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Intrinsic ultralow lattice thermal conductivity of the unfilled skutterudite FeSb$_3$

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It is generally accepted that unfilled skutterudites process high lattice thermal conductivity ($\kappa_l$) that can be efficiently reduced upon filling. Here by using first principles Boltzmann-Peierls transport calculations, we find pure skutterudite of FeSb$_3$ with no filler in fact has an intrinsic ultralow $\kappa_l$ smaller than that of CoSb$_3$ by one order of magnitude. The value is even smaller than those of most of the fully filled skutterudites. This finding means that with FeSb$_3$ as a reference, filling does not necessarily lower $\kappa_l$. The ultralow $\kappa_l$ of FeSb$_3$ is a consequence of lowering in frequency of optical phonon branches associated with the weakly bonded Sb$_4$ rings. They overlap more with heat-carrying acoustic phonons and significantly increase the phase space for three-phonon anharmonic scattering processes. This provides an alternative non-filling related mechanism for lowering the $\kappa_l$ of skutterudites.

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I. INTRODUCTION

Skutterudites are an important class of high performance thermoelectrics$^{1-20}$ as the embodiment of the Slack’s “electron-crystal phonon-glass” idea$^{21}$. The existence of two isosahedral voids in their crystal structures allows for filling in a variety of cations (e.g., rare earth, alkali earth or alkali metals). This offers dual advantages for good thermoelectrics: first, according to the Zintl concept the additional electrons transferred from the electronegative fillers to the CoSb$_3$ framework make possible flexible control of $n$-type doping$^{4,7,8,10,11,15-17,22,23}$, and provides compensating change to the $p$-type doping with Co replaced by electron deficient Fe$^{1,2,5,8,14,24}$. Second, and more importantly, filling strongly lowers lattice thermal conductivity ($\kappa_l$)$^{1-4,7,8,10,11,15-17,19-22}$ and optimizing the filling to lower $\kappa_l$ both in terms of filling fraction and by using suitable mixtures of filled cations plays a central role in the optimization of high performance skutterudite thermoelectrics$^4$.

The physical mechanism responsible for the reduction of $\kappa_l$ in filled skutterudites remains elusive after two decades of intensive research. There are several debated aspects about the nature and role of the vibrations associated with the filled “rattling” atoms: (i) whether the motion of the rattling atoms is incoherent and non-correlated$^{21,25-27}$ or coherently couples with the host framework$^{28,29}$; (ii) whether there exists anharmonic interaction between the localized rattling modes and the propagating phonons of the host framework$^{30-34}$; (iii) whether the reduction of $\kappa_l$ originates from the energy dissipation caused by the resonant scattering of the rattling atoms$^{2,21,25,35}$ or the enhanced conventional anharmonic (Umklapp) scattering processes$^{29,33,34,36}$.

Despite controversy over the mechanism, there is consensus that the filling should reduce the $\kappa_l$ of skutterudites. In this Article we report the finding via first principles transport calculations that the recently reported skutterudite FeSb$_3$$^{37-42}$ often presumed to be closely related to CoSb$_3$ in fact has an very low $\kappa_l$ without filling, even lower than those of most of the fully filled skutterudites (e.g., with Ba, La and Ce). This unexpected finding indicates that filling does not necessarily lower $\kappa_l$ of skutterudites. The ultralow $\kappa_l$ in the unfilled FeSb$_3$ skutterudite is a consequence of lowering in frequency of optical phonon branches that take the role of the rattling modes in the filled skutterudites. The resultant optical-acoustic phonons overlapping increases three-phonon anharmonic scattering channels, thus significantly reducing phonon lifetimes and $\kappa_l$. Our result provides an alternative non-filling related routine to realize low $\kappa_l$ in skutterudites.

