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Resistance oscillations of two-dimensional electrons in crossed electric and tilted magnetic fields.

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Effect of dc electric field on transport of highly mobile 2D electrons is studied in wide GaAs single quantum wells placed in tilted magnetic fields. The study shows that in perpendicular magnetic field resistance oscillates due to electric field induced Landau-Zener transitions between quantum levels that corresponds to geometric resonances between cyclotron orbits and periodic modulation of electron density of states. Magnetic field tilt inverts these oscillations. Surprisingly the strongest inverted oscillations are observed at a tilt corresponding to nearly absent modulation of the electron density of states in regime of magnetic breakdown of semiclassical electron orbits. This phenomenon establishes an example of quantum resistance oscillations due to Landau quantization, which occur in electron systems with a constant density of states.

The quantization of electron motion in magnetic fields generates a great variety of fascinating transport phenomena observed in condensed materials. Shubnikov-de Haas (SdH) resistance oscillations\(^1\) and Quantum Hall Effect (QHE)\(^2\) are famous examples related to the linear response of electrons. Finite electric fields produce remarkable nonlinear effects. At small electric fields Joule heating strongly modifies the 2D electron transport\(^3\)\(^-\)\(^6\) yielding exotic electronic states in which voltage (current) does not depend on current\(^7\)\(^-\)\(^9\) (voltage\(^10\)). Application of a stronger electric field \(E\) produces spectacular resistance oscillations.\(^11\)\(^-\)\(^16\) The oscillations are periodic with the electric field and obey the following relation:

\[
\gamma e R_c E = j\hbar \omega_c, \tag{1}
\]

where \(e\) is electron charge, \(R_c\) is the radius of cyclotron orbits of electrons at Fermi energy \(E_F\), \(j\) is a positive integer and factor \(\gamma \approx 2\). These oscillations are related to impurity assisted Landau-Zener transitions between Landau levels tilted by the electric field\(^11\) and can be treated as geometrical resonances between cyclotron orbits and spatially modulated density of states.\(^17\)\(^,\)\(^18\)

2D electron systems with multiple populated subbands exhibit additional quantum magnetoresistance oscillations.\(^19\)\(^-\)\(^26\) These magnetointersubband oscillations (MISO) are due to an alignment between Landau levels from different subbands \(i\) and \(j\) with corresponding bottom energies \(E_i\) and \(E_j\). The level alignment produces resistance maximums at the condition

\[
\Delta_{ij} = k\hbar \omega_c, \tag{2}
\]

where \(\Delta_{ij} = E_j - E_i\) and the index \(k\) is a positive integer.\(^27\)\(^-\)\(^30\). At a half integer \(k\) Eq.(2) corresponds to resistance minimums occurring at nearly constant density of states (DOS) for broad levels.\(^29\)\(^,\)\(^30\)

An application of in-plane magnetic field to the multi-subband systems creates significant modifications of electron spectra leading to fascinating beating pattern of SdH oscillations and magnetic breakdown of semiclassical orbits\(^31\)\(^-\)\(^37\). Recently it was shown that MISO are strongly modified by the in-plane magnetic field leading to a spectacular collapse of the beating nodes due to magnetic breakdown\(^40\).

In this paper we present investigations of the effect of the electric field on electron transport in three-subband electron systems placed in tilted magnetic fields. The study reveals that the in-plane magnetic field inverts the electric field induced resistance oscillations described by Eq.(1). The strongest inverted oscillations are observed at the HF-MISO nodes in the regime of magnetic breakdown, in the absence of the modulations of the density of states at the fundamental frequency \(1/\hbar \omega_c\). At these conditions the dissipative resistance reaches a minimum value, which is smaller than the resistance at zero magnetic field.

