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Exotic Quantum Critical Point on the surface of $3d$ Topological Insulator

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In the last few years a lot of exotic and anomalous topological phases were constructed by proliferating the vortex like topological defects on the surface of the $3d$ topological insulator (TI) [1–5]. In this work, rather than considering topological phases at the boundary, we will study quantum critical points driven by vortex like topological defects. In general we will discuss a $(2+1)d$ quantum phase transition described by the following field theory: $\mathcal{L} = \bar{\psi}\gamma_\mu(\partial_\mu - ia_\mu)\psi + |(\partial_\mu - ika_\mu)\phi|^2 + r|\phi|^2 + g|\phi|^4$, with tuning parameter r, arbitrary integer k, Dirac fermion ψ and complex scalar bosonic field ϕ which both couple to the same $(2+1)d$ dynamical noncompact U(1) gauge field a_μ. The physical meaning of these quantities/fields will be explained in the text. Making use of the new duality formalism developed in Ref. 6–8, we demonstrate that this quantum critical point has a quasi self-dual nature. And at this quantum critical point, various universal quantities such as the electrical conductivity, and scaling dimension of gauge invariant operators can be calculated systematically through a $1/k^2$ expansion, based on the observation that the limit $k \to +\infty$ corresponds to an ordinary $3d$ XY transition.

PACS numbers:

— *Introduction*

Although it is well-known that the boundary state of a noninteracting $3d$ topological insulator (TI) is described by one or odd number of free $(2+1)d$ Dirac fermions [9–11], curiosity drives theorists to look for all possible boundary states of $3d$ TI under strong interaction. It was demonstrated that under strong interaction the boundary of a $3d$ TI can have various topological orders that cannot be realized in a pure $2d$ system [1–5]. And the general procedure of obtaining these topological orders, is to first drive the boundary into the so-called Fu-Kane superconductor [12], then restore the U(1) symmetry by condensing a bosonic vortex of the superconductor, for example a vortex of 8 fold vorticity (or a vortex that would trap $8\frac{hc}{2e}$ flux once the fermion is coupled to the external electromagnetic field). In the condensate of the $8-$fold vortex, all symmetries of the system are preserved, the boundary remains gapped, but the ground state has topological order with nonabelian anyon excitations [1–5].

More recent theoretical exploration has concluded that the charge neutral $4-$fold vortex is a fermion, and it is doublet that transforms under time-reversal symmetry as $\mathcal{T} : \psi \to i\sigma^y\psi^\dagger$. This fermionic $4-$fold vortex provides a dual description of the boundary of $3d$ TI, which is a $(2+1)d$ quantum electrodynamics (QED$_3$) with $N = 1$ flavor of Dirac fermion:

$$\mathcal{L}_{dual} = \bar{\psi}\gamma_\mu(\partial_\mu - ia_\mu)\psi + \frac{1}{e^2}f_{\mu\nu}^2,$$
$$\gamma^0 = \sigma^y, \quad \gamma^1 = \sigma^x, \quad \gamma^2 = \sigma^z, \qquad (1)$$

where a_μ is the dual of the Goldstone mode of the Fu-Kane superconductor, and the flux quantum of a_μ carries half of the physical electric charge [6–8], thus a_μ is a noncompact gauge field. This duality is a fermionic version of the well-known duality between the $3d$ XY model and the bosonic QED [13, 14]. And based on this duality, recently it was demonstrated that QED$_3$ with $N = 2$ is self-dual [15], which is a fermionic analogue of the self-duality of the noncompact CP^1 theory with easy-plane anisotropy [16–18].

Ref. 6–8 demonstrated that the dual theory Eq. 1 is the parent state of many known strongly interacting boundary states of $3d$ TI, and these boundary states can also be constructed using the original physical Dirac fermion (electron). It is tempting to claim that Eq. 1 is exactly dual to the free Dirac fermion (or weakly interacting Dirac fermion), which is a very simple $(2+1)d$ conformal field theory (CFT). Recently a coupled wire construction of the duality further supports this idea [19]. In this paper we will assume this duality is exact: namely Eq. 1 is indeed a CFT in the infrared that is dual to the noninteracting $(2+1)d$ Dirac fermion, and we will use this assumption to explore other possible behaviors of the boundary.