II. METHOD

We perform first principles calculations of $\kappa_l$ for FeSb$_3$ and fully filled skutterudites of La/CeFe$_4$Sb$_12$ by iteratively solving the linearized Boltzmann-Peierls transport equation of phonons with the SHENGBTE package$^{43}$. The equilibrium crystal structures and interatomic force constants (IFCs) are obtained from DFT calculations with the plane-wave projector-augmented-wave method$^{44}$, as implemented in the VASP code$^{45}$. We employ the local density approximation (LDA) as exchange-correlation functional. A ferromagnetic configuration for FeSb$_3$ is used, which is the lowest-energy magnetic configuration at the LDA level. We did calculations with both LDA and generalized gradient approximation (GGA) functionals, and find in both cases that ferromagnetic ordering is preferred over near neighbor antiferromagnetic order. As noted in prior work by Rassander and co-workers$^{41}$ magnetism is important for the stability of this compound. Structural optimization is done with the kinetic energy cutoffs of 350 eV or more and the $8\times8\times8$
k-point mesh, which ensures the residual forces smaller than 1x10^{-4} eV/Å. The resulted equilibrium lattice constants are slightly smaller than the experimental data (by 2.03%, 1.78%, and 2.00% for FeSb_3, LaFe_3Sb_12 and CeFe_3Sb_12, respectively) as summarized in Table I. The agreements are reasonably good by consideration of the usual underestimation of lattice constants in the DFT-LDA calculations. The harmonic and third-order anharmonic IFCs are calculated by using the real-space supercell approach \cite{43,46}, in a 3x3x3 supercell with a 2x2x2 k-point mesh and a 2x2x2 supercell with a 3x3x3 k-point mesh, respectively. The phonon momenta q-mesh of 15x15x15 is used in solving the transport equation to ensure \( \kappa_l \) converged at the 1x10^{-6} W/mK level.

### Table I. Optimized structure parameters by total energy minimization for FeSb_3, CoSb_3, LaFe_3Sb_12 and CeFe_3Sb_12, compared with available experimental data\cite{12,24,37}.

<table>
<thead>
<tr>
<th>Material</th>
<th>Theory</th>
<th>Exp</th>
<th>Fractional coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>FeSb_3</td>
<td>8.968</td>
<td>9.154</td>
<td>a = 0.329, x = 0.161</td>
</tr>
<tr>
<td>CoSb_3</td>
<td>8.918</td>
<td>9.0345</td>
<td>a = 0.334, x = 0.158</td>
</tr>
<tr>
<td>LaFe_3Sb_12</td>
<td>8.985</td>
<td>9.148</td>
<td>a = 0.335, x = 0.164</td>
</tr>
<tr>
<td>CeFe_3Sb_12</td>
<td>8.957</td>
<td>9.140</td>
<td>a = 0.333, x = 0.164</td>
</tr>
</tbody>
</table>

By solving the linearized Boltzmann-Peierls transport equation of phonons, all the contributions from two-phonon and three-phonon scattering processes responsible for intrinsic \( \kappa_l \) of crystalline materials are included. Briefly the isotopic \( \kappa_l \) of cubic skutterudites at temperature T can be represented as the sum of contributions over each phonon mode \( \lambda \) (with branch p and wave vector q):

\[
\kappa_l \equiv \kappa_l^{\alpha \beta} = \frac{1}{NV} \sum_{\lambda} C_{\lambda} v_{\lambda}^{\alpha} e_{\lambda}^{\alpha} T_{\lambda},
\]

and

\[
C_{\lambda} = \frac{\partial f_{\lambda}(\omega_{\lambda}, T)}{\partial T},
\]

where N is the number of q points uniformly sampled in the phonon Brillouin zone, V is the unit cell volume, \( C_{\lambda} \) is the phonon mode heat capacity, \( f_{\lambda}(\omega_{\lambda}, T) \) is the Bose-Einstein distribution form that is the function of the phonon frequency \( \omega_{\lambda} \) and T, \( e_{\lambda}^{\alpha} \) is the phonon group velocity along the \( \alpha \) direction, and \( T_{\lambda} \) is the phonon lifetime. In bulk materials without impurities, \( T_{\lambda} \) is determined by the processes of two-phonon scattering from isotopic disorder and three-phonon anharmonic scattering. In the relaxation time approximation, \( \tau_{\lambda} \) is equal to a sum of the isotope scattering rate (\( 1/\tau_{\lambda^{iso}} \)) and the anharmonic scattering rate (\( 1/\tau_{\lambda^{anh}} \)). For most of materials, \( 1/\tau_{\lambda^{iso}} \) is at least two-order smaller in magnitude than \( 1/\tau_{\lambda^{anh}} \) can be calculated as the sum over \( \lambda \) of the three-phonon transition probabilities \( \Gamma_{\lambda \lambda' \lambda''}^{\pm} \), which can be expressed as\cite{43,47-49}:

\[
\Gamma_{\lambda \lambda' \lambda''}^{\pm} = \frac{\hbar}{8N} \sum_{i \neq j} \left( \omega_{\lambda} \pm \omega_{\lambda'} - \omega_{\lambda''} \right) |V_{\lambda \lambda' \lambda''}^{\pm}|^2,
\]