Selectively doped GaAs single quantum well of width \(d = 56\) nm was grown by molecular beam epitaxy on a semi-insulating (001) GaAs substrate. The heterostructure has three populated subbands with energies \(E_1, E_2, E_3\) at the bottoms of the subbands. The energy diagram are schematically shown in the insert to Figure 1. Hall bars with width \(W = 50\) \(\mu\)m (\(y\)-direction) and distance \(L = 250\) \(\mu\)m (\(x\)-direction) between potential contacts demonstrating electron mobility \(\mu \approx 1.6 \times 10^6\) cm\(^2\)/Vs and total density \(n \approx 8.8 \times 10^{15}\) m\(^{-2}\) were studied at temperature 4.2 Kelvin. The magnetic field, \(B,\) was directed at different angles \(\alpha\) relative to normal to the samples and perpendicular to the electric current. Hall resistance \(R_H = B_2/(en\mu\tau)\) yields the angle \(\alpha\), where \(B_2 = B \cos(\alpha)\) is the perpendicular magnetic field. Current \(I_{ac} = 1\) \(\mu\)A at 133 Hz was applied through the current...
contacts and the longitudinal and Hall ac voltages ($V_{xx}^{ac}$ and $V_{yy}^{ac}$) were measured in response to a variable dc bias $I_{dc}$ applied through the same current leads. The measurements were done in the linear regime in which the ac voltages are proportional to $I_{ac}$ yielding differential resistance $r_{xx}(I_{dc}) = V_{xx}^{ac}/I_{ac}$. Samples A and B with slightly different gaps: $\Delta_{12}(A)=0.43$ meV and $\Delta_{12}(B)=0.50$ meV were studied.

Figure 1 presents a dependence of the resistance $R_{xx}$ on the perpendicular magnetic field at different angles $\alpha$ as labeled. At $\alpha=0^\circ$ the resistance shows low frequency (LF-MISO) and high frequency (HF-MISO) MISO.$^{38,39}$ LF-MISO correspond to the scattering between the two lowest symmetric (1) and antisymmetric (2) subbands and obey the relation $\Delta_{12} = k\hbar\omega_{c}$. HF-MISO corresponds to scattering between either lowest and the third subband. Due to the mismatch between gaps: $\Delta_{13} - \Delta_{23} = \Delta_{12}$, HF-MISO show a beating pattern correlated with LF-MISO. In particular the nodes of HF-MISO beating are located at LF-MISO minima. A parallel magnetic field, $B_\parallel$, moves nodes at $k=1/2$ and $k=3/2$ toward each other leading to collapse at $\alpha=9.5^\circ$. Insert to Fig.1 shows that odd $k$ LF-MISO maximums are bounded by the nodal lines.$^{40}$

Figure 2 presents dependencies of the differential resistance $r_{xx}$ on the electric field $E$ at $B_\parallel=0.2$ T and different in-plane magnetic fields as labeled taken along the white arrow shown in the left insert to Fig.1.$^{41}$ At $B_\parallel=0$ T the black solid line shows three maximums at $j=1,2$ and 3, which obey Eq.(1). The gray solid line presents the dependence taken at the end of the white arrow in the vicinity of the nodal line. This dependence is inverted with respect to the black line and demonstrates maxima at $j=1/2,3/2$ and $5/2$. These maxima also obey Eq.(1) with the same fundamental periodicity $1/\hbar\omega_{c}$, but at the half integer values of the index $j$. The dashed line presents the dependence at an intermediate field, which does not display considerable oscillations. The insert to Fig.2 demonstrates the evolution of the electric field induced resistance oscillations taken along the black arrow shown in Fig.1. This evolution is due to variations of the perpendicular magnetic field, $B_\perp$, at $B_\parallel=0$ T. These curves do not display an inversion. In contrast to the previous case at $k=3/2$ node the resistance oscillations cease at the fundamental frequency ($1/\hbar\omega_{c}$) and only weak oscillations at second harmonics ($2/\hbar\omega_{c}$) are visible. This behavior is expected. Indeed in accordance with Eq.(2) $k=3/2$ LF-MISO minimum and HF-MISO node correspond to the condition $\Delta_{12} = (3/2)\hbar\omega_{c}$. At this condition symmetric and antisymmetric subband Landau levels are shifted by $3/2\hbar\omega_{c}$ with respect to each other and, therefore, are equally spaced by $\hbar\omega_{c}/2$ near the Fermi energy.$^{40}$ At $k=3/2$ the fundamental harmonic of the density of electron states (DOS) at frequency $1/\hbar\omega_{c}$ is absent. Due to a small Dingle factor the amplitude of the second harmonic of DOS is exponentially small producing very weak geometric resonances with cyclotron orbits at frequency $\sim 2/\hbar\omega_{c}$. $^{42}$

