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Scaling Properties of the Plateau Transitions in the Two-Dimensional Hole Gas System

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The behavior of phase coherence is studied in two-dimensional hole gas through the integer quantum Hall plateau to plateau transition. From the plateau transition as a function of temperature, scaling properties of multiple transitions are analyzed. Our results are in good agreement with the assumption of the zero-point fluctuations of the coherent holes, and support the intrinsic saturation of the coherence time at low temperature limit. The critical exponent p can also be determined under the scheme of the zero-point fluctuations. The similarity and difference in experimental observations between quantum Griffiths singularity and plateau transition is discussed. The spin-orbit coupling effect's influence on the plateau transition is explored by comparing the results from different transitions.

Introduction

The phase coherence time τ_φ is influenced by inelastic scattering processes and it is expected to diverge at zero temperature in theory [1]. However, τ_φ saturates at real experimental measurements [2], which raises an issue that whether the saturation of coherence time is intrinsic or not. There have been substantial amounts of studies of τ_φ in different systems [2-13] including quasi-one-dimensional wires, thin films, two-dimensional electron gas (2DEG) and three-dimensional polycrystalline. Although extrinsic mechanisms may explain the coherence time saturation at ultra-low temperatures [13-15], intrinsic mechanisms cannot be excluded [11,16]. The phase coherence time in 2DEG can be determined from the low field magneto-transport measurements [2,3,7,8,10,11].

At higher magnetic field, the integer quantum Hall (IQH) effect may occur if 2DEG mobility is high enough and temperature is low enough. When the quantum Hall plateaus appear, the system is within the localized states. Due to the existence of localized states under magnetic field, there can be quantum plateau-plateau transitions both for short-range and long-range disorders [17,18]. If there is only one extended state at a critical energy E_C between two neighborhood plateaus, the localization length ξ diverges with $\xi \propto \left|B - B_C\right|^{-\nu}$, where B is the magnetic field, and B_c is the field corresponding to E_c [17-20]. At the same time, the quantum phase coherence length l_φ scales with temperature T as $l_\varphi \propto T^{-p/2}$. In consequence, the slope of the Hall resistance also scales with temperature, as $dR_{xy}/dB\big|_{B_C} \propto T^{-\kappa}$, where $\kappa = p/2\nu$.

Experimentally, the transitions between different filling factors are studied in 2DEG through different systems, such as GaAs/AlGaAs, InGaAs/InP and Si [17,21-24] since the discovery of the IQH effect, and it is still under investigation as an interesting probe for critical phenomenon, coherence and interactions [19,20,25-29]. For example, as we will show later in the text, the coherence length is one of the characteristic lengths from the plateau transition, and the information of τ_ϕ can be further derived from the plateau transition measurements.

In this work, we study the plateau transition in two dimensional hole gas (2DHG) at different filling factors. The scaling properties of the plateau transitions are analyzed. The violation of the scaling law at the low temperature limit is more than the finite size effect and it agrees with the zero temperature limit fluctuations of the holes. Through the temperature dependence, we are able to determine the exponent p and ν from a new method by assuming the zero temperature limit fluctuations. By comparing transitions at different fillings, we argue that the scaling behavior is universally independent on filling factors, but the spin-orbit coupling (SOC) effect in 2DHG need to be considered.

Sample and Plateau Transitions

The measurement were made on GaAs/AlGaAs heterostructures, with a hole density of 2.3×10^{11} cm^{-2} and a mobility of 2.9×10^5 cmV^{-1}s^{-1}. The 2DHG is 62 nm below the surface and the Hall bar's geometry is illustrated in Fig. 1's inset. The sample was illuminated by a red light-emitting diode at 4 K and 20 μA for 1 h. The measurements were carried out in a dilution fridge with a mixing chamber temperature below 6 mK. The base electron temperature of this fridge was around 25 mK. The Hall resistance was measured with a standard lock-in technique at 17 Hz and with an ac excitation current of 1 nA.

