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Quantum Tunneling of the Magnetic Moment in the S/F/S Josephson φ_0 -Junction

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We show that the S/F/S Josephson φ_0 -junction permits detection of macroscopic quantum tunneling and quantum oscillation of the magnetic moment by measuring the ac voltage across the junction. Exact expression for the tunnel splitting renormalized by the interaction with the superconducting order parameter is obtained. It is demonstrated that magnetic tunneling may become frozen at a sufficiently large φ_0 . The quality factor of quantum oscillations of the magnetic moment due to finite ohmic resistance of the junction is computed. It is shown that magnetic tunneling rate in the φ_0 -junction can be controlled by the bias current, with no need for the magnetic field.

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I. INTRODUCTION

Quantum tunneling of the magnetic moment has been subject of intensive research due to the fundamental interest in the phenomenon and because of its potential applications for quantum information technology. Early work focused on non-thermal magnetic relaxation in nanoparticles [1]. Later on, the focus switched to molecular magnets [2]. Due to identical structure of the building blocks of crystals made of magnetic molecules, they permit macroscopic studies of quantum tunneling and quantum oscillations of molecular magnetic moments. On the contrary, the reliable observation and quantitative analysis of the quantum tunneling of the magnetic moment in nanoparticles is hampered by the practical impossibility to make identical nanoparticles. Best samples available still have distribution of sizes and other parameters of the particles within at least 20%. Due to the exponential dependence of the tunneling rate on the size, this translates into the distribution of tunneling rates within many orders of magnitude.

Early on, the difficulty mentioned above prompted researchers to look into the possibility to observe magnetic tunneling in individual nanoparticles. Such measurements of 10-nm ferrimagnetic particles of total spin $S \sim 10^5$, deposited on a nanobridge of a dc-SQUID, were pioneered by Wernsdorfer et al. [3]. The energy barrier was controlled by the external magnetic field. At very small barriers the evidence of non-thermal magnetic relaxation below 1K was obtained. After a preliminary success, however, these efforts were largely abandoned in favor of detecting spin tunneling in better characterized individual magnetic molecules. Measurements of transport current through magnetic molecules bridged between conducting leads [4] and through molecules grafted on carbon nanotubes [5] produced convincing evidence of the effect. Observation and control of quantum tunneling of the magnetic moment in nanoscale magnetic clusters, however, remains a challenging experimental task.

In recent years broad interest has emerged to the inter-

action of spin polarization with superconducting currents in nanostructures. Experimental and theoretical work in this area has given rise to the field of superconducting spintronics [6, 7]. In S/F/S nanostructures ferromagnetism can affect superconductivity via the magnetic field it generates [8–10]. Somewhat stronger influence of ferromagnetism on superconductivity at the F/S interface may occur due to the proximity effect [11]. The opposite influence of the superconductivity on the ferromagnetism is typically weak. This can be understood from the fact that the exchange interaction responsible for magnetic ordering is typically of the order of hundreds or thousands of kelvin while interactions responsible for conventional superconductivity are in the ball park of a few kelvin. One should notice, however, that relativistic interactions responsible for the orientation of the magnetic moment in ferromagnets – the magnetic anisotropy energy is also in the kelvin, or can be even in the subkelvin, range. Thus, the coupling of the superconducting order parameter to the orientation of the magnetic moment can, in principle, produce a formidable torque on the moment.

