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# Theory of Magnon Motive Force in Chiral Ferromagnets

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We predict that magnon motive force can lead to temperature dependent, nonlinear chiral damping in both conducting and insulating ferromagnets. We estimate that this damping can significantly influence the motion of skyrmions and domain walls at finite temperatures. We also find that in systems with low Gilbert damping moving chiral magnetic textures and resulting magnon motive forces can induce large spin and energy currents in the transverse direction.

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Emergent electromagnetism in the context of spintronics [1] brings about interpretations of the spin-transfer torque [2, 3] and spin-motive force (SMF) [4–10] in terms of fictitious electromagnetic fields. In addition to providing beautiful interpretations, these concepts are also very useful in developing the fundamental understanding of magnetization dynamics. A time-dependent magnetic texture is known to induce an emergent gauge field on electrons [5]. As it turns out, the spin current generated by the resulting fictitious Lorentz force (which can also be interpreted as dynamics of Berry-phase leading to SMF) influences the magnetization dynamics in a dissipative way [5, 6, 11–16], affecting the phenomenological Gilbert damping term in the Landau-Lifshitz-Gilbert (LLG) [17] equation. Inadequacy of the simple Gilbert damping term has recently been seen experimentally in domain wall creep motion [18]. Potential applications of such studies include control of magnetic solitons such as domain walls and skyrmions [19–31], which may lead to faster magnetic memory and data storage devices with lower power requirements [32–34]. Recently, phenomena related to spin currents and magnetization dynamics have also been studied in the context of energy harvesting and cooling applications within the field of spin caloritronics [35–39].

Magnons, the quantized spin-waves in a magnet, are present in both conducting magnets and insulating magnets. Treatment of spin-waves with short wavelengths as quasiparticles allows us to draw analogies from systems with charge carriers. For instance, the flow of thermal magnons generates a spin transfer torque (STT) [40–42] and a time-dependent magnetic texture exerts a magnon motive force. According to the Schrödinger-like equation which governs the dynamics of magnons in the adiabatic limit [41], the emergent “electric” field induced by the time-dependent background magnetic texture exerts a “Lorentz force” on magnons, which in turn generates a current by “Ohm’s law” (see Fig. 1). Despite the similarities, however, the strength of this feedback current has important differences from its electronic analog: it is inversely proportional to the Gilbert damping and grows with temperature.

In this paper, we formulate a theory of magnon feedback damping induced by the magnon motive force. We find that this additional damping strongly affects the dynamics of magnetic solitons, such as domain walls and skyrmions, in systems with strong Dzyaloshinskii-Moriya interactions (DMI). We also find that the magnon motive force can lead to magnon accumulation (see Fig. 1), non-vanishing magnon chemical potential, and large spin and energy currents in systems with low Gilbert damping. To demonstrate this, we assume diffusive transport of magnons in which the magnon non-conserving relaxation time,  $\tau_\alpha$ , is larger compared to the magnon conserving one,  $\tau_m$  ( $\tau_\alpha \gg \tau_m$ ). For the four-magnon thermalization,  $\tau_m = \hbar/(k_B T)(T_c/T)^3$ , and for LLG damping,  $\tau_\alpha = \hbar/\alpha k_B T$ , this leads to the constraint  $\alpha(T_c/T)^3 \ll 1$  [43, 44].

*Emergent electromagnetism for magnons.* We initially assume that the magnon chemical potential is zero. The validity of this assumption is confirmed in the last section. Effects related to emergent electromagnetism for magnons can be captured by considering a ferromagnet well below the Curie temperature. We use the stochastic LLG equation:

$$s(1 + \alpha \mathbf{n} \times) \dot{\mathbf{n}} = \mathbf{n} \times (\mathbf{H}_{\text{eff}} + \mathbf{h}), \quad (1)$$