The Hall resistance as a function of the magnetic field is shown in Fig. 1. Four IQH plateaus are shown and their widths change with temperature. Hall traces of different temperatures seem to cross each other at one point and its magnetic field is determined to be B_C. The slope of R_{xy} as a function of temperature is shown in Fig. 2 with three different transitions. As expected, dR_{xy}/dB is linear with temperature in the relative high temperature range on a log-log plot and κ can be determined from the red color fits in Fig. 2.

Comparison with Zero-Point Fluctuations

The slopes of dR_{xy}/dB in all three transitions saturate in the low temperature limit. A possible reason is that the phase coherence length l_φ is larger than the Hall bar finite width 160 μm at low temperatures. In our experiments, the saturation temperature of ~70 mK at this geometry is comparable to data reported in other systems [22,26] and previous studies of finite size effect on plateau transition gave different number of p [22,26]. It should be noted that the saturation number of dR_{xy}/dB at low temperature limit is different by up to 19% for different transitions in our work, which is hard to understand only by the finite size effect. All above suggests that finite size effect may not be the only cause of the saturation.

A second possibility is that the phenomenological relation $l_\varphi \propto T^{-p/2}$ breaks down in the low-temperature limit, originating from the zero-point fluctuations of holes with l_φ becoming temperature independent at zero temperature limit [2]. This zero temperature limit dephasing mechanism can also lead to the saturation of the slope of dR_{xy}/dB and an empirical relation between the slope of the Hall resistance and the temperature, which is derived as follows. When critical field B_C is being approached, the localization length ξ is supposed to diverge by a power law $\xi \propto |B - B_C|^{-\nu}$ with correlation exponent ν. Based on the one-parameter scaling theory, R_{xy} is supposed to be a function of L/ξ with L denoting the effective size of the system, i.e. $R_{xy} \equiv f(L/\xi) = f(L|B-B_C|^{\nu}) \equiv g(L^{1/\nu}|B-B_C|)$. In the quantum Hall effect, R_{xy}~B and the effective system size is determined by the phase coherence length L_φ, so that $(dR_{xy}/dB)\big|_{B_C} \propto L_\varphi^{1/\nu}$. Previously, it's believed that $L_\varphi \propto T^{-p/2}$ and thus the slope follows $(dR_{xy}/dB)\big|_{B_C} \propto T^{-\kappa}$, where $\kappa = p/2\nu$. Here, we adopt the assumption of zero-temperature fluctuations from reference [2] with the dephasing time following the functional form $\tau_\varphi \propto \tanh(\text{constant} \times T^{-p})$, and we can get $L_\phi = \sqrt{D\tau_\varphi} \propto [\tanh(\text{constant} \times T^{-p})]^{\frac{1}{2}}$. Therefore, $(dR_{xy}/dB)\big|_{B_C} \propto L_\varphi^{1/\nu} \propto [\tanh(\text{constant} \times T^{-p})]^{\frac{1}{2\nu}} \propto \tanh(\text{constant} \times T^{-P})^{\kappa/P}$. Note that in the relatively high temperature range this relation recovers to the previous $dR_{xy}/dB\big|_{B_C} \propto T^{-\kappa}$.

The zero temperature limit dephasing effect is in good agreement with the experimental data, as shown in Fig. 2 for all transitions. We speculate that zero-temperature fluctuations play an important role in our sample. Since this study is carried out in GaAs/AlGaAs system at a relatively high magnetic field, the presence of magnetic impurities and the Kondo effect are not critical here as suggested [5,6]. The lowest electron temperature is only 25 mK with our 1 nA excitation current, so the heating effect is not important in our analysis which only involves data above 40 mK.

Influence of Long-Range Coulomb Interaction

It's usually believed that p=2, ν=7/3 and κ=0.43 [24,25,30]. However, for samples with long-range Coulomb interaction, the plateau-plateau transition happens in the strong coupling regime of the non-linear σ model, and reliable method to calculate κ is yet to be developed [31]. In the experimental aspect, the scaling exponent κ has been found to have different values from 0.15 to 0.81 [17,21,22,24-27,32-34]. Some studies found κ depends on the sample properties [22] and the Al concentration in the GaAs/AlGaAs heterostructures [25].