Such a possibility was recently proposed by Buzdin who noticed that spin-orbit interaction in a ferromagnet without the inversion symmetry provides the coupling between the direction of the magnetic moment and the superconducting order parameter [12, 13]. Very generally, the symmetry breaking results in the anomalous Josephson effect [14]. In a non-centrosymmetric ferromagnetic junction, that Buzdin called the φ_0 -junction, the time reversal symmetry is broken, and the current-phase relation becomes $I = I_c \sin(\varphi - \tilde{\varphi}_0)$, where the phase shift $\tilde{\varphi}_0$ is proportional to the component of the magnetic moment perpendicular to the gradient of the asymmetric spin-orbit potential. Theory of anomalous Zeeman effect and spin-galvanic effect in φ_0 -junctions has been further developed in Refs. 15 and 16. Experimental realization of the φ_0 -junction has been recently reported by Szombati et al. [17]. In this paper we argue that such Josephson junction is ideally suited for the study of quantum tunneling of the magnetic moment. The magnetic tun-

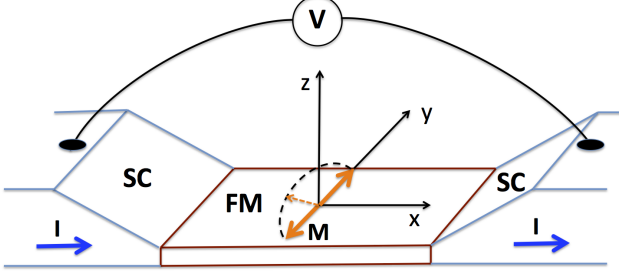


Figure 1: Schematic picture of the Josephson φ_0 -junction.

neling would show in the ac voltage across the junction and it can be controlled by the bias current applied to the junction.

The paper is structured as follows. The model of spin tunneling in a φ_0 -junction is formulated in Section II. Oscillations of the voltage generated by coherent quantum oscillations of the magnetic moment are derived in Section III. Damping of quantum oscillations due to the finite resistance of the junction is discussed in Section IV. Section V demonstrates possibility of the control of the magnetic tunneling rate by the bias current applied to the junction. Section VI contains some numerical estimates and suggestions for experiment.

II. THE MODEL

Following Buzdin [13] we consider a S/F/S Josephson φ_0 -junction depicted in Fig. 1, with the potential energy

$$U = E_J \left\{ 1 - \cos[\varphi - \tilde{\varphi}_0(\mathbf{M})] - \frac{I}{I_c} \varphi \right\} + U_M(\mathbf{M}) \quad (1)$$

Here U_M is the magnetic anisotropy energy with \mathbf{M} being the magnetization of the ferromagnetic layer, $E_J = I_c \Phi_0 / (2\pi)$ is the Josephson energy with Φ_0 being the flux quantum, $\tilde{\varphi}_0$ depends on the direction of \mathbf{M} , and I is the bias current applied to the junction. With the gradient of the spin-orbit potential normal to the layer (that is, along the Z -direction), $\tilde{\varphi}_0$ is given by

$$\tilde{\varphi}_0 = \varphi_0 \left(\frac{M_y}{M_0} \right), \quad (2)$$

where M_0 is the length of the magnetization and $\varphi_0 \ll 1$ depends on the strength of spin-orbit interaction. Notice that at $I = 0$ Eq. (1) has the time reversal symmetry because both ϕ and \mathbf{M} change sign under $t \rightarrow -t$. Consequently, at $I = 0$ the ground state is always degenerate with respect to two opposite orientations of \mathbf{M} determined by the magnetic anisotropy, same as in a conventional ferromagnet at zero external magnetic field. This

degeneracy can be removed by the bias current that may, therefore, switch the direction of \mathbf{M} [12, 13].

Neglecting dissipation the equations of motion for φ and \mathbf{M} are [18, 19]

$$C \left(\frac{\Phi_0}{2\pi} \right)^2 \ddot{\varphi} = -\frac{\partial U}{\partial \varphi} \quad (3)$$

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{B}_{\text{eff}}, \quad \mathbf{B}_{\text{eff}} = -\frac{1}{V} \frac{\partial U}{\partial \mathbf{M}}, \quad (4)$$

with C being the capacitance of the junction and $\mathbf{B}_{\text{eff}}(\varphi, \mathbf{M})$ being the effective field acting on the magnetic moment. To allow for quantum tunneling the junction must be of the smallest possible size, in which case its capacitance C can be safely neglected, so that Eq. (3) reduces to the condition of the energy minimum for the Josephson phase. Imaginary-time solutions of Eq. (4) correspond to the quantum tunneling of \mathbf{M} [1].