where  $s$  is the spin density along  $\mathbf{n}$ ,  $\mathbf{H}_{\text{eff}} = -\delta_{\mathbf{n}} F[\mathbf{n}]$  is the effective magnetic field,  $F[\mathbf{n}] = \int d^3 r \mathcal{F}(\mathbf{n})$  is the free energy and  $\mathbf{h}$  is the random Langevin field. It is convenient to consider the free energy density  $\mathcal{F}(\mathbf{n}) = J(\partial_i \mathbf{n})^2/2 + \hat{D} \mathbf{e}_i \cdot (\mathbf{n} \times \partial_i \mathbf{n}) + \mathbf{H} \cdot \mathbf{n} + K_u n_z^2$  where  $J$  is the exchange coupling,  $\hat{D}$  is a tensor which describes the DMI [47],  $\mathbf{H} = M_s \mu_0 H_a \mathbf{e}_z$  describes the magnetic field,  $K_u$  denotes the strength of uniaxial anisotropy,  $M_s$  is the saturation magnetization,  $H_a$  is the applied magnetic field, and summation over repeated indices is implied. At sufficiently high temperatures, the form of anisotropies is unimportant for the discussion of thermal magnons and can include additional magnetostatic and magnetocrystalline contributions.

Linearized dynamics of magnons can be captured by

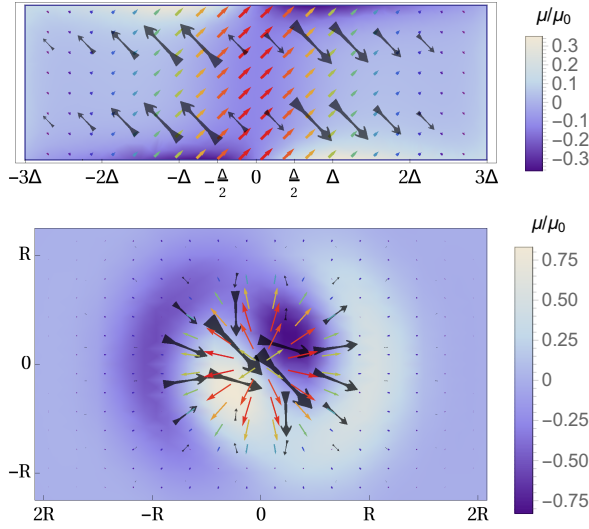


FIG. 1. (Color online) A moving magnetic texture, such as a domain wall (top) or an isolated skyrmion (bottom), generates an emergent electric field and accumulates a cloud of magnons around it. In-plane component of  $\mathbf{n}$  and electric field  $\mathcal{E}$  are represented by small colored arrows and large black arrows, respectively. Magnon chemical potential  $\mu$  is measured in  $\mu_0 = \xi \hbar v D / J \Delta$  for domain wall and  $\mu_0 = \xi \hbar v D / J R$  for skyrmion, where  $\xi$  is the magnon diffusion length. Solitons are moving along  $+x$  axis with velocity  $v$ ,  $n_z = \pm 1$  at  $x = \mp \infty$  for domain wall and  $n_z = 1$  at the center for the isolated hedgehog skyrmion. Material parameters for Co/Pt ( $J = 16 \text{ pJ/m}$ ,  $D = 4 \text{ mJ/m}^2$ ,  $M_s = 1.1 \text{ MA/m}$ ,  $\alpha = 0.03$ , at room temperature [45]) were used for domain wall leading to  $\Delta \approx 7 \text{ nm}$ , and  $\text{Cu}_2\text{OSeO}_3$  parameters ( $J = 1.4 \text{ pJ/m}$ ,  $D = 0.17 \text{ mJ/m}^2$ ,  $s = 0.5 \hbar / a^3$ ,  $a = 0.5 \text{ nm}$  with  $\alpha = 0.01$ , at  $T \sim 50 \text{ K}$  [46]) for skyrmion leading to  $R \approx 50 \text{ nm}$ . System size is taken to be  $6\Delta \times 2\Delta$  for domain wall and  $6R \times 6R$  for skyrmion.

