



# CHORUS

This is the accepted manuscript made available via CHORUS. The article has been published as:

## FFLO strange metal and quantum criticality in two dimensions: Theory and application to organic superconductors

Francesco Piazza, Wilhelm Zwerger, and Philipp Strack

Phys. Rev. B **93**, 085112 — Published 5 February 2016

DOI: [10.1103/PhysRevB.93.085112](https://doi.org/10.1103/PhysRevB.93.085112)

# FFLO strange metal and quantum criticality in two dimensions: theory and application to organic superconductors

Francesco Piazza<sup>1,2,\*</sup>, Wilhelm Zwerger<sup>2,†</sup> and Philipp Strack<sup>3,4‡</sup>

<sup>1</sup>*Institut für Theoretische Physik, Universität Innsbruck, A-6020 Innsbruck, Austria*

<sup>2</sup>*Physik Department, Technische Universität München, 85747 Garching, Germany*

<sup>3</sup>*Institut für Theoretische Physik, Universität zu Köln, D-50937 Cologne, Germany and*

<sup>4</sup>*Department of Physics, Harvard University, Cambridge MA 02138*

Increasing the spin imbalance in superconductors can spatially modulate the gap by forming Cooper pairs with finite momentum. For large imbalances compared to the Fermi energy, the inhomogeneous FFLO superconductor ultimately becomes a normal metal. There is mounting experimental evidence for this scenario in 2D organic superconductors in large in-plane magnetic fields; this is complemented by ongoing efforts to realize this scenario in coupled tubes of atomic Fermi gases with spin imbalance. Yet, a theory for the phase transition from a metal to an FFLO superconductor has not been developed so far and the universality class has remained unknown. Here we propose and analyze a spin imbalance driven quantum critical point between a 2D metal and an FFLO phase in anisotropic electron systems. We derive the effective action for electrons and bosonic FFLO pairs at this quantum phase transition. Using this action, we predict non-Fermi liquid behavior and the absence of quasi-particles at a discrete set of hot spots on the Fermi surfaces. This results in strange power-laws in thermodynamics and response functions, which are testable with existing experimental set-ups on 2D organic superconductors and may also serve as signatures of the elusive FFLO phase itself. The proposed universality class is distinct from previously known quantum critical metals and, because its critical fluctuations appear already in the pairing channel, a promising candidate for naked metallic quantum criticality over extended temperature ranges.

PACS numbers: 74.40.Kb, 71.10.Hf, 05.30.Rt, 74.70.Kn

## I. INTRODUCTION

The concept of electronic “quasiparticles” moving through a crystal and scattering off each other at rates smaller than their typical kinetic energy has—together with fermionic quantum statistics—led to a satisfactory understanding of the electrical conductance and the formation of superconductivity observed in many metals at low temperatures. However, an increasing number of observations in technologically relevant compounds indicate intriguing “strange metal” phases<sup>1</sup>, with long-ranged quantum entanglement<sup>2</sup>, where interactions destroy the electronic quasiparticles, and where electrical currents flow at anomalously slow rates<sup>3</sup>.

Here, we propose a new strange metal phase associated with an underlying quantum critical point (red region in Fig. 1) in anisotropic electronic systems at the onset of inhomogeneous FFLO (Fulde-Ferrell-Larkin-Ovchinnikov) superconductivity<sup>7–9</sup> in two spatial dimensions. We argue that organic superconductors<sup>10,11</sup> are promising candidates hosting this new phase of matter in which electronic quasiparticles are destroyed over regions of the Fermi surface from scattering off FFLO waves carrying a finite momentum. As a consequence, specific heat and NMR (nuclear magnetic resonance) relaxation rates become non-Fermi liquid power-laws, whose exponents we compute. We propose that a new round of data-taking with existing experimental set-ups<sup>4,6</sup> on for example the compound  $\kappa$ -(BEDT-TTF)<sub>2</sub>Cu(NCS)<sub>2</sub> could provide a near-future experimental test of our predictions.

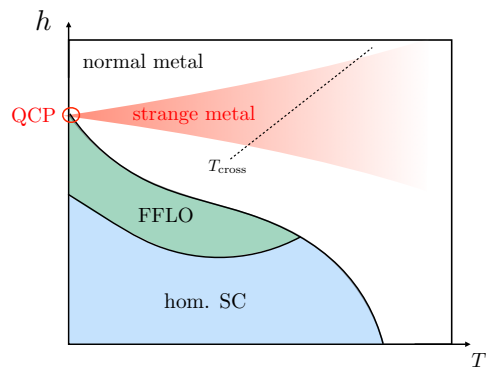


FIG. 1: New strange metal phase with non-Fermi liquid behavior (red region) at the quantum critical point for the onset of inhomogeneous FFLO superconductivity in anisotropic electronic systems in an in-plane magnetic field ( $h$ ) in two spatial dimensions. In the organic conductor  $\kappa$ -(BEDT-TTF)<sub>2</sub>Cu(NCS)<sub>2</sub>, recent evidence places the onset of FFLO correlations (green region) to high in-plane magnetic fields  $h \sim 24 - 30$ T and low temperatures ( $T < 4$ K)<sup>4–6</sup>. The black-dashed line is crossover scale  $T_{\text{cross}}$  explained in the text.

### A. FFLO phase in organic superconductors

Organic superconductors have emerged as leading candidates for observing FFLO superconducting states in electronic systems<sup>4–6,12–15</sup>; complementary efforts are undertaken in heavy Fermion<sup>16</sup> and iron-based<sup>17,18</sup> compounds, as well as in ultracold atoms<sup>19–21</sup>. The crystals of organic superconductors are grown relatively clean,

with a mean-free path larger than the coherence length of the pairs carrying a finite momentum  $\mathbf{Q}_{\text{FFLO}}$ . For in-plane magnetic fields, superconductivity can be destroyed by a Zeeman splitting before orbital effects become relevant. For an isotropic superconductor with no orbital effects, the critical Pauli limit field  $H_P$  is reached when the Zeeman energy  $\mu_B H_P > \Delta/\sqrt{2}$  overcomes the gap<sup>22,23</sup>. A Zeeman driven transition<sup>14</sup> requires an orbital critical field  $H_{\text{orb}}$  larger than  $H_P$ . The Maki parameters  $\alpha = \sqrt{2}H_{\text{orb}}/H_P$  must thus be larger than one, as in  $\kappa$ -(BEDT-TTF)<sub>2</sub>Cu(NCS)<sub>2</sub> where  $\alpha \simeq 8^4$ .

Due to only very weak and/or incoherent electron motion between the two-dimensional layers ( $\epsilon_F/t_{\text{interlayer}} \sim 3700$  for  $\kappa$ -(BEDT-TTF)<sub>2</sub>Cu(NCS)<sub>2</sub><sup>4</sup> and  $t_{\text{in-plane}}/t_{\text{interlayer}} \sim 500$  in the Bechgaard salt (TMTSF)<sub>2</sub>ClO<sub>2</sub><sup>13,24</sup>), out-of-plane orbital currents dissipate quickly. This is essentially true for the two compounds mentioned previously and also in the  $\beta''$ -phase compound  $\beta''$ -(BEDT-TTF)<sub>2</sub>SF<sub>5</sub>CH<sub>2</sub>CF<sub>2</sub>SO<sub>3</sub>, whose phase diagram and NMR spectra do not depend on the in-plane orientation of the  $\mathbf{B}$ -field<sup>52</sup>.