In our experiments, we determined the scaling exponent κ from the linear high temperature behavior of dR_{xy}/dB, and p from the assumption of zero-point fluctuations (Fig. 2). We find that κ slightly exceeds the non-interacting value, which may result from two reasons. In 2DHG, GaAs/AlGaAs is doped with carbon, which results in the long-range Coulomb interaction. The Coulomb interaction can largely change the tunneling density of states and leads to $p \geqslant 2$ [35,36]. On the other hand, the Coulomb impurities in the sample may also introduce spatial inhomogeneity into the system, and give rise to a correlation length exponent ν<7/3 in a finite size system [24]. Therefore, the scaling exponent κ~0.52 associated with all three transitions in our experiments can be attributed to the influence of long-range Coulomb interaction of carbon atoms.

Crossing Point and Quantum Griffiths Singularity

Shown in Fig. 3, different R-B curves at different temperature do not cross each other at exactly the same magnetic field for the 2→3 transition. Although the theory of quantum phase transition expects a single B_C, one can find that a single crossing point of B_C is not common in either experiments or numerical simulations [24], which is due to the irrelevant finite size correction. In scaling theory, a physical quantity F_L scales with the effective system size L with a general scaling functional $F_L = f(c_1 \cdot L^{1/\nu}, c_2 \cdot L^y, \cdots)$ [20]. Here, ν>0 is the critical exponent, y<0 denotes the irrelevant scaling exponent, and c_i is the corresponding scaling variable. Thus, in finite size systems, the irrelevant scaling term adds corrections and leads to multiple crossing points. However, for system in the thermodynamic limit, the irrelevant scaling term $c_2 \cdot L^y$ decreases to zero, and different curves cross at a single critical point. This behavior of multiple crossing points in finite size system also exists in the numerical studies, which is attributed to the irrelevant finite size corrections [20,37]. In experiments at high temperatures, the ratio between phase coherence length l_φ and the localization length ξ is small, and the localization effect is smeared by the conductance fluctuation, which leads to existence of multiple crossing points B_C at different temperatures [24].

On the other hand, based on some experimental observations that quantum Hall plateau transitions and the superconductor insulator transitions have nearly the same value of critical exponents [24,38], researchers presume that the quantum percolation may play an important role in both these two kinds of quantum phase transitions [39]. However, the effects of dissipation can in principle influence the behavior of the quantum percolation model [39], and ultimately change the characteristics of these abovementioned quantum phase transitions [40].

One example demonstrating the influence of dissipation effect on quantum phase transition is a recent study on superconductor-metal transition in Ga thin film, in which the combination effects of quenched disorder and dissipation give rise to an activated quantum scaling behavior associated with the quantum Griffiths singularity [41]. In Ref. [41], similar multiple crossing points of resistivity curves exist because of the irrelevant scaling. Moreover, due to the dissipation effect, the phase coherence length in Ref. [41] also increases much slower than power law behavior but obeying a logarithmic behavior [42]. After similar analysis presented here for the plateau transitions, non-monotonic critical exponent is deduced (Fig. 4). Meanwhile, the crossing points converge to a fixed B_C at low temperature limit. Therefore, the plateau transition is different from the superconductor-metal transition in Ga thin film regardless of their similarities of multiple crossing points in the experimental observation. We suspect that the differences of this paper and Ref. [41] originate from different kinds of dissipation. As justified in Ref. [43], the Ohmic dissipation can give rise to the quantum Griffiths singularity for quantum O(N) symmetry systems with quenched disorder, but a non-Ohmic dissipation smears the quantum Griffiths singularity. In this paper, the phase coherence length tends to saturation, but does not follow the logarithmic law as suggested by the activated scaling [42]. Thus, here the quantum Griffiths singularity is excluded for the quantum Hall plateau transitions. The microscopic origin of the saturated phase coherence length in quantum plateau transition is worth of further investigations.