In this paper we are making three main points. The first is that the interaction between the magnetic moment and the superconducting order parameter in the φ_0 -junction renormalizes the tunnel splitting in a manner that can be exactly computed and measured. The second point is that the Josephson φ_0 -junction permits detection of the quantum tunneling and quantum oscillations of the magnetic moment by measuring the voltage across the junction. The third point is that the φ_0 -junction allows one to control the magnetic tunneling rate by the bias current. Note that the exact form of spin-orbit interaction (Rashba, Dresselhaus, etc.) is important for the concrete form of $\varphi_0(\mathbf{M})$, the rest is determined by the symmetry of the magnetic anisotropy (crystal field).

III. AC VOLTAGE FROM QUANTUM OSCILLATIONS OF THE MAGNETIC MOMENT

To illustrate the first two points we choose a typical form of U_M for a ferromagnetic layer that corresponds to the XY easy magnetization plane with the Y easy axis in that plane,

$$U_M = \frac{1}{2} K_{\perp} V \left(\frac{M_z}{M_0} \right)^2 - \frac{1}{2} K_{\parallel} V \left(\frac{M_y}{M_0} \right)^2, \quad (5)$$

V being the volume of the ferromagnetic layer, and K_{\perp}, K_{\parallel} being the magnetic anisotropy constants. At $I = 0$ the classical degenerate equilibrium corresponds to two opposite orientations of \mathbf{M} along the Y -axis, with the energy barrier between them given by $U_0 = \frac{1}{2} K_{\parallel} V$. We are interested in the quantum oscillations of \mathbf{M} between these two states.

Quantum tunneling of \mathbf{M} is carried out by the instanton solution of Eq. (4) in imaginary time, $\tau = it$, together with the condition $\partial U / \partial \varphi = 0$. In the resulting

semiclassical equations φ is a slave variable that follows the imaginary-time dynamics of \mathbf{M} according to

$$\sin\left(\varphi - \varphi_0 \frac{M_y}{M_0}\right) = \frac{I}{I_c}, \quad (6)$$

making the effect of the junction on \mathbf{M} mathematically equivalent to the effect of the external magnetic field

$$\mathbf{B}_I = \varphi_0 \frac{E_J}{M_0} \left(\frac{I}{I_c}\right) \hat{\mathbf{y}} = \varphi_0 \frac{\Phi_0 I}{2\pi M_0} \hat{\mathbf{y}} \quad (7)$$

At $I = 0$ the instanton solution of Eq. (4) for $\mathbf{M} = M_0(\sin\theta \cos\phi, \sin\theta \sin\phi, \cos\theta)$ is given by [1, 20, 21]

$$\sin\phi = \frac{\sinh(\omega_0\tau)}{\sqrt{\lambda + \cosh^2(\omega_0\tau)}}, \quad \cos\theta = \frac{\sqrt{\lambda} \cos\phi}{\sqrt{1 + \lambda \sin^2\phi}} \quad (8)$$

where $\omega_0 = [\omega_{\parallel}(\omega_{\parallel} + \omega_{\perp})]^{1/2}$, $\lambda = \omega_{\parallel}/\omega_{\perp}$, $\omega_{\parallel,\perp} = 2\gamma K_{\parallel,\perp}/(M_0V)$. The instanton switches the magnetization from $\mathbf{M} = -M_0\hat{\mathbf{y}}$ at $\tau = -\infty$ to $\mathbf{M} = M_0\hat{\mathbf{y}}$ at $\tau = +\infty$. Path integration of $\exp(i \int dt \mathcal{L}/\hbar)$ around the instanton with the Lagrangian