Fig. 2 shows three approximate Fermi surfaces from FFLO-candidate organic superconductors that share flat regions to which  $\mathbf{Q}_{\text{FFLO}}$  would couple to preferentially.  $\mathbf{Q}_{\text{FFLO}}$  are also incommensurate with the underlying crystal momenta. We expect our main results to be (i) universal across the various Fermi surface topologies, as well as (ii) relevant for both uni-directional (discussed later) and bi-directional FFLO modulation provided that the Fermi surface has (at least locally) nested parts which can –and will– be connected by fluctuating  $\mathbf{Q}_{\text{FFLO}}$  order.

## II. MODEL

To understand the main features of prototypical anisotropic 2D Fermi surfaces depicted in Fig. 2, let us consider a simplified model with a spin-dependent single particle dispersion

$$\xi_\sigma(\mathbf{k}) = k_x^2/2m - 2t_y \cos(dk_y) - \mu - \sigma h. \quad (1)$$

Here,  $m$  is the effective mass for the motion along the chains in  $x$ -direction. The chains are coupled by a weak hopping matrix element  $t_y$  as is the case for example in the Bechgaard salt (TMTSF)<sub>2</sub>ClO<sub>2</sub><sup>13,24</sup>. There, the hopping parallel to chains is  $t_x \sim 1340$  K, between the chains  $t_y \sim 134$  K and much weaker in the out-of-plane direction  $t_z \sim 2.6$  K<sup>24</sup>, the latter is therefore neglected in  $\xi_\sigma(\mathbf{k})$ . Orbital effects from  $t_z$ -hopping influence the (material-specific and non-universal) location of  $h_{c2}$ <sup>13,24,25</sup>. For the  $\kappa$ -compound the interlayer motion is very small and for the  $\beta''$ -compounds even incoherent, and plays therefore no role<sup>26</sup>.  $d$  is the distance between the chains ( $\sim 7\text{\AA}$  in (TMTSF)<sub>2</sub>ClO<sub>2</sub>),  $\mu$  is the chemical potential, and  $h$  the Zeeman field which splits the energies for spin-up ( $\sigma = 1$ ) and spin-down electrons ( $\sigma = -1$ ). The key feature of  $\xi_\sigma(\mathbf{k})$  is a nested pair of so called hot spots (indicated by black points in Fig. 2) connected by  $\mathbf{Q}_{\text{FFLO}}$ ; the universal

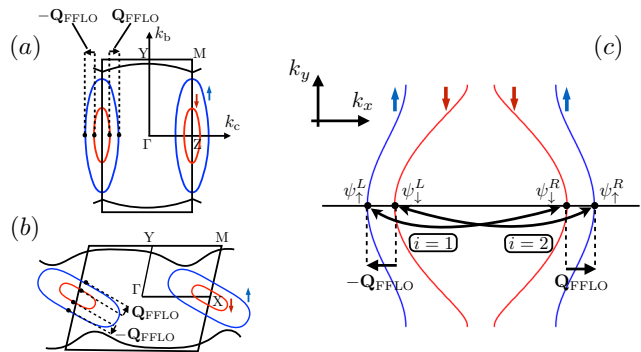


FIG. 2: Approximate Fermi surfaces geometry of candidate materials for the FFLO phase and associated strange metal. (a):  $\kappa$ -(BEDT-TTF)<sub>2</sub>Cu(NCS)<sub>2</sub> with closed elliptic hole pockets corresponding to a orbit frequency of about 600 Tesla<sup>4</sup> and open parts roughly aligned with the  $c$ -axis<sup>26</sup>. Therefore an in-plane magnetic field of  $h = 30$  T results in a  $\sim 5\%$  mismatch (exaggerated in the figure) of the spin-up and spin-down Fermi surfaces. Preferential FFLO ordering possibilities include a uni-directional modulation in  $c$ -direction along the short axis of the hole pocket, where  $\mathbf{Q}_{\text{FFLO}}$  would nest and connect the (Zeeman-split) sides of the hole pocket. (b):  $\beta''$ -(BEDT-TTF)<sub>2</sub>SF<sub>5</sub>CH<sub>2</sub>CF<sub>2</sub>SO<sub>3</sub> with tilted hole pockets. A preferential two-dimensional  $\mathbf{Q}_{\text{FFLO}}$ , with components along the  $b$  and  $c$  axis, would again connect the weakly curved parts of the hole pocket at a right angle. In both the  $\kappa$ - and  $\beta''$ -phase interlayer transport is weak and incoherent such that orbital effects can be neglected. (c): Open Zeeman-split Fermi sheets characteristic for the two-dimensional arrays of conducting chains of the Bechgaard salt (TMTSF)<sub>2</sub>ClO<sub>2</sub><sup>13,24</sup> in an in-plane magnetic field. The precise warping of the Fermi sheets also depends on the applied pressure<sup>27</sup>.

feature it shares with the Fermi surface topologies also of  $\kappa$ -(BEDT-TTF)<sub>2</sub>Cu(NCS)<sub>2</sub> for short-axis FFLO modulation, and  $\beta''$ -(BEDT-TTF)<sub>2</sub>SF<sub>5</sub>CH<sub>2</sub>CF<sub>2</sub>SO<sub>3</sub> with diagonal two-dimensional FFLO modulation as per Fig. 2. The most relevant interaction for the onset of spin-singlet FFLO pairing is a short-range attraction  $g$ :

$$\hat{H}_{\text{int}} = -g \int d^2\mathbf{r} \hat{\psi}_\uparrow^\dagger(\mathbf{r}) \hat{\psi}_\downarrow^\dagger(\mathbf{r}) \hat{\psi}_\downarrow(\mathbf{r}) \hat{\psi}_\uparrow(\mathbf{r}), \quad (2)$$

where the electrons are represented by anti-commuting operators  $\hat{\psi}_\sigma^\dagger, \hat{\psi}_\sigma$ . For our calculations below, we can assume a weak coupling scenario, where the microscopic origin of the pairing interaction plays no role<sup>53,54</sup>. The Fermi surface mismatch between spin-up  $\uparrow$  and spin-down  $\downarrow$  electrons frustrates conventional homogeneous BCS pairing, where the Cooper pairs  $\Delta_{\text{BCS}} = \langle \hat{\psi}_{-\mathbf{k},\uparrow} \hat{\psi}_{\mathbf{k},\downarrow} \rangle$  carry zero net momentum and the superconducting gap function is homogeneous in space. As analyzed in Fig. 3 for the specific  $\xi_\sigma(\mathbf{k})$  given above, the mismatched Fermi surfaces force the electrons to pair at a finite momentum  $\mathbf{Q}_{\text{FFLO}} = (Q_0, 0)$  leading, here in  $x$ -direction along the chains, to a modulated order  $\Delta_{\text{FFLO}} = \Delta_0 \cos(\mathbf{Q}_{\text{FFLO}} \cdot \mathbf{r})$ , like at a liquid crystal transition<sup>28,29</sup>.