Crossing Point and Spin-Orbit Coupling

Although the field theory predicts universal scaling behavior for each plateau-plateau transition [44-46], numerical studies show disagreements that higher Landau levels may exhibit different scaling behaviors from the lowest Landau level, depending on the details of the disorder potential [47] or SOC [48]. Thus, the experimental investigations of multiple plateau-plateau transitions, especially in two-dimensional hole systems with large SOC, can provide important evidence to elucidate the quantum critical properties associated with the plateau-plateau transitions.

The behaviors of critical magnetic field B_C for all the three transitions are summarized in Fig. 5, which provides possible verification of the scaling theory. In both the 2→3 and 4→5 plateau-plateau transitions, the critical fields B_C rapidly smear from the low temperature value with field variation ΔB=30 mT at 200 mK. Such a variation of B_C can be expected from the aforementioned irrelevant finite size correction at high temperature. The similarity between 2→3 and 4→5 transitions is consistent with the general prediction that critical behavior is irrelevant to the Landau level index [24,46]. However, for 3→4 transition, the critical fields B_C is much smoother when increasing the temperature, with the field variation as low as

ΔB=3 mT at 200 mK.

We attribute the remarkable discrepancy to the large SOC effect in the *p*-type carbon-doped GaAs system, which has been known in this system [49]. The Landau levels are spin non-degenerate due to the Zeeman splitting, and the Landau levels with filling factor 2n-1 and 2n are also strongly coupled due to the spin-orbital coupling effect. Because the Landau energy is larger than the Zeeman splitting energy, the inter-level coupling between the 3-rd and 4-th plateau is more effective than the other two transitions, which causes a shorter localization length ξ, weakens the irrelevant finite size correction [24], and leads to a higher precision of the critical B_C. Moreover, although in the theoretical aspect the SOC effect can lead to delocalized states, the large magnetic field change the system to the unitary class [19], which only contains delocalized states at the center on Landau levels.

Our experiments indicate that, apart from the scaling hypothesis, microscopic mechanism can also influence the critical behavior at the plateau-plateau transitions, especially in the *p*-type systems. From this aspect, p=2.1$\pm$0.3 and ν=2.1$\pm$0.3 determined from 3→4 transition are more reliable than those fitted from other two transitions.

Conclusion

In summary, we study the integer quantum Hall plateau transition in the hole system. The scaling properties are observed and the component p and ν are determined by a new method. Our results suggest that the saturation of phase coherence time in quantum Hall system at low temperature limit is intrinsic. In addition, the spin-orbital coupling effect explains the non-universal scaling behaviors for different plateau-plateau transitions.

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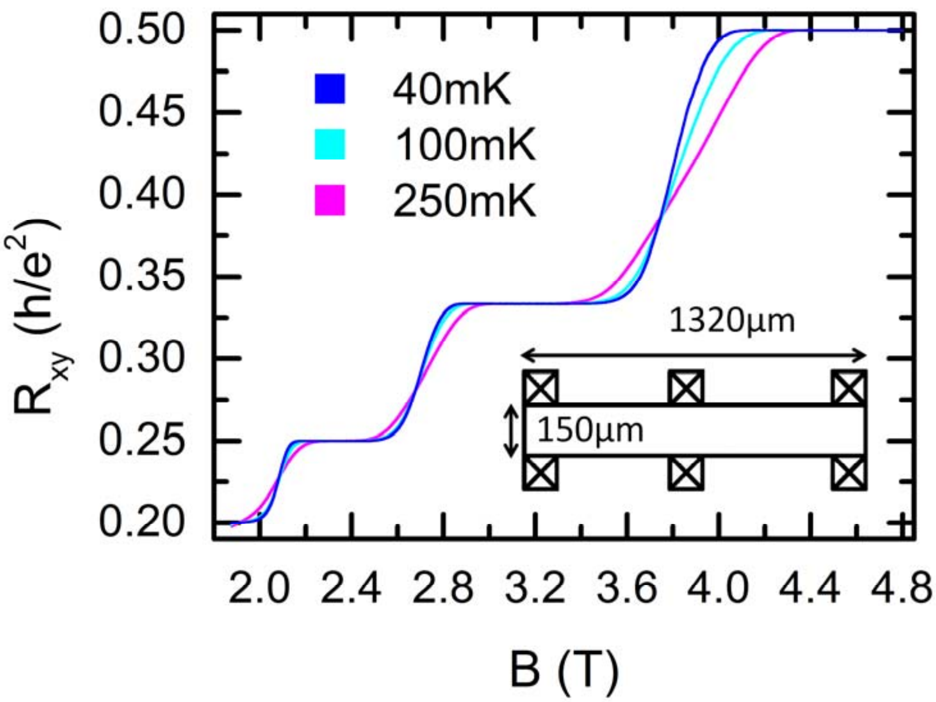