and

\[
V_{\lambda \lambda' \lambda''}^{\pm} = \sum_{i \neq j} \frac{\delta(\omega_{\lambda} \pm \omega_{\lambda'} - \omega_{\lambda''})}{\sqrt{M_i M_j M_k}} \Phi_{ij \lambda}^{\alpha \beta \gamma} \delta_{\lambda' \lambda''}.
\]

where the upper (lower) row in curly brackets corresponds to the +(-) sign of \( \Gamma_{\lambda \lambda' \lambda''}^{\pm} \), which represent three-phonon absorption (two phonons merging into one phonon) and emission (one phonon splitting into two) processes. The scattering matrix elements \( V_{\lambda \lambda' \lambda''}^{\pm} \) can be evaluated with the normalized eigenvectors \( e_{\lambda}^{\alpha} \) of the three phonons involved and the anharmonic interatomic force constants (IFCs) \( \Phi_{ij \lambda}^{\alpha \beta \gamma} \). The contribution of harmonic phonon frequencies to the anharmonic scattering rates can be represented by the three-phonon scattering phase space \( W_{\lambda}^{\pm} \) (+ and - signs corresponding to absorption and emission processes). It is defined as the sum of frequency-containing factors in the expression of three-phonon transition probabilities (Eq. 3), and is written as\cite{51}:

\[
W_{\lambda}^{\pm} = \frac{1}{2N} \sum_{\lambda' \lambda''} \left( \frac{\omega_{\lambda'}}{\omega_{\lambda''}} \right) \left( \frac{\omega_{\lambda''}}{\omega_{\lambda}} \right) |V_{\lambda \lambda' \lambda''}^{\pm}|^2.
\]

### III. RESULTS AND DISCUSSION

Fig. 1 shows calculated \( \kappa_l \) as the function of temperature for unfilled FeSb_3 and CoSb_3\cite{50}, as well as fully filled skutterudites of LaFe_3Sb_12, CeFe_3Sb_12, YbFe_4Sb_12\cite{34}, BaFe_4Sb_12\cite{34} and BaCo_3Sb_12\cite{31}. The excellent agreement between theoretical results of FeSb_3 and available experimental data\cite{12,51} strongly indicate the validity of our calculations. Surprisingly, we find that FeSb_3 exhibits a quite low \( \kappa_l \) of 1.14 W/mK at 300 K, about one order of magnitude lower than 11.6 W/mK of CoSb_3. In the whole temperature range the \( \kappa_l \) of FeSb_3 is apparently much lower than the values of the most filled skutterudites (by more than two-third). The only exception is YbFe_4Sb_12 that owes the lowest theoretical \( \kappa_l \) among reported filled skutterudites\cite{34}. The \( \kappa_l \) of FeSb_3 show a weak temperature dependence, resembling that of YbFe_4Sb_12.
The available experimental data for CoSb$_3$ and LaFe$_4$Sb$_{12}$ are shown for comparison. The agreement between our calculation and the experiments is good. Clearly at most of $\omega$ the rates of FeSb$_3$ are much higher than those of LaFe$_4$Sb$_{12}$, though lower than those of YbFe$_4$Sb$_{12}$. Therefore the observed differences in the anharmonic scattering processes that limit phonon lifetimes indeed account for the discrepancies in $k_l$. Note that the anharmonic scattering rates of FeSb$_3$ are remarkably enhanced in a wide intermediate frequency range between $\sim 1$ and 5 THz. The behavior is similar to those in La/YbFe$_4$Sb$_{12}$ and other filled skutterudites. This affects contributions of phonon modes to $k_l$, as indicated by the cumulative plot of $k_l$ ($k_l'$) that represents the fraction of heat carried by the phonons with less frequencies than $\omega$ (inset of Fig. 4). The higher anharmonic scattering rates correspond to the smaller lifetimes, and thus the less contributions of the phonons to $k_l$. While for CoSb$_3$ $k_l'$ increase rapidly with $\omega$ and the phonons below 2 THz have already contributed to $\sim 80\%$ of $k_l$, $k_l'$ of FeSb$_3$ show a much slower increase and the phonons below 2 THz only contribute to $\sim 50\%$ of $k_l$. The behavior of $k_l'$ of FeSb$_3$ resembles those of filled skutterudites, especially YbFe$_4$Sb$_{12}$. We note that Möchel and co-workers find evidence for lattice softening in this latter compound.