The goal of this paper is to study the quantum phase transition described by the following field theory:

$$\mathcal{L} = \bar{\psi}\gamma_\mu(\partial_\mu - ia_\mu)\psi + \frac{1}{e^2}f_{\mu\nu}^2$$
$$+ |(\partial_\mu - ika_\mu)\phi|^2 + r|\phi|^2 + g|\phi|^4, \qquad (2)$$

with tuning parameter r, arbitrary integer k, Dirac fermion ψ and complex scalar bosonic field ϕ which both couple to the same $(2+1)d$ dynamical U(1) gauge field a_μ. The boson ϕ can be viewed as the $4k$ fold vortex of the Fu-Kane superconductor *bound* with another extra degree of freedom (d.o.f). For even integer k, ϕ is the bound state of $4k$ vortex and an extra boson; while if k is odd, ϕ must contain an extra fermion that transforms in the same way as ψ under $\mathcal{T}$, but neutral under the dynamical gauge field a_μ. The tuning parameter r can be tuned by the mass gap of this extra d.o.f.

Obviously this theory has two phases: when r is sufficiently large, ϕ is gapped, and based on our assumption the boundary is described by Eq. 1, and it is dual to a noninteracting Dirac fermion; while when r is negative and large, ϕ condenses, and it drives the boundary into a topological order with *gapless* Dirac fermion ψ. We are interested in the quantum phase transition between these two phases. Notice that when ϕ condenses, ψ is not automatically gapped, *i.e.* there is no Yukawa type of coupling such as $\phi^* \psi^t \gamma^0 \psi$ in the Lagrangian, which is forbidden by the gauge symmetry for $k \neq 2$. It is easy to show that there is no other obviously relevant couplings in Eq. 2 allowed by the gauge symmetry.

Our goal is to calculate the scaling dimension of gauge invariant order parameters and other universal quantities at the quantum critical point $r = 0$. Let us take the limit $k \to +\infty$ first. In this limit, the gauge field dynamics is completely dominated by its coupling to the scalar field, and the fermions will effectively decouple from the gauge field. More precisely, the fermion decouples from the gauge field at the energy scale below $k^2 e^2$. This effect becomes explicit after we rescale $k a_\mu = \tilde{a}_\mu$. In this case the theory becomes a standard bosonic QED with gauge field $\tilde{a}_\mu$, and it is well-known that this theory is dual to a $3d$ XY transition [13, 14]. We assume that we know *everything* about the $3d$ XY transition, including all of its critical exponents, the scaling dimension of all the composite operators, the operator product expansion, and most importantly, the universal boson conductivity $\tilde{\sigma}$ [20, 21], which we will take as a dimensionless constant, assuming the boson carries charge-1. All these information can be obtained by numerically studying the $3d$ XY transition only. For example, numerically the critical exponent ν has been confirmed to be very close to (slightly larger than) 2/3 [22]. The universal conductivity of the $3d$ XY transition has also been studied with various methods [23–26]. Recent progresses based on conformal bootstrap have determined the value of $\tilde{\sigma}$ very precisely [27], which is highly consistent with the numerical results [25, 26]

— *Scaling dimension of $\mathcal{T}$-breaking order parameter*

Time-reversal symmetry $\mathcal{T}$ is the key symmetry that protects the $3d$ TI. Let us compute the scaling dimension of the time-reversal symmetry breaking order parameter $\bar{\psi}\psi = \psi^\dagger \gamma^0 \psi$. In the large$-k$ limit, because ψ basically decouples from the gauge field (as we argued above), the scaling dimension of $\bar{\psi}\psi$ is the same as that of the free fermion $\Delta[\bar{\psi}\psi] = 2$. The correction to this scaling dimension comes from the gauge fluctuation a_μ, thus we need to know the photon propagator $G^a_{\mu\nu}$ in the large$-k$ limit.