Figure 1: The Hall resistance as a function of magnetic fields at three different temperatures. The inset shows the size of the Hall bar. Four IQH states are involved in this study: filling factor ν=2, 3, 4, 5. The plateau-plateau transition from filling factor k to k+1 is labeled as "k→k+1" in this work.

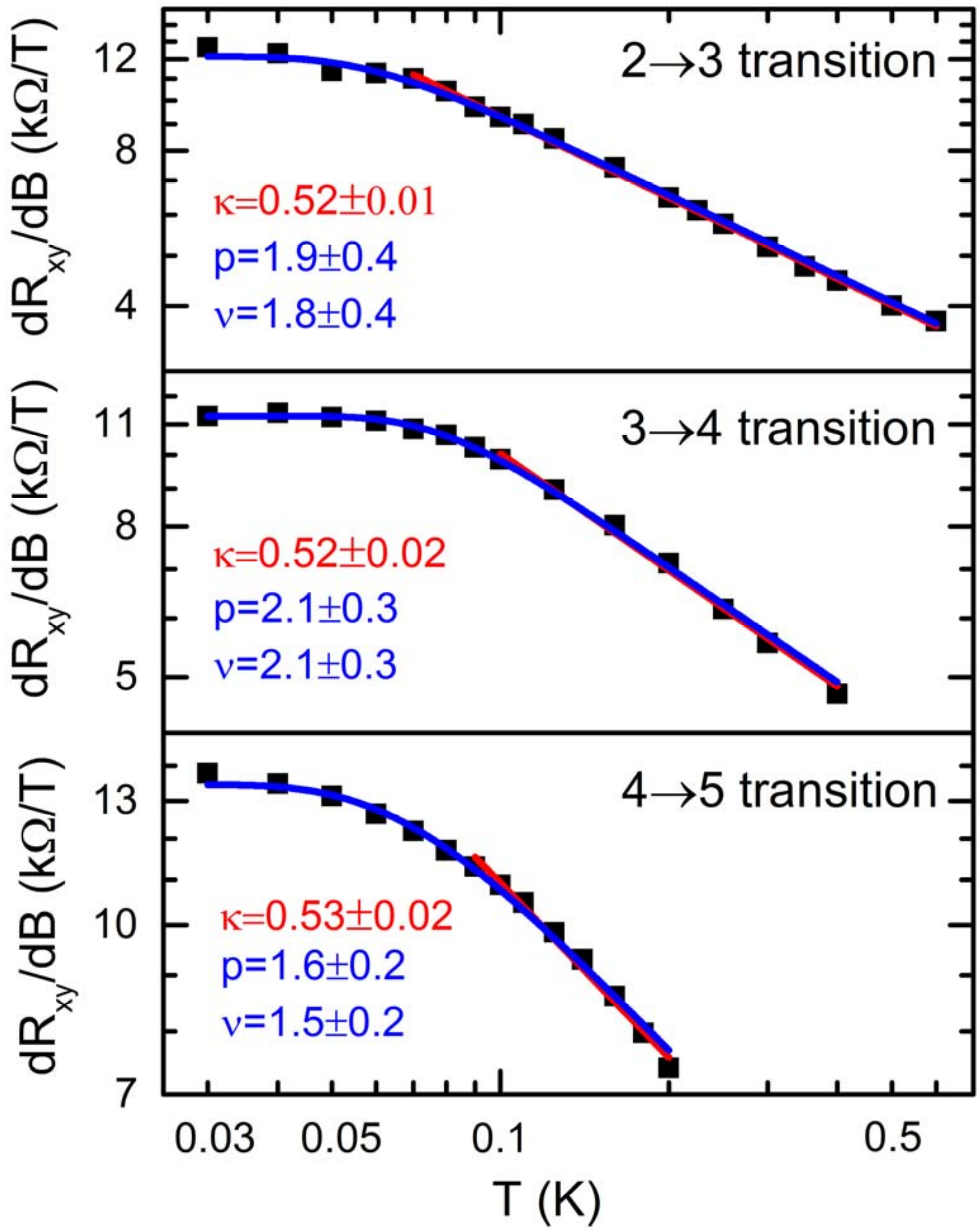

Figure 2: The slope of Hall resistance at different transitions as a function of temperature (log-log plot). The red lines are fitted by $(dR_{xy}/dB)|_{Bc} \propto