We further investigate the roles of harmonic and third-order anharmonic IFCs in enhancing three-phonon anharmonic scattering processes and reducing $k_l$ in FeSb$_3$. In particular we perform calculations of $k_l$ by deliberately interchanging the harmonic/anharmonic IFCs between two different compounds, as shown in Fig. 3c.
For FeSb₃, when replacing the anharmonic IFCs by the ones from CoSb₃ and remaining the other quantities unchanged, we find that κ₁ increases by ~30%, whereas κ₃ of CoSb₃ decreases by ~30% when using the anharmonic IFCs from FeSb₃. The anharmonic scattering rates are generally proportional to the square of the anharmonic IFCs (Eq. 3 and 4). The result means the anharmonic IFCs of FeSb₃ are larger than those of CoSb₃, corresponding to the higher scattering rates in FeSb₃, but far from enough to account for its one-order lower κ₁. For the filled skutterudites of LaFe₄Sb₁₂ and YbFe₄Sb₁₂, when using the anharmonic IFCs from FeSb₃, the resulted κ₁ show ~50% and ~130% increases, respectively. This indicates the anharmonic IFCs of FeSb₃ are smaller, corresponding to the lower scattering rates, which conflicts with the smaller κ₁ of FeSb₃ than that of LaFe₄Sb₁₂. When we interchange the harmonic IFCs between FeSb₃ and CoSb₃, we find κ₁ of FeSb₃ increases by about 8 times, and κ₃ of CoSb₃ decreases by almost the same amount. The changes accord well with the discrepancy of κ₁ between FeSb₃ and CoSb₃. From these results, we can conclude that the main factor responsible for the enhanced anharmonic scattering in FeSb₃ is the harmonic IFCs, rather than the third-order anharmonic IFCs.

The specific way of the harmonic IFCs affecting the anharmonic scattering processes is through phonon frequencies and eigenvectors (Eq. 3 and 4). The phonon eigenvectors contribute to the three-phonon scattering matrix elements. Usually for the same class of materials, the changes of eigenvectors with the varied harmonic IFCs are not as large as the changes of frequencies. It is thus reasonably to assume the scattering matrix elements do not change substantially here. The action of phonon frequencies on the anharmonic scattering is embodied by the three-phonon scattering phase space W⁺ (see Eq. 5), which depicts available three-phonon scattering channels among all modes. It consists of two components corresponding to absorption (W⁺, two phonons merging into one) and emission (W⁻, one splitting into two) processes, respectively. As show in Fig. 3d, the W⁺ of FeSb₃ are distinctly larger than those of CoSb₃ by several times in the low frequency (< 3.5 THz) region, and the W⁻ of FeSb₃ show much larger values in the wide frequency region below 5 THz as well. The origin of the significantly enhanced W± of FeSb₃ lies in its distinct phonon spectrum. Comparing with that of CoSb₃ (Fig. 5b), the phonon spectrum of FeSb₃ (Fig. 5a) shows a clear softening for the lowest optical branch (down to below 2 THz). This optical branch is even lower in frequency than the La-derived rattling mode in LaFe₄Sb₁₂ (Fig. 5c), and is not far away from the extremely low frequency Yb-derived mode in YbFe₄Sb₁₂ (cyan curve in Fig. 5c). Actually it is not only the lowest optical branch, but several adjacent
upper optical branches that show lowering in frequency in FeSb₃. This is unambiguously reflected by a sharp PHDOS peak appearing around 2 THz (Fig. 5e). It is located in the region similar to that of the La-derived rattling modes in LaFe₄Sb₁₂ (blue dash line in Fig. 5e). The low-lying optical phonons overlaps more with the acoustic branches, appreciably increases the phase space W⁺ for three-phonon scattering processes (Fig. 3d). This reduces significantly phonon lifetimes, which is the main root cause for the ultralow κ₁ of FeSb₃.

