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The many-body localization transition (MBLT) between ergodic and many-body localized phase in disordered interacting systems is a subject of much recent interest. Statistics of eigenenergies is known to be a powerful probe of crossovers between ergodic and integrable systems in simpler examples of quantum chaos. We consider the evolution of the spectral statistics across the MBLT, starting with mapping to a Brownian motion process that analytically relates the spectral properties to the statistics of matrix elements. We demonstrate that the flow from Wigner-Dyson to Poisson statistics is a two-stage process. First, fractal enhancement of matrix elements upon approaching the MBLT from the delocalized side produces an effective power-law interaction between energy levels, and leads to a plasma model for level statistics. At the second stage, the gas of eigenvalues has local interaction and level statistics belongs to a semi-Poisson universality class. We verify our findings numerically on the XXZ spin chain. We provide a microscopic understanding of the level statistics across the MBLT and discuss implications for the transition that are strong constraints on possible theories.

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Introduction. Quantum and statistical mechanics represent two seemingly rather different approaches to the description of complex physical systems. Yet these two viewpoints agree for a wide class of isolated quantum systems, which are said to thermalize [1, 2]. Determining the circumstances under which an isolated quantum many-body system becomes its own thermal bath and thermalizes itself, just as Baron Munchausen could pull himself out of a mire by his own hair, perhaps using some kind of fluctuation, is an open question.

Phenomena similar to the emergence of thermalization also occur in few-body quantum systems, which frequently show the emergence of so-called quantum chaos [3]. There, upon changing parameters/number of degrees of freedom, the classical system can go from regular to chaotic behavior. On a quantum level this results in changes of level statistics, which has proven to be a powerful probe of the system properties in the context of quantum chaos. In particular, there exist two standard universal limits: Poisson statistics (PS) and Wigner-Dyson level statistics (WDS) [4]. For few-body systems, PS applies to systems which are classically integrable and do not have any level repulsion. WDS stems from random-matrix theory and holds for generic chaotic systems, where energy levels repel each other (i.e., the energy difference between neighboring levels is statistically unlikely to be small compared to the mean level spacing).

Integrable (non-chaotic) behavior is abundant in the context of few-body physics. On the other hand, in the many-body world the only non-thermalizing *phase* (in the sense of stability to small perturbations) is represented by many-body localized (MBL) systems [5, 6]. Recent progress established that thermalization fails in the MBL phase due to the existence of extensively many conserved quantities [7–10]. On the other hand, it is known that one can tune the system through a phase transition into a thermalizing ergodic phase [11–20]. Below we aim to understand the evolution of the level statistics across the MBL-to-ergodic transition, gaining insights into the breakdown of thermalization.

Crossover between PS and WD statistics has been studied extensively in a single-particle physics context: for quantum kicked rotor [21], integrability breaking perturbations [22, 23], and single-particle Anderson localization transition (ALT) [24–26]. In the many-body problems, PS to WD crossover is also known to occur upon breaking of (quantum) integrability [27]. In most of the examples, the PS and WDS are the only two stable points. The only known exception is the ALT, where *universal statistics* different from PS and WDS emerges at the mobility edge [24].

The spectral statistics in the case of MBL transition was demonstrated to evolve from WDS to PS as one localizes the system [11, 28–30], however not much is known about the intermediate statistics. The common probe used to characterize level statistics across MBLT is an average ratio of the consecutive energy spacings [11–13, 18]. However, this is a single parameter and it does not provide much insight into the intermediate form of the level statistic, nor into physical details of its crossover.

In this paper we study how the spectral statistics changes across the MBL-delocalization transition. In order to build a microscopic understanding of the level statistics we generalize Dyson's Brownian motion model [31], previously applied to the ALT [32], to the many-body case. From the mapping to Brownian motion, we obtain non-trivial relations between fractality [17–20], spectral statistics, and properties of matrix elements across the MBLT [20, 33]. While many features can be simultaneously explained in this analysis, one surprise is that there appear to be two different regimes of intermediate spectral statistics: in one, the effective interaction between energy levels in the plasma model has a variable power-law, while in the other, the effective interaction is short-ranged but over a variable number of levels.