### III. NOVEL EFFECTIVE ACTION

At zero temperature  $T = 0$ , the FFLO state forms out of the imbalanced metal at large magnetic fields upon decreasing  $h$  via a continuous quantum phase transition at  $h = h_{\text{QCP}}$  in Fig. 1 (see also Appendix). Gaps open at the hot spots, i.e. at the discrete points on the Fermi surface connected by  $\mathbf{Q}_{\text{FFLO}}$ . Our aim is now to describe quantum fluctuations of electrons around the hot spots that can scatter off the incipient FFLO order with momentum transfer  $\pm\mathbf{Q}_{\text{FFLO}} + \delta\mathbf{q}$  between hot spots. We parameterize the electron operator  $\hat{\psi}_{\uparrow\downarrow}$  by fermionic quantum fields  $\psi_{\uparrow\downarrow}^{R,L}(\tau, \mathbf{k})$  corresponding to the right and left part of the Fermi surface, as indicated in Fig. 2 (c). The construction goes completely analogously for the Fermi surfaces Fig. 2 (a,b). Fluctuations of the FFLO order parameter are described by one bosonic complex quantum field  $\Delta(\tau, \mathbf{k})_{1,2}$  per hot spot pair, with  $\tau$  the (imaginary) time coordinate. Fluctuations of both, the electrons and the FFLO field, are encoded in the partition function

$$Z = \int D\{\bar{\psi}_{\uparrow\downarrow}^{L,R}, \psi_{\uparrow\downarrow}^{L,R}\} D\{\Delta_{1,2}^* \Delta_{1,2}\} \exp(-\mathcal{S}), \quad (3)$$

where the exponential of the action  $\mathcal{S} = \int_0^\beta d\tau \int d^2\mathbf{r} \mathcal{L}$  weights the field configurations according to:

$$\begin{aligned} \mathcal{L} = & g \sum_{i=1,2} |\Delta_i|^2 + \sum_{\substack{\sigma=\uparrow,\downarrow \\ j=R,L}} \bar{\psi}_\sigma^j \left( \partial_\tau - iv_\sigma^j \partial_x + \frac{\partial_y^2}{2m_y} \right) \psi_\sigma^j \\ & - g [(\Delta_1^* \psi_\downarrow^R \psi_\uparrow^L + \Delta_2^* \psi_\downarrow^L \psi_\uparrow^R) + \text{h.c.}] . \end{aligned} \quad (4)$$

Here we have expanded the electronic dispersion around the hot spots  $\xi_\sigma^{R,L}(\mathbf{k}) = v_\sigma^{R,L} k_x + k_y^2/2m_y$ , with  $x, y$  being the direction orthogonal and parallel to the Fermi surface, respectively.  $m_y$  is (inversely) proportional to the curvature of the Fermi surface (and thus to  $t_y$ ) and related to the applied pressure in the Bechgaard salt (TMTSF)<sub>2</sub>ClO<sub>2</sub><sup>13,24,27</sup>. Further, we note that in the absence of time-reversal symmetry Eq. (4) is not invariant under the spin-flip transformation  $\uparrow \leftrightarrow \downarrow$ . The fermion-boson vertex  $g$  converts bosons with momentum  $\mathbf{Q}_{\text{FFLO}}$  into electron pairs, one at each hot spot of a given pair. The finite curvatures of the Fermi surface allow to restrict Eq. (4) to the hot spots provided temperatures are smaller than  $T_{\text{cross}} \sim t_y h / \epsilon_F$  (cf. Fig. 1).

#### A. Distinct universality class

The electronic propagators in Eq. (4) bear some similarity to those in two-patch theories for metals coupled to nematic and  $U(1)$ -gauge field fluctuations<sup>30,31</sup> as well as for the onset of incommensurate  $2k_F$  antiferromagnetism<sup>32</sup>. An important difference to these latter cases is that the electronic interactions in the particle-hole channel and that they preserve time-reversal

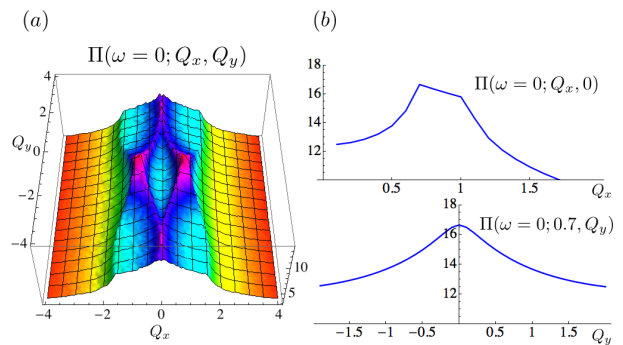


FIG. 3: **(a)**: Pairing susceptibility  $\Pi(\omega_n = 0, \mathbf{Q})$  at zero frequency and finite momentum for the Fermi surfaces c) in Fig. 2. The position of the maximal peaks at  $\mathbf{Q}_{\text{FFLO}} = (Q_0, 0)$  determine the, here unidirectional, modulation wave vector of the FFLO-phase. **(b)**: cuts of the pairing susceptibility around the peak at  $\mathbf{Q}_{\text{FFLO}}$ , displaying the asymmetrically-sloped ridges along  $Q_x$  characteristic of incommensurate symmetry-breaking. To capture quantum fluctuations around the peaks  $\pm\mathbf{Q}_{\text{FFLO}} + \delta\mathbf{q}$ , we later parameterize the pairing fluctuation propagator with a linear slope from the “outside” and a non-analytic square-root from the “inside” ridge. Along  $Q_y$  the peak is symmetric. The susceptibility  $\Pi(\omega_n, \mathbf{q}) = \frac{g^2}{\beta} \sum_{\mathbf{k}} \frac{1 - n_F(\beta\xi_\downarrow(\mathbf{q}-\mathbf{k})) - n_F(\beta\xi_\uparrow(\mathbf{k}))}{\xi_\uparrow(\mathbf{k}) + \xi_\downarrow(\mathbf{q}-\mathbf{k}) - i\omega_n}$  involves a sum  $\sum_{\mathbf{k}}$  over available states in momentum space (numerator) weighted by the inverse of their energies (denominator). Here  $n_F(E) = (1 + \exp(\beta E))^{-1}$  is the Fermi function and  $\beta = 1/k_B T$  the inverse temperature. Parameter values for the plot are  $t_y = 0.5 \hbar^2/md^2$ ,  $\mu = 3.3 \hbar^2/md^2$ ,  $h = 1.0 \hbar^2/md^2$ .

symmetry. In contrast, Eq. (4) defines a new universality class as it describes singular electronic interactions in the (FFLO) pairing channel and the Zeeman field breaks time-reversal symmetry.