Fig. 5d shows the vibration pattern of the lowest optical phonon mode of FeSb₃. It involves torsion of the Sb₄ ring, a typical quasi-molecular motif in skutterudites. The lowering in frequency of this optical mode in FeSb₃ originates from the weaker Sb-Sb bonds of the Sb₄ ring, as demonstrated by the charge density plot in Fig. 6. Clearly the electrons in the Sb₄ ring of FeSb₃ are less localized than the case of CoSb₃. This indicates the rather weaker Sb-Sb bonds in FeSb₃, as expected from its electron deficient nature. This is consistent with the fact that the Young modulus of FeSb₃ is smaller than that of CoSb₃.⁴⁰,⁴¹ As the result, the phonon modes that are mainly dominated by Sb atoms (below 6 THz) show general softening in FeSb₃ (Fig. 5e). The softening also occurs to the acoustic phonons, especially the transverse modes (Fig. 5b), which leads to the moderately reduced averaged phonon velocity of FeSb₃ in Fig. 3b. In addition to the phonon softening, the weaker Sb-Sb bonds in FeSb₃ results in abnormal Grüneisen parameters (γ) of phonons (Fig. 5a). For the low-lying optical branches and the transverse acoustic phonons, the values of γ are negative (in red) and quite large in magnitude. Such phonon modes with the large magnitude γ in principle facilitate high lattice anharmonicity⁵³ and thus low κ₁. The negative sign of γ for the low-lying optical branches implies that they will be further softened under contraction. This is expected to cause the more overlapping with acoustic phonons, more enhanced W⁺ and thus even lower κ₁ at high pressures.

For the filled skutterudites, the electrons of filler transfer to the host framework following the Zintl behavior⁵⁴–⁵⁶. Specifically, the electropositive fillers donate charge to the more covalent transition element – Sb backbone. These electrons primarily distribute on the Sb₄ ring, which considerably strengthens the Sb-Sb covalent bonds, as indicated in Fig. 6c for the LaFe₄Sb₁₂ case. This is consistent with the band structure of skutterudites, which shows a light Sb derived band at the top of the valence bands.⁵⁷ This may also explain why filled Fe-based skutterudites are easier to form than FeSb₃ usually stabilized in films.⁵⁷,⁵⁹,⁶⁰ If one considers only the fact that the strengthened Sb-Sb bonds after La filling lift the Sb-derived optical branches in LaFe₄Sb₁₂, an increase of κ₁ is expected. In fact, the La filler derived rattling modes take the role to remarkably enhance W⁺. As the result κ₁ of LaFe₄Sb₁₂ is still much lower than that of CoSb₃, and only about three times larger than that of FeSb₃. For the case of YbFe₄Sb₁₂, the Yb derived even lower frequency and rather flat optical phonon branches increase W⁺ more significantly, resulting in the more reduced κ₁.⁶⁴ Note that flat optical branches generally have large phase space for Umklapp scattering, but by themselves do not carry substantial heat regardless.
The origin of the ultralow $\kappa_l$ is attributed to the optical phonons with remarkable lowing in frequency associated with the weaker Sb-Sb bonds in FeSb$_3$ owing to its electron deficient nature. These low-frequency optical phonons take the role of the rattling modes in filled skutterudites. Particularly they overlap more with the heat-carrying acoustic phonons and increase the phase space of three-phonon anharmonic scattering processes, resulting in significantly reduced phonon lifetimes and thus ultralow $\kappa_l$. Our results offer new insight into the still debated mechanism responsible for the $\kappa_l$ reduction upon filling in skutterudites and also a new routine for lowering $\kappa_l$ of skutterudite-type thermoelectric materials. Finally we evaluate the maximum mean free path of phonons ($l_{ph}$) for bulk FeSb$_3$, as shown in Fig. 7. The purpose of such calculation is to provide a reference to future experimental measurements for how thick thin-film samples are required to take on the intrinsic $\kappa_l$ of bulk FeSb$_3$ we calculated here. According to our result, the maximum $l_{ph}$ is less than 600 nm. The sample size above this value is needed to safely avoid surface and grain boundary scatterings. Therefore the experimentally synthesized FeSb$_3$ films with the thicknesses of 1.0-1.5 $\mu$m$^{40}$ may be directly applied to measure the intrinsic $\kappa_l$ and verify our theoretical prediction.

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FIG. 7. (color online) The normalized cumulative $\kappa_l$ of FeSb$_3$ as a function of the phonon mean free path ($l_{ph}$) at 300 K. The maximum $l_{ph}$ among all the modes, which corresponds to the onset point of the cumulative $\kappa_l$ becoming constant (equal to 1.0), is indicated.

IV. CONCLUSIONS

To summarize, we report a discovery of ultralow $\kappa_l$ in pure skutterudite FeSb$_3$ with no filler by using first principles Boltzmann-Peierls transport simulations. The calculated $\kappa_l$ is only 1.14 W/mK at room temperature, one order of magnitude lower than that of CoSb$_3$. It is even lower than the $\kappa_l$ of most of fully filled skutterudites. This is in contrast to the widely used approach where filling is used for reduction of $\kappa_l$ in skutterudites.