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# Striped quantum Hall state in a half-filled Landau level

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# Striped quantum Hall state in a half-filled Landau level

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Nature of the fractional quantum Hall state at Landau level filling factor 5/2 remains elusive despite intensive experimental and theoretical work. While the leading theoretical candidates are Moore-Read Pfaffian (Pf) and its particle-hole conjugate anti-Pfaffian (APf), neither received unambiguous experimental support. We show that a state that is intermediate between them, made of alternating stripes of Pf and APf in the bulk, is a viable candidate. Such a state is shown to be incompressible and thus a charge insulator in the bulk, but a heat conductor due to the presence of gapless neutral bulk modes. We argue that properties of such a state is consistent with existing numerical and experimental work, and discuss possible experimental probes of its presence.

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The fractional quantum Hall (FQH) state at Landau level (LL) filling factor $\nu = 5/2$ [1] has been one of the main focuses in experimental and theoretical studies of FQH effect for well over a decade. The intense interest in this state lies in its likely non-Abelian nature and potential application for topological quantum computation [2]. Leading theoretical candidates include Moore-Read Pfaffian (Pf) state [3], which is the very first example of a non-Abelian state of matter, and its closely related particle-hole conjugate, anti-Pfaffian (APf) state [4, 5]. Determining the nature of the 5/2 FQH state experimentally would not only resolve a long-standing issue in the field of quantum Hall effect, but also be of tremendous interest to the physics community in general, in particular if the outcome is an unambiguous demonstration of a non-Abelian topological state of matter for the first time.

The Pf and APf states have identical bulk excitation spectra, and cannot be distinguished by bulk thermoelectric and thermodynamical probes [6–9] (result of an initial attempt appears to be consistent with both [10]). Qualitative differences are in their edge properties, which in principle allow for distinction between them as well as other candidate states through electron and quasiparticle tunneling. Recent experiments [11–13] appear to be more consistent with the APf state, but cannot rule out the Pf state due to possible edge reconstruction [14] or the 331 and other Abelian states [13, 15]. Numerically, Pf and APf states are exactly degenerate in energy in the absence of LL mixing due to particle-hole symmetry for a half-filled LL. LL mixing breaks particle-hole symmetry and is expected to lift this degeneracy, but it remains controversial which state would be favored energetically at this point [16–21].

The purpose of the present work is to point out that a new state made of alternating stripes of Pf and APf

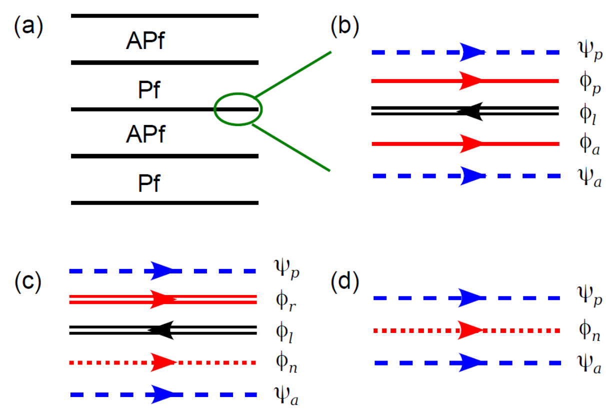


FIG. 1: (Color online) (a) A striped phase containing alternating domains of Pf and APf states. (b) Original edge modes along a Pf-APf domain wall, containing three bosonic modes $\phi_p$, $\phi_a$, $\phi_l$, and two fermionic modes $\psi_p$ and $\psi_a$. (c) The symmetric and antisymmetric linear combinations of the two $\nu = 1/2$ bosonic modes $\phi_p$ and $\phi_a$ form a charge mode $\phi_r$ ($\nu^* = 1$) and a neutral mode $\phi_n$. (d) Electron pair tunneling (see Fig. 2) along with strong Coulomb interaction can gap the counterpropagating charge modes $\phi_l$ and $\phi_r$, leaving the bosonic neutral mode $\phi_n$ and two neutral fermionic modes $\psi_p$ and $\psi_a$ to remain gapless. See text for details.