![Figure 1: Dependence of dissipative resistance on perpendicular magnetic field at different angles $\alpha$ as labeled.](image1)

![Figure 2: (a) Dependence of differential resistance on normalized electric field, $\varepsilon_{dc} = \gamma eR_{xy}/\hbar\omega_{c}$, where $\gamma=1.9$, at different in-plane magnetic fields as labeled, obtained along the white arrow shown in Fig.1. Insert shows the resistance evolution along the black arrow shown in the insert to Fig.1: (b) Positions of resistance maxima in (a) at different magnetic fields $B_\parallel$. Lines present linear fit of the data; (c) Reciprocal slope of the linear fits shown in (b) vs index $j$ indicating agreement with Eq.(1). Sample A.](image2)
described behavior of the DOS is valid along all nodal lines so the observed inversion of resistance oscillations is intriguing.

The absence of the inversion at $B_{\parallel}=0$T suggests that the effect may have a relation to the magnetic breakdown of quasiclassical orbits. Figure 3 supports this proposal. The figure presents an overall behavior of the electric field induced resistance oscillations vs applied dc bias $I_{dc}$ and $B_{\perp}$ taken at two different angles. At $\alpha=0^\circ$ magnetic breakdown is absent and the oscillations obey Eq.(1) with integer indexes $j$. Solid black lines present the theoretical dependence. The magnitude of the dc bias induce resistance oscillations is modulated by MISO. At LF-MISO minimum $k=3/2$ ($B_{\perp}=0.166$ T) the oscillations are almost absent (see also insert to Fig.2) and are strongest in the vicinity LF-MISO maximums at $k=1$ and 2. While at angle $\alpha=9.5^\circ$ similar oscillations are seen in small $B_{\perp}$, the striking inversion of the oscillations is obvious at $B_{\perp} > 0.166$ T. Estimations indicate a $33\%$ probability of magnetic breakdown at $B_{\perp} = 0.3$ T and less than $3\%$ at $B_{\perp} < 0.166$ T.

Figure 4 presents the evolution of the dc bias induced resistance oscillations for sample B taken in a vicinity of $k=2$ LF-MISO maximum at $B_{\perp} = 0.166$ T and different $B_{\parallel}$. The obtained data demonstrate a re-inversion of the resistance oscillations suggesting a periodicity of the inversion with the in-plane magnetic field. Surprisingly oscillations of SdH amplitude in in-plane magnetic fields with a similar period have been recently observed (see Fig.8 in). These amplitude oscillations are related to periodic oscillations of the subband splitting $\Delta_{12}$ in strong magnetic fields. The right panel indicates that at $j \approx 3/4$ almost no resistance oscillations are induced by $B_{\parallel}$. The upper panel shows that this absence of oscillations holds at $j \approx 1/4 + p/2$, where $p$ is a positive integer.

A theory of the observed inversion of dc bias induced resistance oscillations is not available. Below a qualitative model is proposed. Studied wide GaAs quantum wells are considered as two 2D parallel systems separated by a distance $d$ in $z$-direction and the coupling between the systems is treated in tight binding approximation using a tunneling magnitude $t_0$. At $B_{\parallel}=0$ T electrons occupy symmetric (S) and antisymmetric(AS) subbands and move in $x-y$ plane along cyclotron orbits with radius $R_c$ at the Fermi energy. In $B_{\perp}$ the lateral electron motion is quantized and the eigenfunctions can be presented as $|\xi,N\rangle$, where $\xi=S,AS$ and $N=0,1,2,\ldots$ numerates Landau levels. An application of the in-plane magnetic field $B_{\parallel}||E||y$ mixes the symmetric and antisymmetric states. In the vicinity of the nodal line surrounding $k=1$ region eigenfunctions are well approximated by a linear combination of one symmetric and one antisymmetric states (see Fig.10 in), which for simplicity of the presentation we consider to be equally populated: $|l\rangle=(|S,N+1\rangle\pm|AS,N\rangle)/\sqrt{2}$, where index $l$ numerates ascending energy levels. Figure 5(a) presents an evolution of the electron spectrum along the black and white arrows shown in Fig.1. The evolution corresponds to numerical computations of the spectrum in the vicinity of Fermi energy.