the following equation [48]:

$$s(i\partial_t + \mathbf{n}_s \cdot \mathbf{A}_t)\psi = [J(\partial_i/i - \mathbf{n}_s \cdot [\mathbf{A}_i - \mathbf{D}_i/J])^2 + \varphi]\psi, \quad (2)$$

where  $\varphi$  absorbs effect of anisotropies, DMI and the magnetic field,  $\psi = \mathbf{n}_f \cdot (\mathbf{e}'_x + i\mathbf{e}'_y)$  describes fluctuations  $\mathbf{n}_f = \mathbf{n} - \mathbf{n}_s \sqrt{1 - \mathbf{n}_f^2}$  around slow component  $\mathbf{n}_s$  ( $|\mathbf{n}| = |\mathbf{n}_s| = 1$ ,  $\mathbf{n}_s \perp \mathbf{n}_f$ ) in a rotated frame in which  $\mathbf{e}'_z = \mathbf{n}_s$ ,  $\mathbf{D}_i = \hat{D}\mathbf{e}_i$ , and  $\mathbf{A}_\mu \times \equiv \hat{R}\partial_\mu \hat{R}^T$  corresponds to the gauge potential with  $\mu = x, y, z, t$ . Note that in the rotated frame, we have  $\mathbf{n} \rightarrow \mathbf{n}' = \hat{R}\mathbf{n}$  and  $\partial_\mu \rightarrow (\partial_\mu - \mathbf{A}'_\mu \times)$  with  $\mathbf{A}'_\mu \times = (\partial_\mu \hat{R})\hat{R}^T$ . In deriving Eq. 2, we assumed that the exchange interaction is the dominant contribution and neglected the coupling between the circular components of  $\psi$  and  $\psi^\dagger$  due to anisotropies [41, 49].

The gauge potential in Eq. (2) leads to a reactive torque in the LLG equation for the slow dynamics [50]. Alternatively, one can simply average Eq. (1) over the fast oscillations arriving at the LLG equation with the

magnon torque term [40]:

$$s(1 + \alpha \mathbf{n}_s \times) \dot{\mathbf{n}}_s - \mathbf{n}_s \times \mathbf{H}_{\text{eff}}^s = \hbar(\mathbf{j} \cdot \mathbf{D})\mathbf{n}_s, \quad (3)$$

where  $\mathbf{H}_{\text{eff}}^s = -\delta \mathbf{n}_s F[\mathbf{n}_s]$  is the effective field for the slow magnetization calculated at zero temperature [51],  $j_i = (J/\hbar) \langle \mathbf{n}_s \cdot (\mathbf{n}_f \times \partial_i \mathbf{n}_f) \rangle$  is the magnon current and  $\mathcal{D}_i = \partial_i + (\hat{D}\mathbf{e}_i/J) \times$  is the chiral derivative [48, 52, 53].

*Magnon feedback damping.* The magnon current  $\mathbf{j}$  is induced in response to the emergent electromagnetic potential, and can be related to the driving electric field  $\mathcal{E}_i = \hbar \mathbf{n}_s \cdot (\partial_i \mathbf{n}_s \times \mathcal{D}_i \mathbf{n}_s)$  by local Ohm's law  $\mathbf{j} = \sigma \mathcal{E}$  where  $\sigma$  is magnon conductivity. The induced electric field  $\mathcal{E}$  can be interpreted as a magnon generalization of the spin motive force [6]. The magnon feedback torque  $\boldsymbol{\tau} = \hbar \sigma (\mathcal{E} \cdot \mathbf{D})\mathbf{n}_s$  has dissipative effect on magnetization dynamics and leads to a damping tensor  $\hat{\alpha}_{\text{emf}} = \eta(\mathbf{n}_s \times \mathcal{D}_i \mathbf{n}_s) \otimes (\mathbf{n}_s \times \mathcal{D}_i \mathbf{n}_s)$  in the LLG equation with  $\eta = \hbar^2 \sigma / s$  [54].