$$\mathcal{L} = \frac{M_0V}{\gamma} \dot{\phi}(\cos\theta - 1) - U(\varphi, \theta, \phi) \quad (9)$$

gives for the tunnel splitting

$$\Delta_0 = Ae^{-B}, \quad A \sim \hbar\omega_0, \quad B = 2S \ln(\sqrt{\lambda} + \sqrt{1 + \lambda}), \quad (10)$$

where $S = M_0V/(\hbar\gamma)$ is the total spin of the ferromagnetic layer. Strong easy plane anisotropy (in which case $\lambda \ll 1$ and $B = 2\sqrt{\lambda}S$) is required to allow observation of tunneling of a macroscopically large S .

In the low energy domain the problem can be recast as a two-state problem. Projecting Eq. (1) onto the two magnetic states with \mathbf{M} along the Y -axis one obtains a two-state Hamiltonian

$$\begin{aligned} H &= -E_J \cos(\varphi - \varphi_0\sigma_y) - \frac{\Delta_0}{2}\sigma_x \\ &= -E_J \cos\varphi \cos\varphi_0 - \mathbf{b}_{\text{eff}} \cdot \frac{\boldsymbol{\sigma}}{2} \end{aligned} \quad (11)$$

with

$$\mathbf{b}_{\text{eff}} = \Delta_0 \hat{\mathbf{x}} + 2E_J \sin\varphi \sin\varphi_0 \hat{\mathbf{y}} \quad (12)$$

Here $\boldsymbol{\sigma}$ is a spin- $\frac{1}{2}$ operator satisfying

$$\hbar \frac{d\boldsymbol{\sigma}}{dt} = i[H, \boldsymbol{\sigma}] = \boldsymbol{\sigma} \times \mathbf{b}_{\text{eff}} \quad (13)$$

For the components of $\boldsymbol{\sigma}$ one has

$$\hbar \frac{d\sigma_x}{dt} = -2E_J \sin\varphi \sin\varphi_0 \sigma_z \quad (14)$$

$$\hbar \frac{d\sigma_y}{dt} = \Delta_0 \sigma_z \quad (15)$$

$$\hbar \frac{d\sigma_z}{dt} = 2E_J \sin\varphi \sin\varphi_0 \sigma_x - \Delta_0 \sigma_y \quad (16)$$

These equations also hold for the expectation values of the components of $\boldsymbol{\sigma}$. It is easy to see that they conserve the length of $\boldsymbol{\sigma}$ ($|\boldsymbol{\sigma}| = 1$). Linearized near the ground state, $\sigma_x = 1$, in the practical limit of $|\varphi_0| \ll 1$ they describe oscillations of $\sigma_{y,z}$ and φ at the frequency $\omega = \Delta_{\text{eff}}/\hbar$ with

$$\Delta_{\text{eff}} = \sqrt{\Delta_0 (\Delta_0 - 2E_J \varphi_0^2)} \quad (17)$$

Eq. (17) shows that interaction of the magnetic moment with the superconducting order parameter renormalizes the tunnel splitting, $\Delta_0 \rightarrow \Delta_{\text{eff}}$. At $2E_J \varphi_0^2 < \Delta_0$ the system prepared in a state with a definite orientation of \mathbf{M} along the Y -axis begins to oscillate with the expectation values of M_y and φ satisfying

$$M_y = M_0 \cos\left(\frac{\Delta_{\text{eff}}}{\hbar} t\right), \quad \varphi = \varphi_0 \cos\left(\frac{\Delta_{\text{eff}}}{\hbar} t\right) \quad (18)$$

Smallness of φ_0 ensures practicality of such a regime. Oscillations of φ generate the oscillating ac voltage across the junction,

$$\mathcal{V} = \frac{\hbar}{2e} \frac{d\varphi}{dt} = -\varphi_0 \frac{\Delta_{\text{eff}}}{2e} \sin\left(\frac{\Delta_{\text{eff}}}{\hbar} t\right) \quad (19)$$

According to Eq. (17) a sufficiently large φ_0 satisfying

$$2E_J \varphi_0^2 > \Delta_0 \quad (20)$$

freezes magnetic tunneling.