T^{-\kappa}$. The blue lines are fitted by $(dR_{xy}/dB)|_{Bc} \propto \tanh(\text{constant} \times T^{-P})^{\kappa/P}$ and κ is determined by the red fits. The base electron temperature of this measurement system was around 25 mK, and the meaningful temperature for the 30 mK data points may be higher than the thermometer temperature (lattice temperature). Therefore, the blue fittings start from 40 mK.

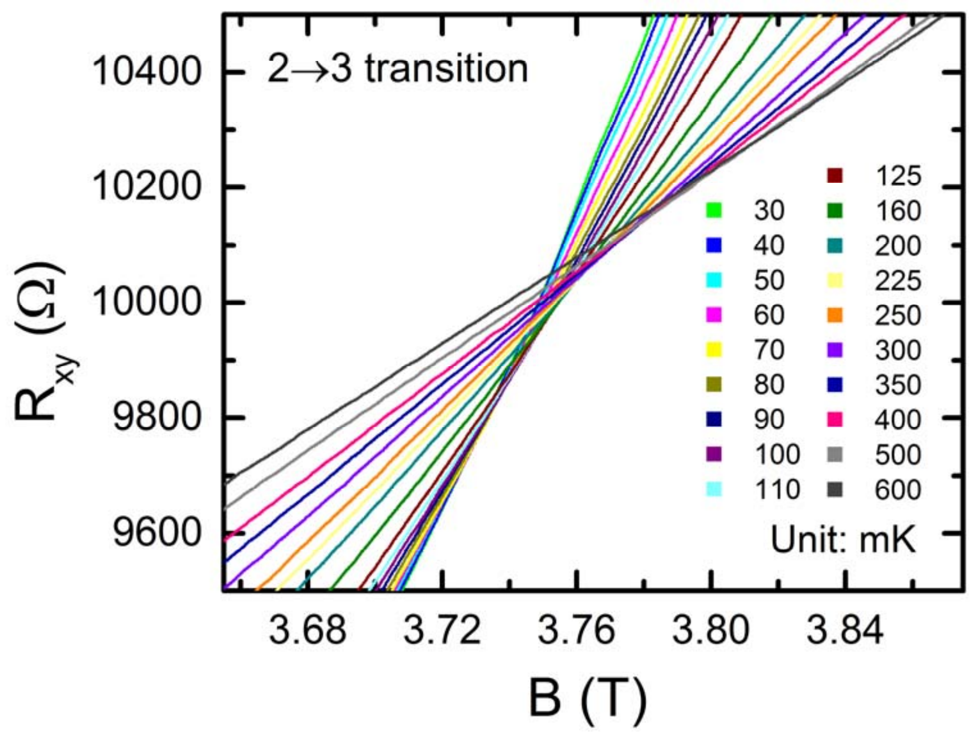

Figure 3: Zoom-in of the crossing region for the 2→3 plateau transition.