A general form of the feedback damping should also include the contribution from the dissipative torque [40]. Here we introduce such  $\beta$ -terms phenomenologically which leads to the LLG equation:

$$s(1 + \mathbf{n}_s \times [\hat{\alpha} + \hat{\Gamma}]) \dot{\mathbf{n}}_s - \mathbf{n}_s \times \mathbf{H}_{\text{eff}}^s = \boldsymbol{\tau}, \quad (4)$$

where  $\boldsymbol{\tau}$  is the magnon torque term and we separated the dissipative  $\hat{\alpha}$  and reactive  $\hat{\Gamma}$  contributions:

$$\hat{\alpha} = \alpha + \hat{\alpha}_{\text{emf}} - \eta \beta^2 \mathcal{D}_i \mathbf{n}_s \otimes \mathcal{D}_i \mathbf{n}_s, \quad (5)$$

$$\hat{\Gamma} = \eta \beta [(\mathbf{n}_s \times \mathcal{D}_i \mathbf{n}_s) \otimes \mathcal{D}_i \mathbf{n}_s - \mathcal{D}_i \mathbf{n}_s \otimes (\mathbf{n}_s \times \mathcal{D}_i \mathbf{n}_s)],$$

where in general the form of chiral derivatives in the  $\beta$ -terms can be different. Given that  $\beta$  and  $\alpha$  are typically small for magnon systems, the term  $\hat{\alpha}_{\text{emf}}$  will dominate the feedback damping tensor. An unusual feature of the chiral part of the damping is that it will be present even for a uniform texture. While the DMI prefers twisted magnetic structures, this can be relevant in the presence of an external magnetic field strong enough to drive the system into the ferromagnetic phase.

In conducting ferromagnets, charge currents also lead to a damping tensor of the same form where the strength of the damping is characterized by  $\eta_e = \hbar^2 \sigma_e / 4e^2 s$  with  $\sigma_e$  as the electronic conductivity [11, 13, 15], which should be compared to  $\eta$  in conducting ferromagnets where both effects are present. Since the magnon feedback damping  $\eta$  grows as  $\propto 1/\alpha$ , the overall strength of magnon contribution can quickly become dominant contribution in ferromagnets with small Gilbert damping. Under the assumption that magnon scattering is dominated by the Gilbert damping such that the relaxation time is given by  $\tau_\alpha = 1/2\alpha\omega$ , magnon conductivity is given by  $\sigma_{3D} \sim 1/6\pi^2 \lambda \hbar \alpha$  in three-dimensions and  $\sigma_{2D} \sim 1/4\pi \hbar \alpha$  in two-dimensions [40] where  $\lambda = \sqrt{\hbar J / s k_B T}$  is the wavelength of the thermal magnons. For  $\text{Cu}_2\text{OSeO}_3$  in ferromagnetic phase, we find  $\eta \approx 2 \text{ nm}^2$ . Similarly,

for a Pt/Co/AlO<sub>x</sub> thin film of thickness  $t = 0.6\text{nm}$  yield  $\eta \approx 1\text{nm}^2$  at room temperature. This shows that the magnon feedback damping can become significant in ferromagnets with sharp textures and strong DMI.

**Domain wall dynamics.** We describe the domain wall profile in a ferromagnet with DMI by Walker ansatz  $\tan(\theta(x,t)/2) = \exp(\pm[x - X(t)]/\Delta)$  where  $X(t)$  and  $\phi(t)$  denote the center position and tilting angle of the domain wall [55],  $\Delta = \sqrt{J/K_0}$  is the domain wall width,  $K_0 = K_u - \mu_0 M_s^2/2$  includes the contributions from uniaxial anisotropy as well as the demagnetizing field and  $\hat{D} = D(\sin\gamma\mathbf{1} + \cos\gamma\mathbf{e}_z \times)$  contains DMI due to bulk and structure inversion asymmetries whose relative strength is determined by  $\gamma$ . After integrating the LLG equation, we obtain the equations of motion for a domain wall driven by external perpendicular field [56]:

$$\Gamma_{XX}\dot{X}/\Delta + \dot{\phi} = F_X, \quad \Gamma_{\phi\phi}\dot{\phi} - \dot{X}/\Delta = F_\phi, \quad (6)$$

where  $\Gamma_{XX} = \alpha + \eta(D/J)^2 \sin^2(\gamma + \phi)/3$  and  $\Gamma_{\phi\phi} = \alpha + \eta[2/3\Delta^2 + (\pi D/2J\Delta) \cos(\gamma + \phi) + (D/J)^2 \cos^2(\gamma + \phi)]$  are dimensionless angle-dependent drag coefficients,  $F_X = H/s$  and  $F_\phi = [K \sin 2\phi - \sin(\gamma + \phi)D\pi/2\Delta]/s$  are generalized “forces” associated with the collective coordinates  $X$  and  $\phi$ ,  $K$  is the strength of an added anisotropy corresponding, e.g., to magnetostatic anisotropy  $K = N_x \mu_0 M_s^2/2$  where  $N_x$  is the demagnetization coefficient. In deriving these equations, we have neglected higher order terms in  $\alpha$  and  $\beta$  [57].