IV. DAMPING OF QUANTUM OSCILLATIONS DUE TO THE FINITE RESISTANCE OF THE JUNCTION

Decoherence of quantum spin oscillations due to various mechanisms has been studied at length. It involves interaction with phonons, nuclear spins, and other microscopic degrees of freedom. These effects are usually weak and are expected to induce weak decoherence of quantum oscillations of the voltage as well. Here we focus on the mechanism of decoherence that is pertinent to the φ_0 -junction. It is related to the finite ohmic resistance R that can be associated either with the resistance of the junction itself or the resistance of the junction together with a resistive shunt connected in parallel with the junction. In the presence of the finite resistance the dynamical equation for φ projected onto the two low-energy spin states becomes

$$\begin{aligned} \frac{1}{R} \left(\frac{\Phi_0}{2\pi}\right)^2 \frac{d\varphi}{dt} &= -\frac{\partial U}{\partial \varphi} = \\ &= -E_J (\sin\varphi \cos\varphi_0 - \cos\varphi \sin\varphi_0 \sigma_y). \end{aligned} \quad (21)$$

Linearized equations (14) - (16) and (21) read

$$\frac{1}{R} \left(\frac{\Phi_0}{2\pi}\right) \frac{d\varphi}{dt} = -I_c (\varphi - \varphi_0 \sigma_y) \quad (22)$$

$$\hbar \frac{d\sigma_y}{dt} = \Delta_0 \sigma_z, \quad \hbar \frac{d\sigma_z}{dt} = \frac{I_c \Phi_0}{\pi} \varphi_0 \varphi - \Delta_0 \sigma_y \quad (23)$$

Writing $\varphi, \sigma_{y,z} \propto \exp(-i\omega t)$ and equating the determinant of the resulting equations to zero we get

$$\left[\frac{-i\omega}{R} \left(\frac{\Phi_0}{2\pi} \right) + I_c \right] [(\hbar\omega)^2 - \Delta_0^2] + \frac{I_c^2 \Phi_0}{\pi} \phi_0^2 \Delta_0 = 0 \quad (24)$$

The solution is $\omega = \Delta_{\text{eff}}/\hbar - i\Gamma$ with Δ_{eff} of Eq. (17) and the decoherence rate $\Gamma \ll \Delta_{\text{eff}}/\hbar$ given by

$$\Gamma = \left(\frac{\varphi_0^2}{\hbar R} \right) \left(\frac{\Phi_0}{2\pi} \right)^2 \frac{\Delta_{\text{eff}}}{\hbar}. \quad (25)$$

It describes the decaying quantum oscillations of the voltage when the φ_0 -junction was initially prepared in a state with a definite orientation of \mathbf{M} along the Y -axis,

$$\mathcal{V} = -\varphi_0 \frac{\Delta_{\text{eff}}}{2e} e^{-\Gamma t} \sin \left(\frac{\Delta_{\text{eff}}}{\hbar} t \right). \quad (26)$$

The corresponding quality factor of the oscillations is given by

$$Q = \left(\frac{2\pi}{\Phi_0} \right)^2 \left(\frac{\hbar R}{\varphi_0^2} \right) \quad (27)$$