Within the picture of Brownian motion [31, 32], the level statistics is controlled by the effective interaction between energy levels, see Fig. 1. In particular, deep in the ergodic phase, the WD statistics emerges from the partition function of a one-dimensional Coulomb gas, where particles interact with a logarithmic potential $U(s) = -\log|s|$. At a first stage, upon approaching the MBL transition, the effective interaction starts to decay as a power-law: $U(s_i - s_j) = |s_i - s_j|^{-\gamma}$ when $|s_1 - s_2| \geq N_{\rm erg}$. The power-law interaction changes tails of the level statistics, so it can be approximately described by the plasma model, and is intermediate between PS and WDS case. At the second stage, when exponent γ becomes bigger than one, the interaction becomes effectively short-ranged, and level spacing distribution tends to the semi-Poisson distribution [34]. In this regime it is the range of the interaction which changes with disorder/system size. As soon as the range of interactions reaches zero, we arrive at Poisson statistics.

Before discussing implications of the above picture of the level statistics, we justify the proposed cartoon using both analytic and numeric arguments. In particular, we argue that the parameter γ introduced above can be extracted from the properties of the many-body matrix elements which decay as a power-law with energy separation between eigenstates, where $\gamma \leq 1$ is the same power which controls level statistics. The power-law behavior of matrix elements can be viewed as a generalization of the Chalker-Daniell scaling of wave function overlap [35] to the many-body case, and it is consistent with fractality of wave functions near MBLT [17–20].

Plasma model for level correlations. In the random matrix theory, the joint probability density for random

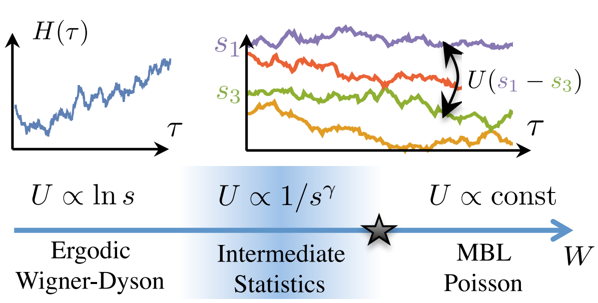

FIG. 1. (top) Random walk in a space of Hamiltonians induces a stochastic process on the eigenenergies. The interaction between eigenlevels is set by a potential energy $U(s_i - s_j)$. (bottom) Evolution of the interaction between levels $U(s)$ across the MBL transition determines the level statistics.

matrix ensembles reads

$$P(\{s_i\}) = \frac{e^{-\beta H}}{Z}, \; H = \sum_i W(s_i) + \sum_{i<j} U(s_i - s_j), \quad (1)$$

where $\beta = 1$ for orthogonal matrix ensemble which will be of primary interest. The confining potential $W(s) = s^2/2$ is parabolic, and interaction is $U(s_i - s_j) = -\ln|s_i - s_j|$. As Dyson demonstrated in his pioneering work [31], this distribution function may be viewed as a stationary distribution of the stochastic random walk in a space of matrices (Hamiltonians).

To derive the joint distribution of eigenenergies from a random walk, one can start from the eigenbasis and perform a stochastic step in the space of Hamiltonians, induced by ΔH. Then, we get the energy correction in a form

$$\Delta s_n = V_{nn} + \sum_{m\neq n} \frac{V_{mn}V_{nm}}{s_n - s_m}, \quad V_{mn} = \langle m|\Delta H|n\rangle, \quad (2)$$

which is the shift of eigenenergies induced by the perturbation ΔH up to second order. For Gaussian ensembles of random matrices, using $\langle V_{nm}V_{mn}\rangle = \frac{2}{\beta}\Delta\tau$ and $\langle V_{nn}V_{mm}\rangle = \delta_{mn}\Delta\tau$ one can derive Fokker-Planck equation (see Supplemental Material [36] for more details). Its stationary (equilibrium) solution is given by Eq. (1) with logarithmic interaction. Note, that in what follows we omit the damping term which keeps the bandwidth fixed [36].