Time-averaged domain wall velocity obtained from numerical integration of the equations of motion for a Co/Pt interface with Rashba-like DMI is shown in Fig. 2. Thermal magnon wavelength at room temperature ( $\approx 0.3\text{nm}$ ) is much shorter than the domain wall size  $\Delta = \sqrt{J/K_0} \approx 7\text{nm}$ , so the quasiparticle treatment of magnons is justified. We observe that damping reduces the speed at fixed magnetic field, and this effect is enhanced with increasing DMI strength  $D$  and diminishing the Gilbert damping  $\alpha$  (see Fig. 2).

Another important observation is that in the presence of the feedback damping, the relation between applied field and average domain wall velocity becomes nonlinear. This is readily seen from steady state solution of the equations of motion before the Walker breakdown with  $\phi = \phi_0$  which solves  $\sin(\gamma + \phi_0)D\pi/2s\Delta = [H/s][\alpha + \eta(D/J)^2 \sin^2(\gamma + \phi_0)/3]^{-1}$  (noting that  $D/\Delta \gg K$ , implying a Néel domain wall [56, 58]) and  $X = vt$ , leading to the cubic velocity-field relation  $(sv/\Delta)(\alpha + \eta[2sv/J\pi]^2/3) = H$  for field-driven domain wall motion. The angle  $\phi_0$  also determines the tilting of  $\mathcal{E}$  as seen in Fig. 1.

**Skyrmion dynamics.** Under the assumption that the skyrmion retains its internal structure as it moves, we treat it as a magnetic texture  $\mathbf{n}_s = \mathbf{n}_s(\mathbf{r} - \mathbf{q}(t))$  with  $\mathbf{q}(t)$  being the time-dependent position (collective coordinate [61]) of the skyrmion. We consider the motion

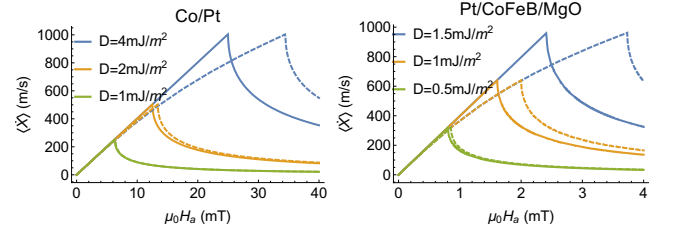


FIG. 2. (Color online) Domain wall velocity as a function of the magnetic field and varying strength of DMI for Co/Pt and Pt/CoFeB/MgO films. Solid (dashed) lines correspond to dynamics at zero (room) temperature. We used material parameters  $M_s = 1.1\text{MA/m}$ ,  $J = 16\text{pJ/m}$ ,  $K_0 = 0.34\text{MJ/m}^3$ ,  $\alpha = 0.03$  [45] for Co/Pt, and  $M_s = 0.43\text{MA/m}$ ,  $J = 31\text{pJ/m}$ ,  $K_0 = 0.38\text{MJ/m}^3$ ,  $\alpha = 4 \times 10^{-3}$  [31, 59, 60] for Pt/CoFeB/MgO.