V. CONTROL OF MAGNETIC TUNNELING BY THE BIAS CURRENT

In this Section we are interested in the possibility to control the rate of magnetic tunneling by the bias current applied to the ϕ_0 -junction. As long as the dissipation is weak it does not change the tunneling rate and can be neglected for our purpose. To illustrate our point we consider the simplest case of a uniaxial magnetic anisotropy with an easy axis along the Z -direction,

$$U_M = -\frac{1}{2} K_{\parallel} V \left(\frac{M_z}{M_0} \right)^2 \quad (28)$$

studied in Ref. 13. With account of Eq. (6) one obtains the following dynamical equations for the spherical angles describing the orientation of \mathbf{M}

$$\frac{d\phi}{dt'} = - \left(\sin \theta - \frac{I}{I_0} \sin \phi \right) \frac{\cos \theta}{\sin \theta} \quad (29)$$

$$\frac{d\theta}{dt'} = - \frac{I}{I_0} \cos \phi, \quad (30)$$

where $t' = \omega_{\text{FMR}} t$ with $\omega_{\text{FMR}} = \gamma K_{\parallel}/M_0$ being the frequency of the ferromagnetic resonance at $I = 0$, and

$$I_0 = \frac{K_{\parallel} V}{\varphi_0 E_J} I_c \quad (31)$$

Note that depending on the values of parameters entering Eq. (31) I_0 can be smaller or greater than I_c . At $I = 0$ the degenerate ground state corresponds to the magnetic moment parallel ($\theta = 0$) or antiparallel ($\theta = \pi$)

to the Z -axis. At $I < I_0$ (assuming also that $I < I_c$) the degenerate ground states are

$$\phi = \frac{\pi}{2}, \quad \sin \theta = \frac{I}{I_0}, \quad \cos \theta = \pm \sqrt{1 - \left(\frac{I}{I_0} \right)^2} \quad (32)$$

At $I_0 < I < I_c$ the non-degenerate ground state is $\phi = \pi/2, \theta = \pi/2$, corresponding to the magnetic moment looking in the Y -direction.

In the case of $I_0 < I_c$, when I is close but smaller than I_0 , the energy barrier between the degenerate states in Eq. (32) is small,

$$U(I) = \frac{1}{2} K_{\parallel} V \epsilon^2, \quad \epsilon = 1 - \frac{I}{I_0} \ll 1 \quad (33)$$

We are interested in the quantum tunneling between degenerate classical ground states: $\phi = \pi/2, \theta = \pi/2 \pm \sqrt{2}\epsilon$. Substituting $\theta = \pi/2 + \beta$ and $\phi = \pi/2 + \alpha$, with $|\alpha|, |\beta| \ll 1$, into Eqs. (29) and (30) one obtains

$$\frac{d\alpha}{dt'} = \left(\epsilon - \frac{\beta^2}{2} \right), \quad \frac{d\beta}{dt'} = \alpha \quad (34)$$

Here we have taken into account (see below) that $\alpha \sim \epsilon$ while $\beta \sim \sqrt{\epsilon}$, making $\alpha \ll \beta$. Combining the two equations, introducing $\bar{\beta} = \beta/\sqrt{2}\epsilon$ and the imaginary time $\bar{\tau} = it'\sqrt{\epsilon/2}$ we get

$$\frac{d\bar{\beta}}{d\bar{\tau}} = 1 - \bar{\beta}^2 \quad (35)$$

which has the instanton solution $\bar{\beta} = \tanh \bar{\tau}$,

$$\beta(\tau) = \sqrt{2}\epsilon \tanh \left(\sqrt{\frac{\epsilon}{2}} \omega_{\text{FMR}} \tau \right) \quad (36)$$

that switches β between $-\sqrt{2}\epsilon$ at $\tau = -\infty$ to $\sqrt{2}\epsilon$ at $\tau = \infty$.

Path integration of $\exp(i \int dt \mathcal{L}/\hbar)$ around the instanton with the Lagrangian given by Eq. (9) yields for the tunnel splitting $\Delta = A e^{-B}$ with

$$A \sim \hbar \omega_{\text{FMR}} \left(1 - \frac{I}{I_0} \right)^{1/2}, \quad B = \frac{4\sqrt{2}}{3} S \left(1 - \frac{I}{I_0} \right)^{3/2} \quad (37)$$

Notice that the tunneling rate in this case depends exponentially on the bias current. However, contrary to the previously studied case of biaxial magnetic anisotropy, quantum oscillations of the magnetic moment between classically degenerate states $\phi = \pi/2, \theta = \pi/2 \pm \sqrt{2}\epsilon$ formed by the uniaxial anisotropy along the Z -axis and a superconducting current in the X direction (see Fig. 1) do not produce oscillations or any other time dependence of φ . Consequently, in the limit of infinite resistance, they do not generate any voltage across the junction.