states in the bulk, is a possible candidate of the FQH state at 5/2 [see Fig. 1(a) for an illustration]. We show that such a striped state, unlike those realized in high LLs [22–26], is *incompressible* and thus a FQH state. On the other hand it supports *gapless* neutral modes in the bulk, and thus has the novel property of being a bulk heat conductor. We argue that its properties are qualitatively consistent with existing experiments and numerical studies, and suggest new experiments to probe its presence.

While the FQH state at 5/2 was discovered experimentally in 1987 [1] and the Pf state first written down in 1991 [3], the connection between them started to be taken seriously only after numerical work by Morf in 1998 [27] and Rezayi and Haldane (RH) in 2000 [28]. At the time of these seminal numerical works the APf state was not known. In Morf's work a shift that specifically favors the Pf state was chosen on the sphere, while in that of RH the Pf state was particle-hole symmetrized by hand on the torus when comparing with numerical ground state obtained by exactly diagonalizing the Coulomb Hamiltonian of a half-filled first-excited LL. Intriguingly, RH also found that the (particle-hole symmetrized) Pf state is actually stable only in a very narrow region in their phase diagram, sandwiched between the (compressible) composite fermion Fermi liquid-like state and a striped state. With the additional insight of the existence of the APf state, we now propose that the striped state is nothing but a mixture made of alternating stripes of Pf and APf states. It thus appears very naturally in the phase diagram very close to the (particle-hole symmetrized) Pf state. More importantly, the incompressible nature (as we will demonstrate below) of this striped FQH state explains the robustness of the experimentally observed FQH state at 5/2, despite the numerical finding of RH that the particle-hole symmetrized Pf state is stable only in a narrow region of parameter space.

*Domain wall between Pf and APf states* — Pf and APf states are both incompressible. The striped state [as illustrated in Fig. 1(a)] contains domain walls separating Pf and APf states. Such domain walls are expected to support *gapless* modes, just like edges. One thus might expect there are gapless charge modes in the bulk due to the presence of these domain walls, and the state would be compressible (just like striped states in higher LLs). We now show that in the present case the domain wall only supports neutral modes but the charge modes are gapped due to presence of electron tunneling between different modes and strong Coulomb interaction. As a result we arrive at a novel incompressible state that supports gapless neutral modes in the bulk.

The Lagrangian density of the Pf-APf interface includes terms corresponding to bosonic and fermionic modes [29]:

$$\begin{aligned} L \;=\; & \frac{1}{4\pi}[\partial_t\phi_l\partial_x\phi_l - 2(\partial_t\phi_p\partial_x\phi_p + \partial_t\phi_a\partial_x\phi_a)] \\ - \; & [i\psi_p\partial_t\psi_p + i\psi_a\partial_t\psi_a] - H[\phi,\psi], \end{aligned} \tag{1}$$

where $\phi_l$ is the (left moving) bosonic edge field of the $\nu = 1$ background in which the embedded holes forming the Pf state (thus resulting in the APf state), $\phi_p$ and $\phi_a$ are the edge bosonic fields of the Pf state for the electrons and holes respectively, and $\psi_p$ and $\psi_a$ are the corresponding edge Majorana fermion fields, as illustrated in Fig. 1(b). $H$ is the Hamiltonian density.

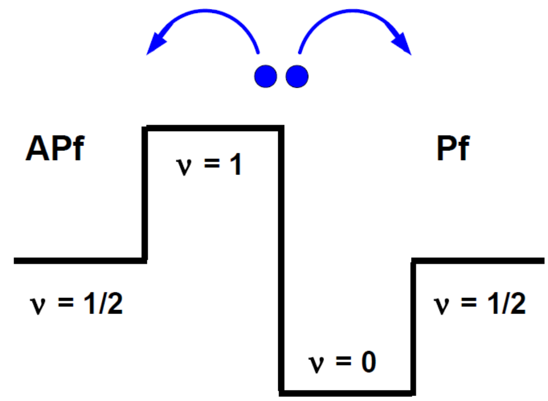


FIG. 2: (Color online) The relevant momentum-conserving tunneling process. A pair of electrons tunnel from the $\nu = 1$ edge into the Pf edges of electrons and holes, respectively.