### IV. ANOMALOUS QUANTUM DYNAMICS

The first step toward understanding quantum fluctuations is to compute the dynamics of the bosonic FFLO pairing field  $D_{i=1}(\tau, \mathbf{r}) = \langle \hat{\Delta}_1(\tau, \mathbf{r}) \hat{\Delta}_1^\dagger(0, \mathbf{0}) \rangle$ , which it picks up from emission and absorption of electron pairs. In frequency and momentum space, the interest lies in the slopes and direction dependence of the pairing susceptibility close to the maxima in Fig. 3. The leading contribution to  $D_1(\omega_n, \mathbf{k})$  is given by the one-loop bubble diagram evaluated in the Appendix. For the critical properties of interest here, it suffices to expand the (massless) fluctuation propagator directly at the quantum critical point for small frequencies  $\omega_n$  and momenta  $\pm\mathbf{Q}_{\text{FFLO}} + \delta\mathbf{k}$ . The result in the physically relevant small imbalance limit,  $\delta v = v_\uparrow - v_\downarrow \ll v = (v_\uparrow + v_\downarrow)/2$  (recall that an in-plane magnetic field of 30 T leads to a  $\sim 5\%$  mismatch in spin-up and down Fermi surfaces in

$\kappa$ -(BEDT-TTF)<sub>2</sub>Cu(NCS)<sub>2</sub><sup>4</sup>) is:

$$D_1^{-1}(\omega_n, \mathbf{k}) = \frac{V\sqrt{2m_y}}{4\pi v|\delta v/v|} \left[ 2\text{Re} \sqrt{-\frac{k_y^2}{2m_y} + \delta v k_x + \frac{\delta v}{v} i\omega_n} + B \frac{k_y^2}{2m_y v} + C \frac{\delta v}{v} k_x \right]. \quad (5)$$

The key feature here is the square root, introducing a non-analyticity; the usual poles of coherent excitations are substituted by a branch cut in the complex plane. Physically, this reflects the overdamped, dissipative dynamics of the order parameter due to decay into electron pairs. The subleading corrections to the square root singularity proportional to  $B, C$  are the first analytic corrections which capture the asymmetry in the slopes of the peaks in Fig. 3. Eq. (5) bears some resemblance to the one-loop results for the order parameter dynamics in the incommensurate antiferromagnetic and charge order case<sup>32,33</sup>, which are singular fluctuations in the particle-hole channel of different physical origin.

### A. Non-Fermi liquid behavior of electrons

The peculiar dynamics Eq. (5) of the FFLO pairing fluctuations strongly back-acts on the electrons close to the hot spots. This renormalizes the electron propagator, say, at the  $L \uparrow$  hotspot,  $G_{L\uparrow}(\tau, \mathbf{r}) = \langle \hat{\psi}_{L\uparrow}(\tau, \mathbf{r}) \hat{\psi}_{L\uparrow}^\dagger(0, \mathbf{0}) \rangle$  whose Fourier transform reads  $[G_{L\uparrow}(\nu_n, \mathbf{k})]^{-1} = i\nu_n - \xi_{L\uparrow}^L(\mathbf{k}) - \Sigma_{L\uparrow}(\nu_n, \mathbf{k})$ . The inverse quasi-particle lifetime  $\Gamma^{-1}$  at the hot spots for propagators of the form (5) can be computed analytically<sup>33</sup>. At the QCP, evaluation of the one-loop self-energy with the dynamical pairing propagator yields (see Appendix

$$\Gamma_{\text{FFLO}}^{-1} = \text{Im} \Sigma_{L\uparrow}^{\text{ret}}(\omega, \mathbf{q}_{\text{HS}}) = \frac{1}{\sqrt{3}} \left( \frac{|\delta v/v| |\omega|}{B} \right)^{2/3}, \quad (6)$$

exhibits non-Fermi liquid behavior with a fractional exponent  $2/3$  due to the interactions with the singular FFLO pairing fluctuations (recall that in a Fermi liquid  $\Gamma_{\text{FL}}^{-1} \sim \omega^2$ ). The strength of the anomalous decay in Eq. (6) is proportional to the difference  $\delta v$  of the Fermi velocities which itself is proportional to the applied Zeeman field. In the language of critical phenomena Eq. (6) implies an anomalous frequency dimension  $\eta_\tau = 1/3$  as the bare propagator is linear in frequency<sup>34-36</sup>. Probably weaker non-Fermi liquid behavior may also appear in homogeneous Fermi systems with Zeeman-imbalance<sup>37-39</sup>. A similar computation for the electron self-energy in  $k_x$  and  $k_y$  direction yields a much weaker renormalization such that the corresponding  $\eta_k$ 's vanish at the one-loop level. For direct loop computations (in absence of infrared cutoffs), the fermionic dynamic exponent is defined as  $z^f = \frac{1-\eta_k}{1-\eta_\tau}$  and deviates from the Fermi liquid value ( $z_{\text{FL}}^f = 1$ ) and instead attains the value  $z_{\text{FFLO, one-loop}}^f =$

$\frac{3}{2}$ . We note this value is also obtained in nematic metals at the one-loop level<sup>40</sup> but receives quantitative corrections at higher loops<sup>31,41</sup>. We expect diagrammatic expansions of the path integral of Eq. (4) to be different to the nematic metal, however. One-loop corrections to the fermion-boson vertex, for example, vanish here in the FFLO case.

## V. PREDICTIONS FOR NEW EXPERIMENTS

The general scaling form of the electronic specific heat in the quantum critical ‘‘fan’’ in Fig. 1 is<sup>42,43</sup>

$$C_e/T \sim T^{\frac{d-\theta}{z^f}-1} = T^{-0.33}, \quad (7)$$

where the last equality has used  $z_{\text{FFLO, one-loop}}^f = 3/2$ ,  $d = 2$ , and  $\theta = 1$  in case hyperscaling is violated. Recall that in a Fermi liquid  $C_e/T$  is independent of temperature. For  $\kappa$ -(BEDT-TTF)<sub>2</sub>Cu(NCS)<sub>2</sub>, the specific heat has been measured by Lortz *et al.*<sup>4,44</sup> over a range of B-fields including values towards the upper phase-boundary of the FFLO phase: an upswing with decreasing  $T$  incompatible with Fermi liquid behavior can be seen at  $h = 28$  T but new rounds of data taking at high magnetic fields are needed to resolve this.

A more recent experiment, where signatures of FFLO order have been detected in the compound  $\kappa$ -(BEDT-TTF)<sub>2</sub>Cu(NCS)<sub>2</sub> was performed by Mayaffre *et al.* who have measured the NMR relaxation rate  $1/(T_1 T)$  at high magnetic fields including data set at  $h = 22, 27$  Tesla (see Fig. 2 of Ref. 6). They associate the observed upswing at  $h = 22$  Tesla with bound states of quasiparticles at the nodes of an FFLO order parameter. At  $h = 27$  Tesla, however,  $1/(T_1 T)$  seems to decrease for temperatures smaller than 3 Kelvin. Our strange metal scenario predicts a power law decrease of the form  $1/(T_1 T) \sim T^{2/3}$  which is suppressed compared to a constant Fermi liquid relaxation rate. New rounds of data taking over extended temperature and higher magnetic fields should resolve this. The power-law exponent in  $1/(T_1 T)$  is determined by the scaling behavior of the density of states

$$\frac{1}{T_1 T} \sim \frac{1}{T} \int d\omega N_\uparrow^{\text{hot}}(\omega) N_\downarrow^{\text{hot}}(\omega) n_F(\omega) [1 - n_F(\omega)] \quad (8)$$

with  $N_\sigma^{\text{hot}}(\omega) = \int \frac{d^2 \mathbf{k}}{(2\pi)^2} A_\sigma^{\text{hot}}(\omega, \mathbf{k})$ . For the critical electrons at the hot spots the associated spectral function is expected to fulfill  $\omega/T$  scaling<sup>42,45</sup>

$$A_\sigma^{\text{hot}}(\omega, \mathbf{k}) \sim \frac{c_0}{|\omega|^{1-\eta_\tau}} \mathcal{F}_\sigma \left( \frac{c_1 \omega}{(k_x + k_y^2)^{z_f}}, \frac{\omega}{T} \right), \quad (9)$$