In the large$-k$ limit, since this quantum phase transition belongs to the $3d$ XY universality class, we assume that the universal conductivity of the boson degrees of freedom which carries the global U(1) symmetry of the $3d$ XY transition is a known dimensionless constant $\tilde{\sigma}$. We know that in the momentum-frequency space of the Euclidean space-time, the Kubo formula gives us the following relation between the correlation function of the boson current $J^\mu(p)$ and the universal conductivity $\tilde{\sigma}$:

$$\langle J_\mu(p) \ J_\nu(-p) \rangle = \tilde{\sigma} |p| \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right). \quad (3)$$

Then because the boson current $J_\mu = \frac{k}{2\pi} \epsilon_{\mu\nu\rho} \partial_\nu a_\rho$, the photon propagator at the quantum critical point in the large$-k$ limit reads

$$G^a_{\mu\nu}(p) = \frac{\tilde{\sigma}(2\pi)^2}{k^2 |p|} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right). \quad (4)$$

Or in other words Eq. 2 reduces to a bosonic QED in the large$-k$ limit, and Eq. 4 describes the fully dressed gauge field propagator. Throughout the paper we will choose the gauge $\partial_\mu a_\mu = 0$.

The rest of the calculation is pretty standard: because the photon propagator carries a factor $1/k^2$, a systematic expansion controlled by small factor $1/k^2$ can be carried out. By combining the vertex correction and the wave function renormalization together, the scaling dimension of $\bar{\psi}\psi$ at the $1/k^2$ order reads

$$\Delta[\bar{\psi}\psi] = 2 - \frac{16\tilde{\sigma}}{3k^2}. \quad (5)$$

A similar calculation of scaling dimension of fermion bilinear operators of the standard QED$_3$ with large$-N$ flavors of fermions can be found in Ref. 28–30. But let us stress that in our case we only have one flavor of fermion and boson field each.

— *Scaling dimension of four-fermion interaction term*

A weak short range four-fermion interaction would be irrelevant for a $(2+1)d$ gapless Dirac fermion. However, gauge fluctuation potentially could change the scaling dimension of the four-fermion interactions, and make them relevant. In our system Eq. 2, because there is only one flavor of Dirac fermion ψ, there is only one allowed four fermion interaction term without spatial derivatives:

$$g(\bar{\psi}\psi)^2 = -\frac{1}{3} g(\bar{\psi}\gamma_\mu\psi)^2. \quad (6)$$

The scaling dimension of this four fermion interaction term can again be calculated with a $1/k^2$ expansion. All the Feynman diagrams that contribute at this $1/k^2$ order are listed in Fig. 1. The final result is

$$\Delta[(\bar{\psi}\psi)^2] = 4 + \frac{16\tilde{\sigma}}{3k^2}. \quad (7)$$

Thus the gauge fluctuation makes the four-fermion interaction term even more irrelevant than it is at the free Dirac fermion CFT. This calculation supports that Eq. 2

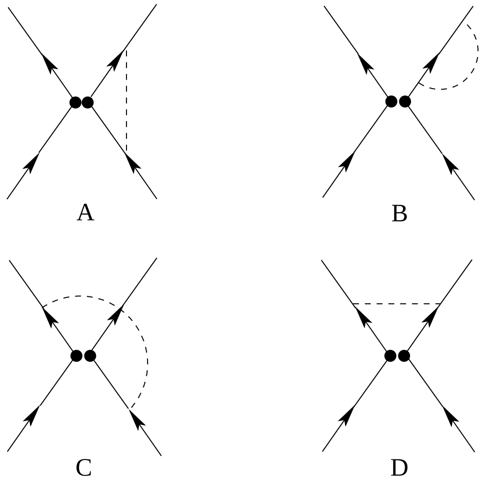

FIG. 1: The Feynman diagrams that will contribute to the the scaling dimension of the four fermion interaction term at the $1/k^2$ order.

describes a continuous quantum phase transition, since the four fermion interaction is likely not rendered relevant by gauge fluctuation for any k at the quantum critical point $r = 0$.