of a skyrmion under the temperature gradient  $\nabla\chi = -\nabla T/T$ , which exerts a magnon torque:

$$\boldsymbol{\tau} = (1 + \beta_T \mathbf{m} \times)(L \nabla\chi \cdot \mathbf{D})\mathbf{m}, \quad (7)$$

where  $L$  is the spin Seebeck coefficient and  $\beta_T$  is the “ $\beta$ -type” correction. These are given by  $L_{3D} \sim k_B T/6\pi^2 \lambda \alpha$  and  $\beta_T \approx 3\alpha/2$  in three-dimensions and  $L_{2D} \sim k_B T/4\pi \alpha$  and  $\beta_T \approx \alpha$  in two-dimensions within the relaxation time approximation [41, 42]. Multiplying the LLG equation Eq. (4) with  $\int d^2r \partial_{q_j} \mathbf{n}_s \cdot \mathbf{n}_s \times$  and substituting  $\dot{\mathbf{n}}_s = -\dot{q}_i \partial_{q_i} \mathbf{n}_s$ , we obtain the equation of motion for  $\mathbf{v} = \dot{\mathbf{q}}$ :

$$s(W - Q\mathbf{z} \times) \mathbf{v} + (\beta_T \eta_D - Q\mathbf{z} \times) L \nabla\chi = \mathbf{F}. \quad (8)$$

Above,  $W = \eta_0 \alpha + \eta \alpha_0$  can be interpreted as the contribution of the renormalized Gilbert damping,  $Q = \int d^2r \mathbf{n}_s \cdot (\partial_x \mathbf{n}_s \times \partial_y \mathbf{n}_s)/4\pi$  is the topological charge of the skyrmion,  $\eta_0$  is the dyadic tensor,  $\eta_D$  is the chiral dyadic tensor which is  $\sim \eta_0$  for isolated skyrmions and vanishes for skyrmions in SkX lattice [48] (detailed definitions of these coefficients are given in the Supplemental Material [62]). The “force” term  $\mathbf{F} = -\nabla U(\mathbf{q})$  due to the effective skyrmion potential  $U(\mathbf{q})$  is relevant for systems with spatially-dependent anisotropies [63], DMI [64], or magnetic fields. In deriving this equation, we only considered the dominant feedback damping contribution  $\hat{\alpha}_{\text{emf}}$  which is justified for small  $\alpha$  and  $\beta$ . For temperature gradients and forces along the  $x$ -axis we obtain velocities:

$$v_x = \frac{-L \partial_x \chi (Q^2 + W \beta_T \eta_D) + F_x W}{s(Q^2 + W^2)},$$

$$v_y = \frac{-L \partial_x \chi Q (\beta_T \eta_D - W) + F_x Q}{s(Q^2 + W^2)}. \quad (9)$$

The Hall angle defined as  $\tan \theta_H = v_y/v_x$  is strongly affected by the renormalization of  $W$  since  $\tan \theta_H = Q/W$  for a “force” driven skyrmion and  $\tan \theta_H \approx (\beta_T \eta_0 - W)/Q$  for a temperature gradient driven skyrmion. Similar to the domain wall velocity in Fig. 2, the Hall effect

will depend on the overall temperature of the system. We find that for a skyrmion driven by  $\partial_x \chi$ , the Hall angle  $\theta_H$  may flip the sign in magnets with strong DMI as the temperature increases. We estimate this should happen in  $\text{Cu}_2\text{OSeO}_3$  at  $T \sim 50\text{K}$  using a typical radial profile for a rotationally symmetric skyrmion given by Usov ansatz  $\cos(\theta/2) = (R^2 - r^2)/(R^2 + r^2)$  for  $r \leq R$  and  $R \approx 2\pi J/D \approx 52\text{nm}$ .

*Magnon pumping and accumulation.* The motion of skyrmions induces a transverse magnon current across the sample. This effect can be quantified by the average magnon current due to magnon motive force per skyrmion:

$$\mathbf{j} = \sigma \int d^2r \mathbf{E} / \pi R^2 = (\mathbf{v} \times \mathbf{e}_z) 4\sigma \hbar^2 Q / R^2. \quad (10)$$

The current can only propagate over the magnon diffusion length; thus, it can be observed in materials with large magnon diffusion length or small Gilbert damping.