where the anomalous scaling in spatial directions  $x$  and  $y$  can take different finite values but do not influence  $z_f$  at the one-loop level. The same temporal anomalous dimension  $\eta_\tau = 1/3$  that appears in the self-energy Eq. (6) leads to  $N_\sigma^{\text{hot}}(\omega) \sim \omega^{1/3}$  (see Appendix) and thus to the

exponent  $2/3$  in Eq. (8) for electrons from the immediate vicinity of the hot spots. As in the nematic metal, we expect higher-loop corrections to further suppress the density of states<sup>31</sup>. In contrast to the nematic metal, however, where all points on the Fermi surface can be taken to be equivalent<sup>31,45</sup>, here in the FFLO strange metal a discrete pair of hot spots is singled out. To account for electrons from “colder” parts of the Fermi surface one should add a constant Fermi liquid piece to the density of states  $N_\sigma(\omega) = N_{0\sigma}^{\text{cold}} + N_\sigma^{\text{hot}}(\omega)$  and NMR relaxation rate when comparing with experiments. A proper analysis of crossovers and potential contributions violating  $\omega/T$  scaling<sup>46,47</sup> require momentum-resolved renormalization group techniques.

## VI. SUMMARY

The FFLO strange metal phase proposed here provides a new universality class for the study of critical thermodynamics and transport without electronic quasi-

particles. Our results open up the possibility to targeted new rounds of data taking in the organic superconductors and may facilitate access to a naked quantum critical metal with critical fluctuations in the pairing channel over extended temperature ranges.

## Acknowledgments

We thank Jochen Wosnitza for discussions and for providing us with the raw specific heat data of Ref. 4. Moreover, we thank Peter Fulde, Tobias Holder, Matthias Vojta, and Tarik Yefsah for discussions and gratefully acknowledge useful comments on the manuscript by Andreas Eberlein and Walter Metzner. PS acknowledges support by the Leibniz prize of A. Rosch, a MURI grant W911NF-14-1-0003 from ARO, a NSF grant DMR-1360789, and the MIT-Harvard Center for Ultracold Atoms. FP acknowledges support by the APART fellowship of the Austrian Academy of Sciences.

\* Electronic address: francesco.piazza@ph.tum.de

† Electronic address: zwerger@tum.de

‡ Electronic address: strack@thp.uni-koeln.de

<sup>1</sup> S. Sachdev, “Strange and stringy,” *Scientific American*, vol. 308, p. 44, Dec 2013.

<sup>2</sup> J. Shao, E.-A. Kim, F. D. M. Haldane, and E. H. Rezayi, “Entanglement entropy of the  $\nu = 1/2$  composite fermion non-fermi liquid state,” *Phys. Rev. Lett.*, vol. 114, p. 206402, May 2015.

<sup>3</sup> J. A. N. Bruin, H. Sakai, R. S. Perry, and A. P. Mackenzie, “Similarity of scattering rates in metals showing t-linear resistivity,” *Science*, vol. 339, p. 804, Feb 2013.

<sup>4</sup> R. Lortz, Y. Wang, A. Demuer, P. H. M. Böttger, B. Bergk, G. Zwicknagl, Y. Nakazawa, and J. Wosnitza, “Calorimetric evidence for a fulde-ferrell-larkin-ovchinnikov superconducting state in the layered organic superconductor  $\kappa$ -(BEDT-TTF)<sub>2</sub>Cu(NCS)<sub>2</sub>,” *Phys. Rev. Lett.*, vol. 99, p. 187002, Oct 2007.

<sup>5</sup> B. Bergk, A. Demuer, I. Sheikin, Y. Wang, J. Wosnitza, Y. Nakazawa, and R. Lortz, “Magnetic torque evidence for the fulde-ferrell-larkin-ovchinnikov state in the layered organic superconductor  $\kappa$ -(BEDT-TTF)<sub>2</sub>Cu(NCS)<sub>2</sub>,” *Phys. Rev. B*, vol. 83, p. 064506, Feb 2011.

<sup>6</sup> H. Mayaffre, S. Krämer, M. Horvatić, C. Berthier, K. Miyagawa, K. Kanoda, and V. F. Mitrovic, “Evidence of andreev bound states as a hallmark of the fflo phase in  $\kappa$ -(BEDT-TTF)<sub>2</sub>Cu(NCS)<sub>2</sub>,” *Nature Physics*, vol. 10, p. 928, Oct 2014.

<sup>7</sup> P. Fulde and R. A. Ferrell, “Superconductivity in a strong spin-exchange field,” *Phys. Rev.*, vol. 135, p. A550, 1964.

<sup>8</sup> A. I. Larkin and Y. N. Ovchinnikov, “Inhomogeneous state of superconductors,” *Sov. Phys. JETP*, vol. 20, p. 762, 1965.

<sup>9</sup> R. Casalbuoni and G. Nardulli, “Inhomogeneous superconductivity in condensed matter and qcd,” *Rev. Mod. Phys.*, vol. 76, pp. 263–320, Feb 2004.

<sup>10</sup> T. Ishiguro, Yamaji, and G. Saito, *Organic Superconductors*. Springer, 1998.

<sup>11</sup> A. G. Lebed, *The Physics of Organic Superconductors and Conductors*. Springer, 2008.

<sup>12</sup> J. Shinagawa, Y. Kurosaki, F. Zhang, C. Parker, S. E. Brown, D. Jérôme, J. B. Christensen, and K. Bechgaard, “Superconducting state of the organic conductor (TMTSF)<sub>2</sub>clo<sub>4</sub>,” *Phys. Rev. Lett.*, vol. 98, p. 147002, Apr 2007.

<sup>13</sup> S. Yonezawa, S. Kusaba, Y. Maeno, P. Auban-Senzier, C. Pasquier, K. Bechgaard, and D. Jérôme, “Anomalous in-plane anisotropy of the onset of superconductivity in (TMTSF)<sub>2</sub>clo<sub>4</sub>,” *Phys. Rev. Lett.*, vol. 100, p. 117002, Mar 2008.

<sup>14</sup> J. A. Wright, E. Green, P. Kuhns, A. Reyes, J. Brooks, J. Schlueter, R. Kato, H. Yamamoto, M. Kobayashi, and S. E. Brown, “Zeeman-driven phase transition within the superconducting state of  $\kappa$ -(BEDT-TTF)<sub>2</sub>Cu(NCS)<sub>2</sub>,” *Phys. Rev. Lett.*, vol. 107, p. 087002, Aug 2011.

<sup>15</sup> R. Beyer, B. Bergk, S. Yasin, J. A. Schlueter, and J. Wosnitza, “Angle-dependent evolution of the fulde-ferrell-larkin-ovchinnikov state in an organic superconductor,” *Phys. Rev. Lett.*, vol. 109, p. 027003, Jul 2012.

<sup>16</sup> C. Pfeleiderer, “Superconducting phases of  $f$ -electron compounds,” *Rev. Mod. Phys.*, vol. 81, pp. 1551–1624, Nov 2009.

<sup>17</sup> K. Cho, H. Kim, M. A. Tanatar, Y. J. Song, Y. S. Kwon, W. A. Coniglio, C. C. Agosta, A. Gurevich, and R. Prozorov, “Anisotropic upper critical field and possible fulde-ferrell-larkin-ovchinnikov state in the stoichiometric pnictide superconductor lifeas,” *Phys. Rev. B*, vol. 83, p. 060502, Feb 2011.