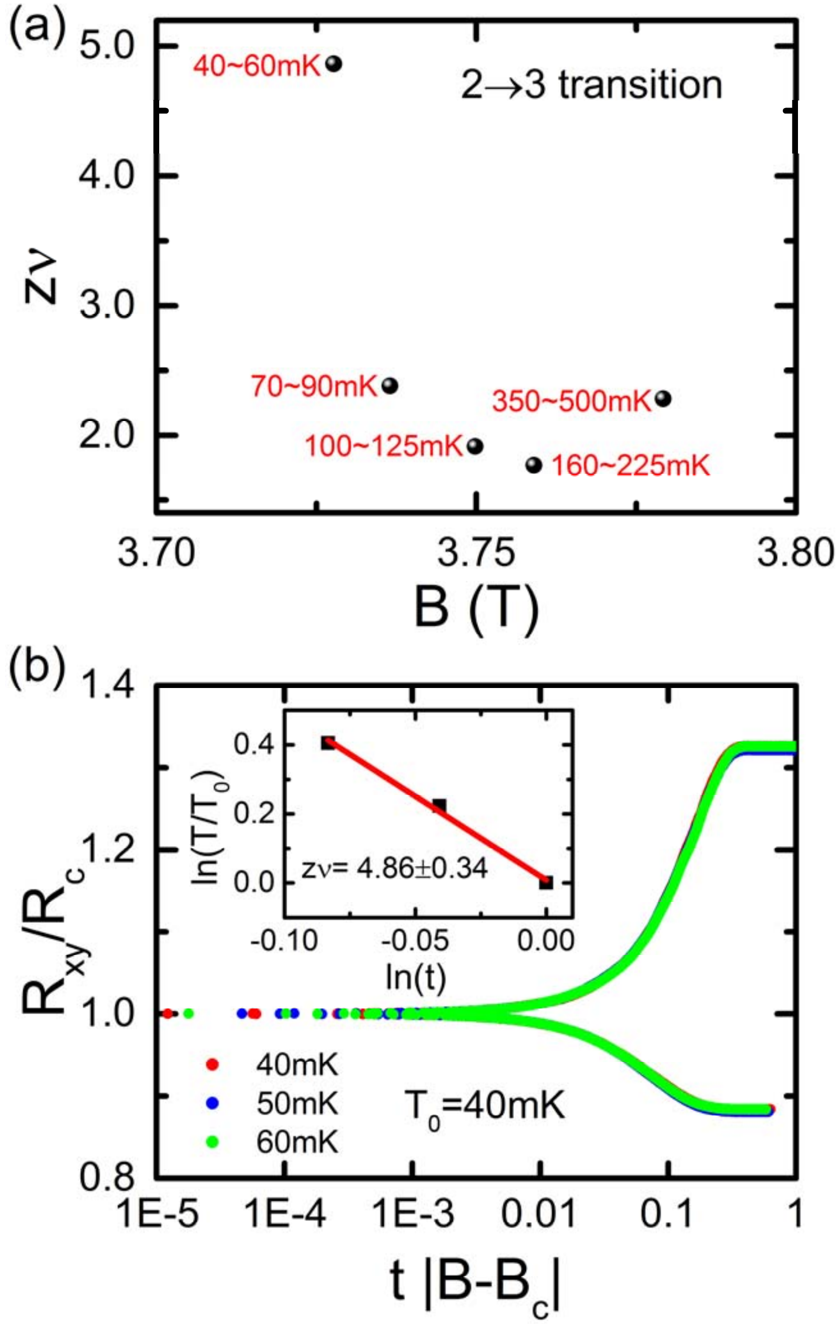

Figure 4: (a) Scaling exponent as a function of magnetic field, extracted from procedure shown in Fig. 4b. Similar analysis can be found in reference [41]. (b) An example of three-curve analysis results (40~500 mK). Normalized resistance as a function of the scaling variable $t\left|B - B_c\right|$, where $t = (T / T_0)^{-\frac{1}{z\nu}}$. The insets are the corresponding temperature behavior of the scaling parameter *t*, so that $\ln(T / T_0) = -z\nu \ln(t)$. We use *z*ν to be in consistent with reference [41] and *z*ν is equal to 1/κ.

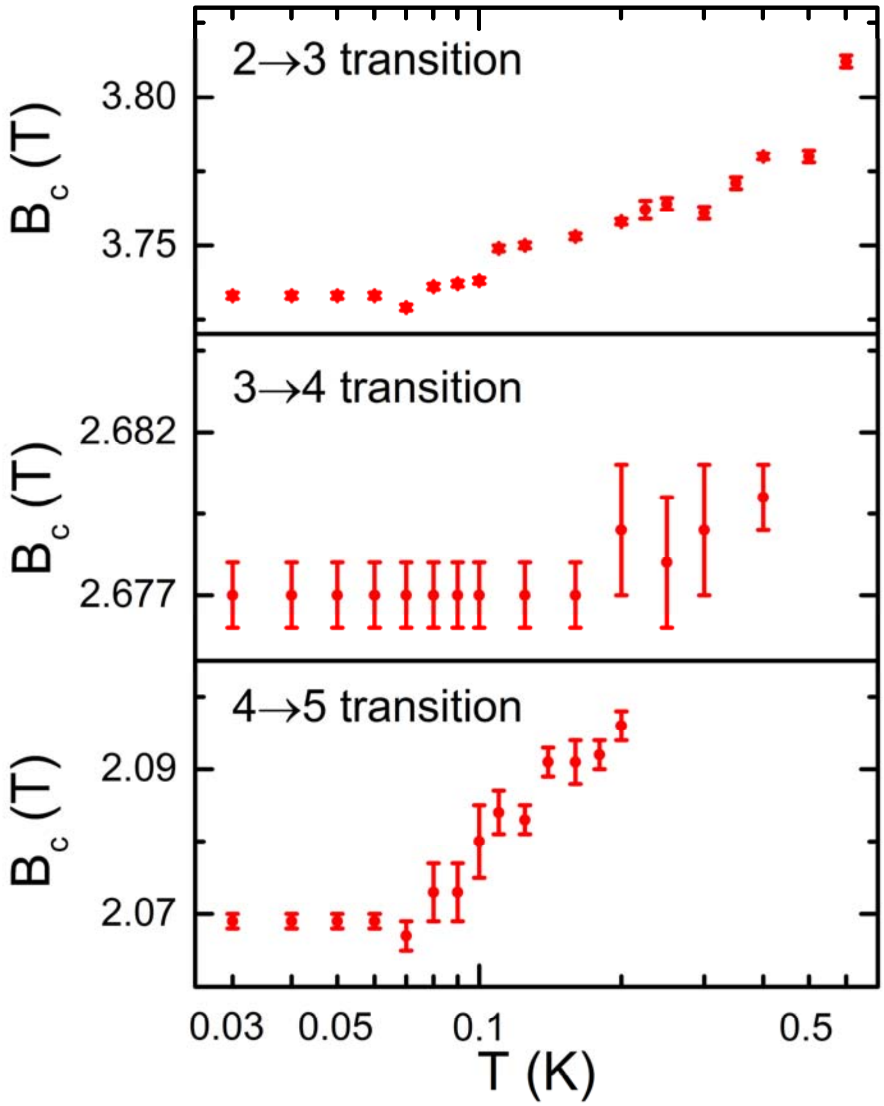

Figure 5: Temperature dependence of the crossing points at different filling plateau transitions (log-linear plot).