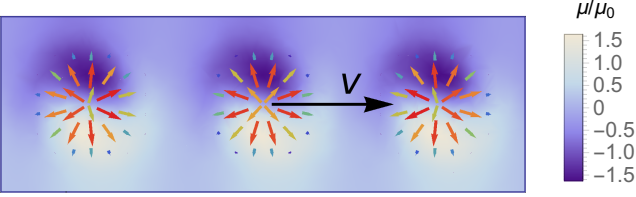


FIG. 3. (Color online) An array of moving skyrmions (only 3 shown in the figure) induces a transverse current and accumulation of magnons along the edges.  $\mu$  is obtained by numerically solving the diffusion equation using material parameters for Pt/CoFeB/MgO given in the caption of Fig. 2 with  $R = 35\text{nm}$ . System height and interskyrmion distance are taken to be  $3R$ .

So far, we have assumed a highly compressible limit in which we disregard any build up of the magnon chemical potential  $\mu$ . In a more realistic situation the build up of the chemical potential will lead to magnon diffusion. To illustrate the essential physics, we consider a situation in which the temperature is uniform. For slow magnetization dynamics in which magnons quickly establish a stationary state (i.e.  $R/v \gg \tau_\alpha$  for skyrmions and  $\Delta/\dot{X} \gg \tau_\alpha$  for domain walls, which is satisfied at high enough temperatures) we write a stationary magnon diffusion equation:

$$\nabla^2 \mu = \frac{\mu}{\xi^2} + \nabla \cdot \mathbf{E}, \quad (11)$$

where  $\xi = \lambda/2\pi\alpha$  is the magnon diffusion length and we used the local Ohm's law  $-\nabla\mu = \mathbf{j}/\sigma - \mathbf{E}$ . Renormalization of magnon current in Eq. (3) then follows from solution of the screened Poisson equation  $\mathbf{j} = \sigma \mathbf{E} + (\sigma/4\pi) \nabla \int d^3r' (\nabla' \cdot \mathbf{E}) e^{-|\mathbf{r}-\mathbf{r}'|/\xi} / |\mathbf{r}-\mathbf{r}'|$  in three dimensions and  $\mathbf{j} = \sigma \mathbf{E} + (\sigma/2\pi) \nabla \int d^2r' (\nabla' \cdot \mathbf{E}) K_0(|\mathbf{r}-\mathbf{r}'|/\xi)$

in two dimensions where  $K_0$  is the modified Bessel function for an infinitely large system [65]. By analyzing the magnon current due to magnon accumulation analytically and numerically, we find that renormalization becomes important when the length associated with the magnetic texture is much smaller than the magnon diffusion length.

Finally, we numerically solve Eq. (11) for isolated solitons (see Fig. 1) and for an array of moving skyrmions (see Fig. 3). Given that the width of the strip in Fig. 3 is comparable to the magnon diffusion length one can have substantial accumulation of magnons close to the boundary. Spin currents comparable to the estimate in Eq. (10) can be generated in this setup and further detected by the inverse spin Hall effect [66]. From Eq. (10), for a skyrmion with  $R = 35\text{nm}$  moving at  $10\text{m/s}$  in Pt/CoFeB/MgO with  $D = 1.5\text{mJ/m}^2$  [31], we obtain an estimate for spin current  $j_s = j\hbar \sim 10^{-7} \text{ J/m}^2$  which roughly agrees with the numerical results. This spin current will also carry energy and as a result will lead to a temperature drop between the edges.

*Conclusion.* We have developed a theory of magnon motive force in chiral conducting and insulating ferromagnets. The magnon motive force leads to temperature dependent, chiral feedback damping. The effect of this damping can be seen in the non-linear, temperature dependent behavior of the domain wall velocity. In addition, observation of the temperature dependent Hall angle of skyrmion motion can also reveal this additional damping contribution. We have numerically confirmed the presence of the magnon feedback damping in finite-temperature micromagnetic simulations of Eq. (1) using MuMax3 [67].

Magnon pumping and accumulation will also result from the magnon motive force. Substantial spin and energy currents can be pumped by a moving chiral texture in systems in which the size of magnetic textures is smaller or comparable to the magnon diffusion length. Further studies could concentrate on magnetic systems with low Gilbert damping, such as yttrium iron garnet (YIG), in which topologically non-trivial bubbles can be realized.

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