<sup>18</sup> A. Ptok and D. Crivelli, “The fulde-ferrell-larkin-ovchinnikov state in pnictides,” *Journal of Low Temperature Physics*, vol. 172, no. 3-4, pp. 226–233, 2013.

<sup>19</sup> M. W. Zwierlein, A. Schitzorek, C. H. Schunck, and

- W. Ketterle, “Fermionic superfluidity with imbalanced spin populations and the quantum phase transition to the normal state,” *Science*, vol. 311, p. 492, 2006.
- <sup>20</sup> Y. Liao, A. Rittner, T. Paprotta, L. Wenhui, G. B. Partridge, R. G. Hulet, S. K. Baur, and E. J. Mueller, “Spin-imbalance in a one-dimensional fermi gas,” *Nature*, vol. 467, p. 567, Mar 2010.
- <sup>21</sup> W. Ong, C. Cheng, I. Arakelyan, and J. E. Thomas, “Spin-imbanced quasi-two-dimensional fermi gases,” *Phys. Rev. Lett.*, vol. 114, p. 110403, Mar 2015.
- <sup>22</sup> B. S. Chandrasekhar, “A note on the maximum critical field of high-field superconductors,” *Appl. Phys. Lett.*, vol. 1, 1962.
- <sup>23</sup> A. M. Clogston, “Upper limit for the critical field in hard superconductors,” *Phys. Rev. Lett.*, vol. 9, pp. 266–267, Sep 1962.
- <sup>24</sup> A. G. Lebed and S. Wu, “Larkin-ovchinnikov-fulde-ferrell phase in the superconductor  $(\text{TMTSF})_2\text{ClO}_4$ : Theory versus experiment,” *Phys. Rev. B*, vol. 82, p. 172504, Nov 2010.
- <sup>25</sup> N. Dupuis and G. Montambaux, “Superconductivity of quasi-one-dimensional conductors in a high magnetic field,” *Phys. Rev. B*, vol. 49, pp. 8993–9008, Apr 1994.
- <sup>26</sup> J. Wosnitzer, “Quasi-two-dimensional organic superconductors,” *Journal of Low Temp. Phys.*, vol. 146, p. 641, Mar 2007.
- <sup>27</sup> A. Sedeki, D. Bergeron, and C. Bourbonnais, “Extended quantum criticality of low-dimensional superconductors near a spin-density-wave instability,” *Phys. Rev. B*, vol. 85, p. 165129, Apr 2012.
- <sup>28</sup> L. Radzihovsky and A. Vishwanath, “Quantum liquid crystals in an imbalanced fermi gas: Fluctuations and fractional vortices in larkin-ovchinnikov states,” *Phys. Rev. Lett.*, vol. 103, p. 010404, Jul 2009.
- <sup>29</sup> K. V. Samokhin, “Spectrum of goldstone modes in larkin-ovchinnikov-fulde-ferrell superfluids,” *Phys. Rev. B*, vol. 83, p. 094514, Mar 2011.
- <sup>30</sup> S.-S. Lee, “Low-energy effective theory of fermi surface coupled with  $u(1)$  gauge field in  $2 + 1$  dimensions,” *Phys. Rev. B*, vol. 80, p. 165102, Oct 2009.
- <sup>31</sup> M. A. Metlitski and S. Sachdev, “Quantum phase transitions of metals in two spatial dimensions. i. ising-nematic order,” *Phys. Rev. B*, vol. 82, p. 075127, Aug 2010.
- <sup>32</sup> B. L. Altshuler, L. B. Ioffe, and A. J. Millis, “Critical behavior of the  $t = 0$   $2k_f$  density-wave phase transition in a two-dimensional fermi liquid,” *Phys. Rev. B*, vol. 52, pp. 5563–5572, Aug 1995.
- <sup>33</sup> T. Holder and W. Metzner, “Non-fermi-liquid behavior at the onset of incommensurate  $2k_f$  charge- or spin-density wave order in two dimensions,” *Phys. Rev. B*, vol. 90, p. 161106(R), Oct 2014.
- <sup>34</sup> A. Abanov and A. V. Chubukov, “Spin-fermion model near the quantum critical point: One-loop renormalization group results,” *Phys. Rev. Lett.*, vol. 84, pp. 5608–5611, Jun 2000.
- <sup>35</sup> J. Lee, P. Strack, and S. Sachdev, “Quantum criticality of reconstructing fermi surfaces in antiferromagnetic metals,” *Phys. Rev. B*, vol. 87, p. 045104, Jan 2013.
- <sup>36</sup> S. Sur and S.-S. Lee, “Quasilocal strange metal,” *Phys. Rev. B*, vol. 91, p. 125136, Mar 2015.
- <sup>37</sup> K. V. Samokhin and M. S. Mar’enko, “Quantum fluctuations in larkin-ovchinnikov-fulde-ferrell superconductors,” *Phys. Rev. B*, vol. 73, p. 144502, Apr 2006.
- <sup>38</sup> A. J. A. James and A. Lamacraft, “Non-fermi-liquid fixed point for an imbalanced gas of fermions in  $1 + \epsilon$  dimensions,” *Phys. Rev. Lett.*, vol. 104, p. 190403, May 2010.
- <sup>39</sup> P. Strack and P. Jakubczyk, “Fluctuations of imbalanced fermionic superfluids in two dimensions induce continuous quantum phase transitions and non-fermi-liquid behavior,” *Phys. Rev. X*, vol. 4, p. 021012, Apr 2014.
- <sup>40</sup> W. Metzner, D. Rohe, and S. Andergassen, “Soft fermi surfaces and breakdown of fermi-liquid behavior,” *Phys. Rev. Lett.*, vol. 91, p. 066402, Aug 2003.
- <sup>41</sup> T. Holder and W. Metzner, “Anomalous dynamical scaling from nematic and  $u(1)$ -gauge field fluctuations in two dimensional metals,” *arXiv:1503.05089*, March 2015.
- <sup>42</sup> T. Senthil, “Critical fermi surfaces and non-fermi liquid metals,” *Phys. Rev. B*, vol. 78, p. 035103, Jul 2008.
- <sup>43</sup> S. A. Hartnoll and A. Karch, “Scaling theory of the cuprate strange metals,” *Phys. Rev. B*, vol. 91, p. 155126, Apr 2015.
- <sup>44</sup> R. Beyer and J. Wosnitzer, “Emerging evidence for ffo states in layered organic superconductors,” *Low Temp. Phys.*, vol. 39, p. 225, March 2013.
- <sup>45</sup> S. Sachdev, *Quantum Phase Transitions*. Cambridge University Press, 2011.
- <sup>46</sup> A. Abanov, A. V. Chubukov, and J. Schmalian, “Quantum-critical theory of the spin-fermion model and its application to cuprates: Normal state analysis,” *Advances in Physics*, vol. 52, no. 3, pp. 119–218, 2003.
- <sup>47</sup> L. Dell’Anna and W. Metzner, “Fermi surface fluctuations and single electron excitations near pomeranchuk instability in two dimensions,” *Phys. Rev. B*, vol. 73, p. 045127, Jan 2006.
- <sup>48</sup> M. M. Parish, S. K. Baur, E. J. Mueller, and D. A. Huse, “Quasi-one-dimensional polarized fermi superfluids,” *Phys. Rev. Lett.*, vol. 99, p. 250403, Dec 2007.
- <sup>49</sup> A. Lüscher, R. M. Noack, and A. M. Läuchli, “Fulde-ferrell-larkin-ovchinnikov state in the one-dimensional attractive hubbard model and its fingerprint in spatial noise correlations,” *Phys. Rev. A*, vol. 78, p. 013637, Jul 2008.
- <sup>50</sup> A. E. Feiguin and F. Heidrich-Meisner, “Pair correlations of a spin-imbanced fermi gas on two-leg ladders,” *Phys. Rev. Lett.*, vol. 102, p. 076403, Feb 2009.
- <sup>51</sup> R. M. Lutchyn, M. Dzero, and V. M. Yakovenko, “Spectroscopy of the soliton lattice formation in quasi-one-dimensional fermionic superfluids with population imbalance,” *Phys. Rev. A*, vol. 84, p. 033609, Sep 2011.
- <sup>52</sup> J. Wosnitzer, private communication
- <sup>53</sup> Microscopically, the attractive interactions may thus be either due to a phonon-exchange mechanism or due to spin-fluctuation mediated pairing.
- <sup>54</sup> Note also that a model with the simple single particle dispersion  $\xi_\sigma(\mathbf{k})$  of Fig. 2(c) and an adjustable attraction can be realized with ultracold atoms, where both the band structure and strength of the attraction can be engineered via optical lattices and Feshbach resonances<sup>20,21,48–51</sup>.