Dyson's mapping was generalized to the case of disordered problems [32]. For such problems, it is natural to perform a random walk (RW) in a space of Hamiltonians by changing realizations of disorder. As we are going to concentrate on properties of a spin chain in a random magnetic field, which is coupled to the z component of a spin S_i^z, we take $\Delta H = \sum_{i=1}^L h_i(\tau) S_i^z$, with $\langle h_i(\tau)h_j(\tau')\rangle = v^2\delta(\tau - \tau')\delta_{ij}$. Similar to the case of random matrices [3, 31, 36], the two correlators which determine the level dynamics are:

$$\langle V_{nn}V_{mm}\rangle = \delta d_{nm} = \langle n|S_i^z|n\rangle\langle m|S_i^z|m\rangle, \quad (3)$$
$$\langle V_{nm}V_{mn}\rangle = \delta c_{nm} = |\langle m|S_i^z|n\rangle|^2, \quad (4)$$

where we assumed that $v^2 = \delta/L$, where δ is the many-body level spacing, so that s_n represent unfolded energy spectrum. The correlator (3) sets the spectrum of a random noise, while spectral function c_{nm} determines the interaction between levels in the ensemble.

Effective interaction between levels. The RW process depends crucially on two correlators Eqs. (3)-(4). To make analytic progress we use a mean-field like approximation [32], assuming that d_{nm} and c_{nm} can be replaced by their ensemble averages,

$$c(\omega) = \langle c_{nm}\delta(s_n - s_m - \omega)\rangle, \quad (5)$$

(and similar expression for d_{nm}) which now depend only on the energy difference between eigenstates. For the single-particle Anderson localization, the c_{nm} and d_{nm} necessarily coincide with the wave functions overlaps [32], $c_{nm} = d_{nm} \propto \int dx|\psi_n(\tau,x)|^2|\psi_m(\tau,x)|^2$. The fractality of the wave function near the mobility edge results in a power-law enhancement of $c(\omega) \propto A/\omega^\gamma$ [35, 37]. In the case of ALT this enhancement arises because the envelope of wave functions nearby in energy lives on the same multifractal domain [37]. In the many-body case similar enhancement can arise from the fractal structure of the wave function in the Hilbert space in a vicinity of MBLT [17–20].

We view the matrix elements $V_{nm} = \langle n|S_i^z|m\rangle$ as a coefficients of the wave function of excitation created by a local operator S_i^z from an eigenstate $|m\rangle$ [20]. We assume that the inverse participation ratio (IPR), $I_2 = \mathcal{V}\sum_m |V_{nm}|^4 \propto \mathcal{V}^{-d_2}$, where d_2 is generalized fractal dimension, and $\mathcal{V} = \exp(sL)$ is the number of states in the Hilbert space. We translate the IPR into the scaling with the energy separation as $\mathcal{V}^2\langle V_{nn}^2 V_{nk}^2\rangle \propto (\mathcal{V}/\mathcal{R})^{1-d_2}$, where $\mathcal{R} = |n-k| \approx (E_k - E_n)/\delta$. From here, omitting the diagonal matrix element V_{nn} given by the spin expectation value, we arrive to the scaling:

$$c(\omega) \propto \left(\frac{J}{\omega}\right)^\gamma, \qquad \gamma = 1 - d_2. \tag{6}$$

The above argument should be viewed as a phenomenological; at present the microscopic nature of a fractal behavior is not clear, although Griffiths (rare-region) effects [18] in vicinity of MBL transition provide one possible microscopic scenario. Also, relating d_2 to the properties of matrix elements, i.e. exponent κ in the scaling [20, 33], $|V_{nm}| \propto \exp(-(s+\kappa)L)$ is an interesting question.

The correlation between diagonal matrix elements, the function d_{nm} also shows a power-law dependence. However, there is an enhancement of d_{nm} for $n = m$, allowing to approximate $d(\omega)$ as a delta-function, see SM for additional discussion [36].

Implications for spectral statistics. Using power-law form of $c(\omega)$ Eq. (6), and the delta-function form of $d(\omega)$ we can map our model onto the plasma model for the level statistics [38], provided $\gamma < 1$. The plasma model assumes a power-law interaction potential $U(s) = A/|s|^\gamma$ in the joint distribution function (1). It predicts the tails of the level statistics $P(s) \propto s^\beta \exp(-h_\gamma s^{2-\gamma})$ for $s \gg 1$, and variance of the number of levels in a box of size N becomes $\operatorname{var} N \propto N^\gamma$, which is intermediate between WD-like rigidity $\operatorname{var} N \propto \log N$ and Poisson case [3, 4].