— *Universal electrical conductivity*

As was pointed out in Ref. 6–8, in Eq. 2, a 2π flux quantum of a_μ carries half electric charge. Thus the physical electric current density at the $2d$ surface reads $J^e_\mu = \frac{1}{4\pi}\epsilon_{\mu\nu\rho}\partial_\nu a_\rho$. The electrical conductivity σ^e is encoded in the Euclidean space-time correlation function of the current operator:

$$\langle J^e_\mu(p)\ J^e_\nu(-p)\rangle = \frac{\sigma^e}{e^2/\hbar}|p|\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right). \qquad (8)$$

When ϕ is gapped ($r > 0$), the system is described by QED with $N = 1$ flavor of Dirac fermion ψ, which by our assumption is dual to a noninteracting Dirac fermion which is not coupled to any dynamical gauge field. Thus this phase with $r > 0$ is a semimetal with universal electrical conductivity $\sigma^e = \frac{1}{16}\frac{e^2}{\hbar}$. The quantum phase transition we are studying is a transition from an electrical semimetal to an electrical insulator, although the insulator phase is also gapless.

Right at the quantum critical point, the electrical conductivity must be a different universal value. Because we already know that in the large$-k$ limit the photon a_μ propagator is given by Eq. 4, using the photon propagator, we can compute the physical electric current-current correlation function:

$$\begin{aligned}\langle J^e_\mu(p)\ J^e_\nu(-p)\rangle &= \frac{1}{(4\pi)^2}p^2 G^a_{\mu\nu}(p) \\ &= \frac{\tilde{\sigma}}{4k^2}|p|\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right). \qquad (9)\end{aligned}$$

Comparing with Eq. 8, we conclude that in the large$-k$ limit the universal electrical conductivity at the quantum critical point reads

$$\sigma^e = \frac{\tilde{\sigma}}{4k^2}\frac{e^2}{\hbar}. \qquad (10)$$

The leading correction to this value must be at the $1/k^4$ order, which comes from the correction to the a_μ propagator from the Dirac fermion ψ.

If the time-reversal symmetry $\mathcal{T}$ is broken at the $2d$ boundary, *i.e.* the system develops a nonzero expectation value of $\bar{\psi}\psi$, the Hall conductivity at the quantum critical point $r = 0$ will also be at order $\sim \frac{1}{k^4}\frac{e^2}{\hbar}$.

— *Self-duality*

In this subsection we will see that Eq. 2 has a (quasi-)self-dual structure. The duality transformation of the second line of Eq. 2 is rather standard, it is simply the particle-vortex duality:

$$\mathcal{L}_b = |(\partial_\mu - ib^{(1)}_\mu)\Phi|^2 + \tilde{r}|\Phi|^2 + \tilde{g}|\Phi|^4 + \frac{ik}{2\pi}a \wedge db^{(1)}, \qquad (11)$$

where Φ can be viewed as the unit vortex field of ϕ, and it is bound with $2\pi k$ flux of a_μ, because ϕ carries charge$-k$ under a_μ. The duality of the first line of Eq. 2 requires the newly developed (hypothesized) duality in Ref. 6–8:

$$\mathcal{L}_f = \bar{\chi}\gamma_\mu(\partial_\mu - ib^{(2)}_\mu)\chi + \frac{i}{4\pi}a \wedge db^{(2)}, \qquad (12)$$

where now χ transforms under time-reversal as $\mathcal{T} : \chi \to i\sigma^y\chi$. If $\mathcal{L}_b$ in Eq. 11 is ignored, integrating out a_μ in $\mathcal{L}_f$ will gap out $b^{(2)}_\mu$, thus $\mathcal{L}_f$ only has a free Dirac fermion χ in the infrared, which corresponds to the case studied in Ref. 6–8