## Appendix A: Continuous mean-field FFLO transition

We decouple the interaction term (Eq.(1) of the main text) in the pairing channel and write the partition function as the following functional integral

$$Z = \int D(\bar{\psi}_{\uparrow,\downarrow}, \psi_{\uparrow,\downarrow}) D(\Delta^*, \Delta) e^{-S_{\text{tot}}[\bar{\psi}_{\uparrow,\downarrow}, \psi_{\uparrow,\downarrow}, \Delta^*, \Delta]} \quad (\text{A1})$$

for the complex Grassmann field  $\psi$  and the bosonic field  $\Delta$ , with the euclidean interaction action

$$S_{\text{int}} = \int_0^\beta d\tau \int d^2\mathbf{r} \left[ \frac{1}{g} |\Delta(\tau, \mathbf{r})|^2 - (\Delta^*(\tau, \mathbf{r}) \psi_{\downarrow}(\tau, \mathbf{r}) \psi_{\uparrow}(\tau, \mathbf{r}) + \Delta(\tau, \mathbf{r}) \bar{\psi}_{\uparrow}(\tau, \mathbf{r}) \bar{\psi}_{\downarrow}(\tau, \mathbf{r})) \right].$$

An effective Ginzburg-Landau theory for the pairing field is obtained by integrating out the fermionic fields. Written in Fourier space the latter reads

$$S_{\text{eff}} = \frac{\beta}{g} \sum_{\omega_n, \mathbf{k}} |\Delta(\omega_n, \mathbf{k})|^2 - \text{Tr} \ln \beta \underline{G}^{-1}, \quad (\text{A2})$$

where  $\text{Tr} = \sum_{\nu_n, \mathbf{k}} \text{tr}$ , with  $\text{tr}$  meaning the trace in the 2x2 spinor space, the bosonic(fermionic) Matsubara frequency  $\omega_n = 2\pi n/\beta$  ( $\nu_n = \pi(2n+1)/\beta$ ),  $\beta$  the inverse temperature, and with the matrix propagator

$$\underline{G}^{-1}(\nu_n, \mathbf{k}; \nu'_n, \mathbf{k}') = \begin{pmatrix} \delta_{\nu_n, \nu'_n} \delta_{\mathbf{k}, \mathbf{k}'} (i\nu_n - \xi_{\uparrow}(\mathbf{k})) & \Delta(\nu_n - \nu'_n, \mathbf{k} - \mathbf{k}') \\ \Delta^*(\nu'_n - \nu_n, \mathbf{k}' - \mathbf{k}) & \delta_{\nu_n, \nu'_n} \delta_{\mathbf{k}, \mathbf{k}'} (i\nu_n + \xi_{\downarrow}(-\mathbf{k})) \end{pmatrix}, \quad (\text{A3})$$

where  $\xi_{\sigma}(\mathbf{k})$  is the fermionic dispersion relation. Now, upon expanding the logarithm in powers of  $\Delta$ , we can compute the coefficients of the effective Ginzburg-Landau theory. In particular, the second order term contains the pairing susceptibility

$$\Pi(\omega_n, \mathbf{q}) = \sum_{\mathbf{k}} \frac{1 - n_F(\beta \xi_{\downarrow}(\mathbf{q} - \mathbf{k})) - n_F(\beta \xi_{\uparrow}(\mathbf{k}))}{\xi_{\uparrow}(\mathbf{k}) + \xi_{\downarrow}(\mathbf{q} - \mathbf{k}) - i\omega_n}. \quad (\text{A4})$$

Specifying now to the mean-field FFLO transition with favoured momentum  $|\mathbf{Q}_{\text{FFLO}}|$  we can express the pairing field as

$$\Delta(\omega_n, \mathbf{q}) \rightarrow \Delta_0(0, \mathbf{Q}_{\text{FFLO}}) = \beta \delta_{\omega_n, 0} [d_{+\mathbf{Q}_{\text{FFLO}}} \delta_{\mathbf{q}, \mathbf{Q}_{\text{FFLO}}} (2\pi)^2 + d_{-\mathbf{Q}_{\text{FFLO}}} \delta_{\mathbf{q}, -\mathbf{Q}_{\text{FFLO}}} (2\pi)^2]. \quad (\text{A5})$$

The  $d$ 's are the amplitudes of the FFLO-order parameter, that are chosen to be equal and real.

The quadratic Landau coefficient is thus given by

$$a_2 = 1/g - \Pi(\omega_n = 0, \mathbf{Q}_{\text{FFLO}}).$$

The quartic Landau  $a_4$  coefficient results from the expansion of the logarithm to fourth order in  $\Delta$  and contains all possible contractions of four fermionic propagators with the four external momenta fixed at  $\pm \mathbf{Q}_{\text{FFLO}}$  and zero frequency. It must be numerically calculated. All contractions diverge when  $T \rightarrow 0$ , indicating the need to devise a renormalization group procedure for the problem at very low temperatures. For generic choices of parameters

$$\lim_{T \rightarrow 0} a_4 > 0 \quad (\text{A6})$$

indicative of a continuous phase transition at least at the mean-field level.

## Appendix B: Pairing fluctuation propagator, fermion propagator and density of states

In this section we compute the one-loop corrections to the pairing and fermion propagators, leading to the Eqs.(4) and (5) of the main text. We use the low-energy Lagrangian from Eq.(3) of the main text, whose Fermi surface structure and one-loop diagrams are shown in Fig. 4.