VI. DISCUSSION

We have investigated quantum dynamics of the magnetic moment in a ferromagnetic Josephson junction with broken inversion symmetry. Spin-orbit interaction in such a junction results in the anomalous Josephson effect in which a phase difference acquires an additional term φ_0 that depends on the magnetization. Quantum tunneling of the magnetic moment has been studied. Renormalization of the tunnel splitting by the interaction with the superconductor and decoherence due normal resistance of the junction have been computed. We have shown that quantum oscillation of the magnetic moment generates quantum oscillations of the voltage across the junction through coupling to the superconducting phase difference. This suggests an alternative method of detection of coherent quantum oscillations of a spin as compared to probing Rabi oscillations by electron spin resonance [22]. As for any spin-coherence measurements such experiments are challenging as they have to be conducted with the smallest junctions at low temperatures in order to freeze thermal superparamagnetic behavior. However, magnetic tunneling in nanoparticles has been studied with the help of Josephson junctions at millikelvin temperatures before [23]. Weak magnetic field of a nanoparticle deposited on the top of the bridge of a nano-SQUID was measured. The φ_0 -junction has a much stronger coupling of the magnetic moment to the Josephson dynamics. It, therefore, provides an interesting novel tool for the studies of magnetic tunneling.

The absolute value of the anomalous phase φ_0 determines the strength of the effect. It is given by [13] $\varphi_0 \equiv l(v_{so}/v_F)$, where $v_{so}/v_F \ll 1$ characterizes the strength of the spin-orbit interaction, and $l = 4LE_{ex}/(\hbar v_F)$, with L being the length of the ferromagnetic layer in the X -direction in Fig. 1 and E_{ex} being the energy of the exchange interaction between conducting electrons and localized ferromagnetic spins. Typically $v_{so}/v_F \sim 0.1$, $L < 10\text{nm}$, and $E_{ex} \sim 100 - 500\text{K}$, so φ_0 should be in the range $0.01 < \varphi_0 < 0.1$. At $\varphi_0 \sim 0.1$ the quality factor of quantum oscillations of the voltage in Eq. (27) is of order $Q \sim 0.1R(\Omega)$. Thus, observation of the oscillations requires R in excess of a few tens of ohms. At $\varphi_0 \sim 0.1$ and $\Delta_{\text{eff}} \sim 0.1\text{K}$ the initial ($t = 0$) amplitude of the ac voltage would be in the microvolt range, while the frequency, $\Delta_{\text{eff}}/(2\pi\hbar)$, would be in the GHz range.

Turning to the magnetic tunneling controlled by the bias current, one way to detect this effect would be with the help of a SQUID loop sensitive to a small Z -component of the magnetic moment that changes sign in the tunneling event [23]. Nano-SQUIDs permit detection of the change in the magnetic moment of only a few Bohr magnetons. In our case the change would be much greater. One problem for experiments performed with single-domain magnetic nanoparticles and molecules de-

posited on the bridge of a nano-SQUID was that the energy barrier and the tunneling rate had to be controlled by a strong magnetic field. This negatively affected the performance of the SQUID. The advantage of the φ_0 -junction is that the barrier and the tunneling rate can be accurately controlled by the bias current, with no need for the external magnetic field. We have shown that the current can decrease the tunneling barrier to a sufficiently small value that would make the tunneling rate large enough to observe quantum switching of the magnetic moment on the experimental time scale.

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