In our case, due to the existence of the bosonic matter field, integrating out a_μ induces the following constraint:

$$b^{(2)}_\mu = -2kb^{(1)}_\mu = -2kb_\mu. \qquad (13)$$

Thus the final dual theory reads

$$\begin{aligned}\mathcal{L}_{dual} &= \bar{\chi}\gamma_\mu(\partial_\mu + i2kb_\mu)\chi + \cdots \\ &+ |(\partial_\mu - ib_\mu)\Phi|^2 + \tilde{r}|\Phi|^2 + \tilde{g}|\Phi|^4. \qquad (14)\end{aligned}$$

Here $\tilde{r} \sim -r$: when $\tilde{r} < 0$, Φ is condensed, which is dual to the disordered phase of ϕ, and low energy physics of this phase is either described by a QED with $N = 1$ flavor of fermion ψ, or a single gapless Dirac fermion χ; When $\tilde{r} > 0$, Φ is disordered, and the low energy physics of this phase is described by either a QED with $N = 1$ flavor of fermion χ, or a single gapless Dirac fermion ψ (which is coupled to a gapped discrete gauge field). The dual theory Eq. 14 is very similar to the original theory Eq. 2, the only difference is that now it is the fermionic degree of freedom that carries a large gauge charge.

Again, in the large$-k$ limit, Lagrangian Eq. 14 describes a $3d$ XY transition, because after rescaling $kb_\mu = \tilde{b}_\mu$, Φ is effectively neutral under $\tilde{b}_\mu$ in the large$-k$ limit.

Again, in the large$-k$ limit, the propagator of gauge field b_μ can be calculated exactly, based on the observation that the fermion current $J^\psi_\mu = \bar{\psi}\gamma_\mu\psi = \frac{1}{4\pi}\epsilon_{\mu\nu\rho}\partial_\nu b^{(2)}_\mu = -\frac{k}{2\pi}\epsilon_{\mu\nu\rho}\partial_\nu b_\mu$. In the large$-k$ limit the correlation function of J^ψ_μ can be computed exactly because in this limit ψ decouples from a_μ, and the correlation function of J^ψ in this limit is well-known:

$$\langle J^\psi_\mu(p)\ J^\psi_\nu(-p)\rangle = \frac{1}{16}|p|\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right), \tag{15}$$

this implies that photon b_μ propagator in the large$-k$ limit reads

$$G^b_{\mu\nu} = \frac{\pi^2}{4k^2|p|}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right). \tag{16}$$

In this dual theory, operator $\bar{\chi}\chi$ breaks time-reversal symmetry, and hence it can be identified as $\bar{\psi}\psi$ in the original theory Eq. 2 [19]. Thus the scaling dimension of $\bar{\chi}\chi$ is also

$$\Delta[\bar{\chi}\chi] = 2 - \frac{16\tilde{\sigma}}{3k^2}. \tag{17}$$

— *Critical exponent*

We would also like to calculate the scaling dimension of the tuning parameter r in Eq. 2, which is identified as $\tilde{r}$ in the dual theory, thus the composite operator $|\phi|^2$ is equivalent to $|\Phi|^2$.

To calculate the scaling dimension of $\tilde{r}$, one strategy is to expand Eq. 14 at the Gaussian fixed point of Φ and perform a combined $\epsilon = 4 - D$ and $1/k^2$ expansion. Although this calculation is straightforward, we hope to expand everything at the $3d$ XY fixed point (which we assume to know everything about) in the large$-k$ limit. In order to carry out the renormalization group (RG) calculation, we make use of the operator product expansion (OPE) in the momentum space:

$$\begin{aligned} &\left(\frac{1}{2}\tilde{r}|\Phi|^2 J^\Phi_\mu(\vec{p})J^\Phi_\nu(-\vec{p})\right)G^b_{\mu\nu}(\vec{p}) \\ \sim\ &\left(\frac{1}{2}\tilde{r}|\Phi|^2\frac{C}{p^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\right)G^b_{\mu\nu}(\vec{p}) \\ =\ &\left(\tilde{r}|\Phi|^2\frac{C}{|p|^3}\right)\frac{\pi^2}{4k^2}. \end{aligned} \tag{18}$$