The inverse pairing fluctuation propagator diagrammatically shown in the upper row of Fig. 4(b) contains the bare part, simply given by the Landau coefficient  $a_2$  (vanishing at the critical point), plus the pairing susceptibility:

$$\Pi_{i=1}(\omega_n, \mathbf{q}) = \frac{1}{\beta} \sum_{\nu_n, \mathbf{k}} G_{R\downarrow}^{(0)}(\nu_n, \mathbf{k}) G_{L\uparrow}^{(0)}(\omega_n - \nu_n, \mathbf{q} - \mathbf{k}) = \frac{1}{\beta} \sum_{\nu_n, \mathbf{k}} \frac{1}{i\nu_n - \xi_{\downarrow}^R(\mathbf{k})} \frac{1}{i(\omega_n - \nu_n) - \xi_{\uparrow}^L(\mathbf{q} - \mathbf{k})}, \quad (\text{B1})$$



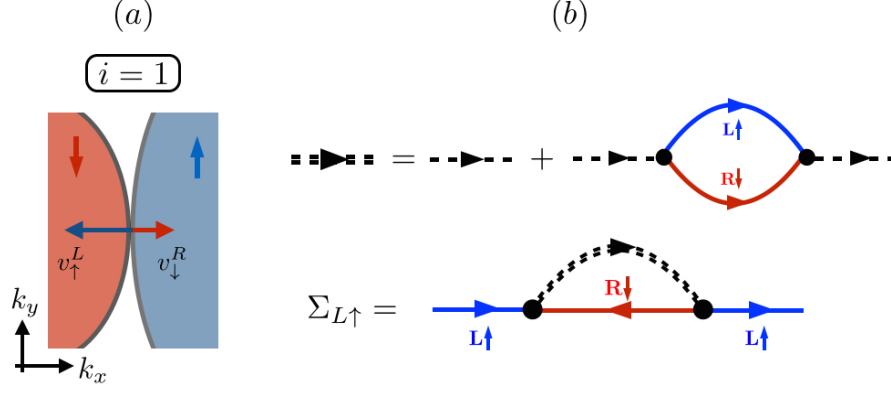


FIG. 4: Elements of the low-energy quantum field theory for a single hot-spot of the Lagrangian given in Eq.(3) of the main text. **(a)**: Fermi surfaces for the two species with dispersion relation  $\xi_{\sigma}^{R,L}(\mathbf{k}) = v_{\sigma}^{R,L} k_x + k_y^2/2m_y$ . A finite curvature is present and the Fermi velocities of the two species are different. **(b)**: Feynman diagrams at one-loop order. Upper row: propagator for the pairing fluctuations, containing the susceptibility given in Eq. (B1). Lower row: self-energy correction to the electron dispersion and quasi-particle lifetime, given in Eq. (B3).

We first perform the Matsubara sum in Eq. (B1). By restricting to  $T = 0$ , we integrate first over  $k_x$  and then over  $k_y$ , always discarding the unphysical UV-divergent terms, and get

$$\begin{aligned} \Pi_{i=1}(\omega_n, \mathbf{k}) = & -\frac{V\sqrt{2m_y}}{2\pi(v_{\uparrow} + v_{\downarrow})} \left[ \frac{1}{|\frac{v_{\uparrow}}{v_{\downarrow}} - 1|} \sqrt{-\frac{v_{\uparrow}}{v_{\downarrow}} \frac{k_y^2}{2m_y} + (\frac{v_{\uparrow}}{v_{\downarrow}} - 1)(v_{\uparrow}k_x + i\omega_n)} + \right. \\ & \left. + \frac{1}{|\frac{v_{\downarrow}}{v_{\uparrow}} - 1|} \sqrt{-\frac{v_{\downarrow}}{v_{\uparrow}} \frac{k_y^2}{2m_y} + (\frac{v_{\downarrow}}{v_{\uparrow}} - 1)(-v_{\downarrow}k_x + i\omega_n)} \right], \end{aligned} \quad (\text{B2})$$

which in the small imbalance limit  $\delta v = v_{\uparrow} - v_{\downarrow} \ll v = (v_{\uparrow} + v_{\downarrow})/2$  gives Eq.(4) of the main text since  $D_{i=1}^{-1}(\omega_n, \mathbf{k}) = -\Pi_{i=1}(\omega_n, \mathbf{k})$  at the critical point. In the main text the first analytical corrections coming from higher order terms in the fermionic dispersion have been included in  $\Pi_{i=1}(\omega_n, \mathbf{k})$ .

In order to compute the non-Fermi-liquid exponent we consider the quasi-particle decay rate given by the imaginary part of the analytically continued self-energy. At the one-loop level the fermionic self-energy is given by (Feynman diagram in the lower row of Fig. 4(b))

$$\Sigma_{L\uparrow}(\nu_n, \mathbf{q}) = \frac{1}{\beta} \sum_{\nu_n, \mathbf{k}} D_{i=1}(\omega_n, \mathbf{k}) G_{R\downarrow}^{(0)}(\omega_n - \nu_n, \mathbf{k} - \mathbf{q}). \quad (\text{B3})$$

After analytical continuing  $i\nu_n \rightarrow \omega + i0^+$  and taking the imaginary part we get

$$\text{Im}\Sigma_{L\uparrow}^{\text{ret}}(\omega, \mathbf{q}) = -\sum_{\mathbf{k}} [n_F(\xi_{R\downarrow}(\mathbf{k})) + n_B(\xi_{R\downarrow}(\mathbf{k}) + \omega)] \text{Im}D_1^{\text{ret}}(\xi_{R\downarrow}(\mathbf{k}) + \omega, \mathbf{k} + \mathbf{q}). \quad (\text{B4})$$

We now set  $T = 0$ ,  $\mathbf{q} = 0$ , and first integrate over  $k_x$  and then over  $k_y$ , to get Eq.(5) of the main text. By computing the self-energy (B3) at finite momentum we obtain no  $\mathbf{q}$ -dependence at leading order, once we expand the pairing propagator (Eq.(4) in the main text) consistently with the small imbalance limit, to get the Landau-damped form

$$D_1^{-1}(\omega_n, \mathbf{k}) \simeq \frac{V\sqrt{2m_y}}{4\pi v|\delta v/v|} \left[ \sqrt{2m_y} \frac{|\delta v/v||\omega_n|}{|k_y|} + B \frac{k_y^2}{2m_y v} \right]. \quad (\text{B5})$$

Within this approximate one-loop framework we have therefore no anomalous dimension for the fermion momenta:  $\eta_x = \eta_y = 0$ .

By adding the self-energy correction from Eq.(5) in the main text to the retarded fermion propagator, we can compute the spectral function  $A_{\uparrow}(\omega, \mathbf{k}) = -\text{Im}G_{L\uparrow}^{\text{ret}}(\omega, \mathbf{k})/\pi$  which satisfies the scaling relation of Eq.(8) in the main text with  $\eta_{\tau} = 1/3$ . The density of states

$$N_{\uparrow}(\omega, T) \sim \omega^{1/3} \quad (\text{B6})$$

is then obtained by integrating the spectral function over  $k_x, k_y$  after rescaling both coordinates in order to eliminate the  $\omega$ -dependence from the integrand. This procedure is based on the idea that we have to put a UV-cutoff on both the  $k_x$  and the  $k_y$  integrals since we are dealing with a low-energy theory in the proximity of the hot-spot. Accordingly, we add to the hot-spot contribution (B6) (called  $N_{\uparrow}^{\text{hot}}$  in the main text) a  $\omega$ -independent shift coming from the “cold” part of the Fermi surface (called  $N_{0\uparrow}^{\text{cold}}$  in the main text), which indeed should behave like a Fermi liquid. However, in this way the exponent in Eq. (B6) can be strongly overestimated.

---