The charge density along the interface

$$\rho(x) = \frac{1}{2\pi}\partial_x(\phi_l + \phi_p + \phi_a) = \frac{1}{2\pi}\partial_x\phi_c, \tag{2}$$

where

$$\phi_c = \phi_l + \phi_p + \phi_a \tag{3}$$

is the total charge field. This naturally suggests the following combination of $\phi_p$ and $\phi_a$:

$$\phi_{r,n} = \phi_p \pm \phi_a, \tag{4}$$

where $\phi_r$ is a right-moving charge field and $\phi_n$ is a right-moving *neutral* bosonic field. In terms of these combinations the dynamical terms involving bosonic fields in $L$ [Eq. (1)] become

$$\frac{1}{4\pi}[(\partial_t\phi_l\partial_x\phi_l - \partial_t\phi_r\partial_x\phi_r) - \partial_t\phi_n\partial_x\phi_n], \tag{5}$$

as illustrated in Fig. 1(c). Note in particular the two terms combined in the bracket above is the same as those of left and right movers in an ordinary (or non-chiral) Luttinger liquid.

All terms allowed by symmetry show up in $H$ with varying magnitudes. Of particular importance to the physics we discuss here are the following.

(i) A pair of electrons tunnel from the $\nu = 1$ edge into the Pf edges of electrons and holes (see Fig. 2), respectively, and its hermitian conjugate (h.c.):

$$T_p \propto \psi_p\psi_a e^{2i(\phi_l+\phi_p+\phi_a)} + h.c. = 2\psi_p\psi_a\cos(2\phi_c). \tag{6}$$

(ii) Strong Coulomb interaction

$$V_c = v_c(\partial_x\phi_c)^2. \tag{7}$$

In the absence of screening (say due to nearby metallic gates) $v_c$ diverges logarithmically in the long-distance limit, due to the long-range nature of Coulomb interaction. In practice this renders $v_c$ much larger than kinetic energy and other interaction terms involving neutral fields. This significantly reduces the scaling dimension of $\cos(2\phi_c)$, making it

$$\Delta_c \ll 1. \tag{8}$$

As a result of this we expect the scaling dimension of $T_p$

$$\Delta_{T_p} = 2\Delta_\psi + \Delta_c = 1 + \Delta_c < 2. \tag{9}$$

This means $T_p$ is a *relevant* perturbation under renormalization group, and develops an expectation value via the usual mechanism as in the sine-Gorden model (or in ordinary Luttinger liquid with back-scattering). This opens up a gap for the charge modes $\phi_l$ and $\phi_r$, leaving us with a single gapless neutral bosonic mode $\phi_n$ and two fermionic modes, as illustrated in Fig. 1(d). Note that they are propagating along the same direction and the total central charge is $c = 1 + 1/2 + 1/2 = 2$.

*Numerical evidence.* Now we revisit the RH work in sphere and torus geometries. The study showed that there is a first-order transition from the symmetrized Pf state to a compressible striped phase. The single Slater determinant that dominates the striped phase has the occupation pattern 0000111100000111100001111, which can be regarded as alternating unit cells of the Pf root configuration 1100 and its particle-hole conjugation 0011. While RH originally interpreted the striped state as being compressible, in analogy to its higher LL counterparts, we note their numerical results only indicate broken translation symmetry but do not provide information on compressibility or charge gap, thus do not contradict the possibility of an incompressible state here.