$J^\Phi_\mu(\vec{p})$ is the current operator of field Φ in Eq. 14. The meaning of this OPE is that, when the momentum $\vec{p}$ of J^Φ_μ and the photon propagator is much larger than the momentum of $|\Phi|^2$, the correlation function between the composite operator $|\Phi|^2 J^\Phi_\mu(\vec{p})J^\Phi_\nu(-\vec{p})$ and another operator can be approximated by the correlation between $|\Phi|^2\frac{C}{|p|^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$ and that operator. We have checked this OPE by comparing the two Feynman diagrams in Fig. 2, and the correlation

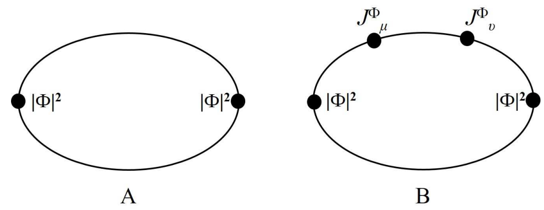

FIG. 2: The diagram on the right hand side is the correlation function $\langle|\Phi^2|_{\vec{q}} J^\Phi_\mu(\vec{p})J^\Phi_\nu(-\vec{p})|\Phi^2|_{-\vec{q}}\rangle$. When $|\vec{p}| \gg |\vec{q}|$ it scales as the left hand side correlation function times factor $\frac{1}{|p|^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$.

function $\langle|\Phi^2|_{\vec{q}} J^\Phi_\mu(\vec{p})J^\Phi_\nu(-\vec{p})|\Phi^2|_{-\vec{q}}\rangle$ indeed scales as $\sim \langle|\Phi^2|_{\vec{q}}\ |\Phi^2|_{-\vec{q}}\rangle\frac{1}{|p|^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$ when $|\vec{p}| \gg |\vec{q}|$.

In this OPE, the dimensionless number C only depends on the $3d$ XY universality class, and as we stated we assume that it can be determined by studying the OPE of the $3d$ XY transition only, through for instance the $1/N$ expansion as in Ref. 31. Although the dimensionless number C is yet to determine, the $1/|\vec{p}|^2$ scaling of this OPE is known, because the scaling dimension of the boson current J^Φ_μ is $\Delta[J^\Phi_\mu] = 2$, which at the $3d$ XY fixed point is unrenormalized compared with the free boson theory because it is a conserved current.

After the standard momentum shell RG calculation, *i.e.* integrating out the degrees of freedom with momentum $\vec{p}$ between $b\Lambda < |p| < \Lambda$, the OPE above will contribute a correction to $\tilde{r}|\Phi|^2$ that is proportional to $\ln(1/b)$. Now we can conclude that the RG equation for $\tilde{r}$ to the $1/k^2$ order reads

$$\frac{d\tilde{r}}{d\ln(1/b)} = \left(\Delta_{xy} + \frac{C}{8k^2}\right)\tilde{r}, \tag{19}$$

which determines the scaling dimension of $\tilde{r}$. Here Δ_{xy} is the scaling dimension of $\tilde{r}$ at the $3d$ XY universality class, which is very close to $3/2$ [22].

— *Summary*

In this work we did our best to study the the quantum phase transition described in Eq. 2, with its dual Lagrangian described by Eq. 14. The self-dual nature of this transition allows us to calculate many quantities in a controlled expansion with $1/k^2$. But it is possible that, with small enough k, the transition becomes first order.

The same techniques used in this work can be applied to other field theories as well. For instance QED_3 with two flavors of Dirac fermions, and one flavor of fermion carries gauge charge-1, while the other flavor of fermion carries a much larger gauge charge$-k$. A similar $1/k^2$ expansion can also be applied to this theory as well.

Ref. 32 has applied the mirror symmetry [33–35] (duality between supersymmetric field theories) to the half-filled Landau level [36], which is a system closely related to the boundary of $3d$ TI [6–8]. Previous study [37] also indicates that the mirror symmetry is related to the "deconfined QCP" [17, 18]. We suspect the QCP discussed in this paper may also have an interesting supersymmetric version. We will leave this to future study.

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