Resistance oscillations are observed at high filling factors and, thus, the semiclassical treatment is appropriate. It is accepted that the main contribution to dc bias induced resistance oscillations comes from electron
Below we analyze the spatial structure of eigenfunctions.

Figure 5(b) shows the wave function |1⟩=(|S,N⟩+|AS,N-1⟩)/√2 for top (z=d/2) and bottom (z=-d/2) 2D layers at N=16. Since N is even the wave function |S,N⟩ (|AS,N-1⟩) is symmetric (antisymmetric) in both y and z-directions. The eigenfunction |1⟩ is a sum of these two functions that leads to the spatial electron distribution P(y) = |Ψ(y)|² shown in Fig.5(c): at the left (right) turning point of the oscillator state |1⟩ an electron is located mostly in the bottom (top) 2D layer at −Rc (Rc). A similar configuration is obtained for state |3⟩ while the electron distribution in state |2⟩ is the distribution in state |1⟩ rotated by 180° around the y=0 axes.

The electric field E tilts the spectrum in y-direction (not shown) that allows horizontal transitions between the levels due to elastic impurity scattering, which is considered as a local perturbation. The impurity backscattering near the turning points changes the direction of electron velocity by π, which is accomplished by an overlap between the incoming state near a turning point and the outgoing state located near the opposite turning point of the oscillator shifted by 2Rc. Illustrating this statement Fig.5(d) indicates that the wave functions of the states |1⟩ and |2⟩ overlap at the opposite turning points, which leads to backscattering while the backscattering between states |1⟩ and |3⟩ is significantly suppressed since these wave functions at the opposite turning points are located in different 2D layers and, thus, the overlap between two functions is exponentially small. Similar consideration indicates the presence (absence) of backscattering between states |l⟩ and |m⟩ with different (the same) parity of indexes: mod2(m−l) = 1 (mod2(m−l) = 0). At nodal lines the energy difference between states with different index parity obeys the relation: δE = Ej − El = ħωc(l + 1/2), that leads to the relation: γeRcE = ħωc(l + 1/2) for the electric field induced resistance oscillations in tilted magnetic field.

At zero dc bias the backscattering occurs inside the same quantum level. Thus in tilted magnetic fields the impurity backscattering in the linear response is suppressed at the nodal lines since the parity of the incoming and outgoing states is the same. This conclusion is in agreement with the experiment. Indeed Fig.1 shows that at the k = 3/2 HF-MISQ node located at B⊥= 0.2 T and B∥= 0.033 T the resistance reaches a value which is less than the value of the resistance both at k = 3/2 at B∥= 0 T and even at zero magnetic field. The data indicates that electron backscattering by impurities is effectively controlled by in-plane magnetic field. This result may have important implications for the field of topological insulators, where electron backscattering is considered to be crucial.

In conclusion the electric field induced resistance oscillations are studied in wide GaAs quantum wells placed in tilted quantizing magnetic field. The oscillations are related to impurity assisted Landau-Zener transitions between quantum levels and in perpendicular magnetic fields obey relation: 2eRcE = ħjωc, where j is a positive integer. A tilt of the magnetic field inverts the oscillations. The strongest inversion occurs at the nodal line of the beating between magnetointersubband resistance oscillations at which the density of electron states is nearly constant. These oscillations obey the relation 2eRcE = ħjωc, where j is a positive half integer. The effect is related to spatial redistribution of eigenfunctions of multi-subband electron systems leading to significant modification of the electron backscattering in tilted magnetic fields.

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