In disk geometry, where there is a boundary, the agreement of the ground state to the Pf or APf is not as good as in a boundaryless geometry in terms of wave function overlap. However, the present authors and co-workers showed that a robust Pf state can be identified by the total angular momentum, edge excitations, and, more importantly, the possible induction of both Abelian and non-Abelian quasiholes in a model with realistic confining potential [30]. When the confining potential becomes much weaker, the ground state can be identified as the APf state based on the total angular momentum and the response of the system to the confining potential tuning [31]. The transition from Pf to APf states driven by background potential, which breaks particle-hole symmetry, has also been confirmed in the presence of LL mixing and finite layer thickness [21].

Interestingly, there is an additional microscopic state *separating* the Pf and APf phases [21, 31], as illustrated in Fig. 3. This intermediate state was identified to be a striped phase[31]. More recently, Zhang *et al.* found that as the edge confining potential is weakened, the Pf state is destabilized by softening of the (neutral) fermionic edge mode [32]. This suggests the difference between the resultant striped and Pf states lie in their low-energy *neutral* modes. This is clearly consistent with our assessment of the nature of the striped phase. While there is no numerical evidence for it, it is not unreasonable to speculate that the instability of the APf state driven by decreasing $d$ is similarly triggered by softening of neutral mode(s). More detailed discussion on the analysis of disc geometry numerics is presented in the Supplemental Material.

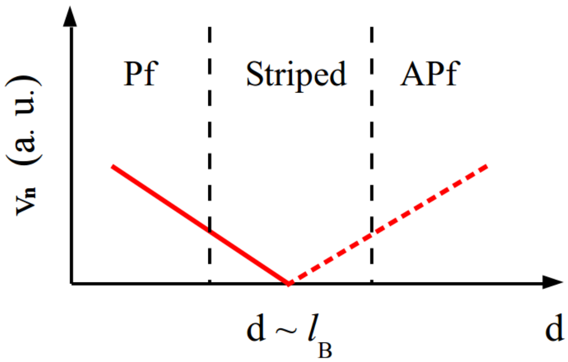


FIG. 3: (Color online) Phase diagram of the Pf, striped, and APf. The striped phase is associated with the smallness of the neutral fermionic edge-mode velocity $v_n$, when the energies of the Pf and APf states are comparible. The transition can be tuned by the confining potential strength, controlled by the setback distance $d$ in the numerical model between the two-dimensional electron layer and the background charge layer. Note that on the APf side we illustrate by a positive $v_n$, but the neutral fermionic mode propagates along the opposite direction to its Pf counterpart.

*Experimental Consequences.* It has been difficult to understand that while the theoritical modelings with long-range Coulomb interaction often show that the Pf or APf state survives in only a small parameter space[28], the FQH effect at $\nu = 5/2$ is observed in essentially all high mobility samples. With the understanding that the adjacent striped phase observed numerically is indeed the striped FQH phase with a charge gap proposed here, the quantized plateau is then expected to be observable in a much larger parameter space, which is clearly consistent with experiments.

As discussed earlier, LL mixing induced by Coulomb interaction breaks the particle-hole symmetry, and can thus drive a Pf-APf transition [35, 36], *if* they are the only two competing phases. No such transition, however, has been observed as the level of LL mixing is varied by changing the electron density [37, 38], or varying other parameters like magnetic field [39]. This can again be easily understood in terms of the striped quantum Hall phase proposed here, which would respond to the changing LL mixing and other changes by varying the relative weight of Pf and APf domains, *without* having a phase transition. The only exception to this is a recent experiment that claimed to have observed a transition with unusually large Landau level mixing [40]. In this experiment, a drastic change of the energy gap dependence on the electron density was interpreted as the signature of a transition, although there is no direct method to identify either the Pf state or the APf state. We believe it is likely that this transition is actually between the striped state to either the Pf or APf state, with the latter being stabilized by unusually large LL mixing due to the low electron density.

