sakurai, Modern Quantum Mechanics (The Benjamin/Cummings Publishing Company, Inc., CA, 1985).

[38] E. P. Wigner, Group Theory and its Application to the Quantum Mechanics of Atomic Spectra. translation from German by J. J. Griffin. (Academic Press, New York, 1959).

[39] D. M. Pepper, Laser Handbook (North-Holland Physics, Amsterdam, 1988).

[40] Q. Lin and S. Fan, Phys. Rev. X **4**, 031031 (2014).

[41] For a detailed derivation, see the supplementary information of Ref.[23].

[42] R. E. Peierls, Z. Phys. **80**, 763 (1933).

[43] D. J. Thouless, Phys. Rev. B 27, 6083 (1983).

[44] Y. E. Kraus, Y. Lahini, Z. Ringel, M. Verbin, and O. Zilberberg, Phys. Rev. Lett. 109, 106402 (2012).