For larger values of $\gamma \geq 1$ the effective interaction in the gas of eigenvalues becomes short range, and mapping to the plasma model no longer works. Instead, spectral properties now are expected to be well-described by a family of semi-Poisson distributions [34], which arise from a gas of eigenvalues with a finite-range interaction. They predict Poisson-like behavior of the tails of $P(s)$ and level compressibility $P(s) \propto s^\beta e^{-(\beta h+1)s}$, and $\operatorname{var} N \propto \chi N$ with $\chi \leq 1$, where h is the range of interactions. Such level statistics has been dubbed "critical" in the literature [39–41] and is believed to describe the level statistics at the ALT [25, 26].

Using the above intuition, we propose the following form of the level spacing distribution and spectral rigidity to interpolate between WDS and PS,

$$P(s;\beta,\gamma_P) = C_1 s^\beta \exp\left(-C_2 s^{2-\gamma_P}\right), \; \operatorname{var} N = \chi N^{\gamma_{\text{var}}}, \tag{7}$$

where the parameter $1 \geq \gamma_P, \gamma_{\text{var}} \geq 0$ controls the tails of the statistics and level rigidity, and $1 \geq \beta \geq 0$ determines the level repulsion. The constants $C_{1,2}$ can be fixed by requiring that $\langle 1\rangle = \langle s\rangle = 1$. When $\gamma_P = 0$, this distribution becomes WD. In the opposite limit, $\gamma_P \to 1$, distribution (7) becomes a semi-Poisson with generic β. For the spectral rigidity our interpolating function also can describe the (semi-)Poisson limit, however failing to capture logarithmic growth of $\operatorname{var} N$ in the WD case.

Numerical results. We use the XXZ spin chain in a random field as a specific model with a previously located MBL transition [11] to test our picture of level statistics. The Hamiltonian is

$$\hat{H}_{\text{XXZ}} = \sum_{\langle ij\rangle} \mathbf{S}_i \cdot \mathbf{S}_j + \sum_i w_i S_i^z, \quad S^{x,y,z} = \frac{1}{2}\sigma^{x,y,z}, \tag{8}$$

where disorder enters via random fields w_i uniformly distributed in the interval $[-W;W]$. We perform exact diagonalization for chains of size $L = 12, \ldots 16(18)$ with periodic boundary conditions to extract properties of matrix elements (spectral statistics). We use central part of the many-body spectrum, which corresponds to energy density $\varepsilon = (E - E_{\min})/(E_{\max} - E_{\min}) = 0.45 \pm 0.1$ and contains 246, 969, 3794, 14316 levels on average for $L = 12, \ldots 18$. The MBL transition at this energy density is believed to occur near $W_c \approx 3.6$ [13]. To unfold levels, we fit the staircase function with a 3rd order polynomial. We use both local and global level unfolding schemes [42].

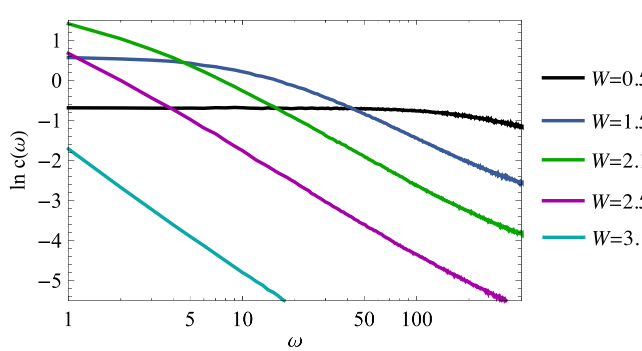

FIG. 2. Averaged function $c(E)$ evolves from being almost flat at low disorder ($W = 0.5$) to a power-law decay. Note that for the intermediate values of disorder, matrix element is enhanced at small energy difference compared to the limit of weak disorder.