More importantly, a distinctive novel property of the striped FQH phase is the presence of gapless neutral modes (but not the charge modes) in the bulk, in sharp contrast to all known quantum Hall or compressible states. In fact, with an intermediate striped phase like the numerics suggested, it is necessary that the bulk neutral modes are gapless; otherwise, the topologically distinct Pf and APf phases would be adiabatically connected. In a clean system at zero temperature, the absence of backscattering of these neutral modes can be attributed to symmetry (breaking) and conservation laws. A direct experimental observable consequence is that this electronic system is a bulk thermal conductor and a bulk charge insulator *at the same time*. This contrasts the known quantum Hall states that transport heat and charge along the edge but not in the bulk, and compressible states in which the bulk conducts both charge and heat. More specifically, in the limit of weak disorder when we expect the stripes are orderly aligned along a particular direction, we expect an anisotropic state that conducts heat along the direction of the stripes but not in the perpendicular direction, in analogy to the anisotropic conducting state near half-filled higher LLs[22–26]. Unlike there, the alternating Pf and APf stripes have the same filling factor, hence incompressible, because no extra charge can be accommodated by tuning the stripe width. However, the location of the gapless neutral modes can adjust to the frustration in interaction, such that the striped phase is potentially more favorable in energy than either the Pf or APF state. Observation of such anisotropic heat conduction on a quantum Hall plateau can be considered definitive evidence of striped quantum Hall state proposed here. Stronger disorder may disorder the stripes, rendering the system an isotropic heat conductor, but still a FQH state. We note existence of bulk neutral modes was found in a very recent experiment [41], in which highly sensitive noise measurement revealed the unexpected heat propagation through incompressible FQH bulk at various filling factors in the lowest LL. The state proposed in this paper is the first example with such properties, and it would be interesting to check whether the bulk heat transport is also present for $\nu = 5/2$.

The striped FQH state has a more complicated edge structure. Depending on how the state terminates, it could either have a Pf edge, APf edge, or more generically, a hybrid between them. The latter would occur when the edge intersects with the domain walls in the bulk, and is illustrated in Fig. 4. In this case there is a *single* charged mode that propagates along the edge, while the neutral modes of the Pf and APf edges merge into the bulk neutral modes at their intersections. The rich variety of edge structures may well be responsible for the lack of consistency in the results of existing edge tunneling experiments, and can have profound implications in interferometry experiments. These will be explored in future work.

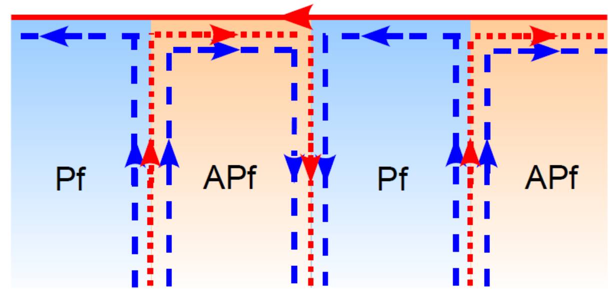


FIG. 4: (Color online) Illustration of the edge structure of the alternating Pf and APf stripes. There is a single charged mode (solid red line) that propagates along the edge, while the neutral modes (dashed and dotted lines) of the Pf and APf edges merge into the bulk neutral modes at their intersections.

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*Note added.* While the paper was being written up we became aware of a recent preprint [42] on the theory of particle-hole conjugated composite fermion Fermi liquid states, motivated by a recent experiment that hinted at a possible particle-hole symmetry breaking near $\nu = 1/2$ [43]. In Ref. [42] the *possibility* of a phase in which the charge modes being gapped at an interface between Pf and APf states was pointed out as a corollary of the authors' detailed analysis of an interface between particle-hole conjugated composite fermion Fermi liquid states, although its physical consequences were not discussed. In comparison to the strong current interest on achieving particle-hole symmetry for a half-filled LL [42, 44–48], the state we propose here is particle-hole symmetric in average, but not in a local sense.


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