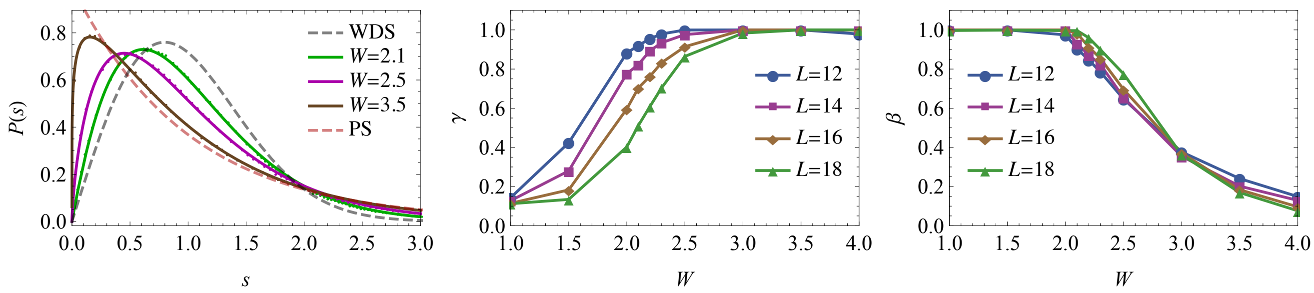

FIG. 3. (a) Evolution of level spacing distributions as system is tuned towards MBL phase. Points represent data, while solid lines are best fits with a two-parameter distribution (7). Red and black dashed lines correspond to Poisson and Wigner-Dyson distribution. (b) The exponent γ_P, controlling tails of level statistics, flows with L for $W \lesssim 2.5$, but is constant in vicinity of MBLT $W_c \approx 3.6$. (c) In contrast, β controlling the level repulsion, remains constant for $W \lesssim 2$, and starts to flow closer to the MBLT.

We start by discussing the numerical results for averaged $c(\omega)$, presented in Fig. 2(a). Upon increasing disorder, we see the crossover of $c(\omega)$ from a constant to a power-law decay. As one may expect, this crossover happens at some scale, N_{erg}, so that $c(\omega < N_{\text{erg}}) \propto \text{const}$, and decays as a power-law beyond $\omega > N_{\text{erg}}$. The additional scale N_{erg} has a meaning similar to the correlation length, over which ergodicity holds. As $N_{\text{erg}} \to 0$, interaction between levels becomes critical even for the smallest separations.

From the power-law form of $c(\omega)$, we expect that level spacing distribution for the XXZ spin chain to be well described by Eq. (7). Fig. 3(a) illustrates that the flow of the level statistics is indeed well captured by Eq. (7). We also considered a number of single-parameter ansatze (in particular, Brody and semi-Poisson distribution); none of those could capture the changes of $P(s)$ across MBLT. The $P(s)$ for disorder $W < 2$ is not shown, as it looks very similar to WD distribution: since $P(s)$ is influenced the most by the interaction between close levels, N_{erg} must become close to zero before we see the flow in the level statistics. In contrast to the level statistics, which is influenced by a non-critical part of $c(\omega)$, the spectral rigidity is expected to be less sensitive to the behavior of $c(\omega)$ at small ω. In SM [36] we show that $\text{var}\, N$ behaves as a power-law (7), and becomes linear for $W \gtrsim 2$. Also, we test that different estimates for exponent γ show reasonable agreement as follows from plasma model.

Finally, we consider the flow of parameters γ_P and β with increasing system size, presented in Fig. 3(b)-(c). While γ_P controlling tails of the level statistics has a strong flow at disorder $W \leq 2.5$, at larger disorders γ_P is very close to one and changes little with L. This further supports the conclusion that for $W \geq 2.5$ the effective interaction between energy levels becomes short-ranged for the largest accessible system sizes. Consistent with our expectation, β shown in Fig. 3(c) changes weakly when statistics is described by plasma model ($W \leq 2$), and begins to flow once level interactions are local.

Discussion and open questions. Using analytical and numerical arguments we described the spectral properties across MBL transition using a two-stage flow picture. Note that we need at least two parameters, γ and N_{erg}, to describe level statistics. This is not surprising if we recall that even the case of ALT, the existence of multifractality means that to describe the universal properties one requires more information beyond the small number of critical indices needed for a simple thermodynamic phase transition [25, 26]. Below we discuss the implications of the proposed picture of the spectral statistics flow.

At the first stage the "correlation length" N_{erg} shrinks to zero, but the exponent responsible for level interactions γ is smaller than one. Intuitively, the levels beyond correlation length become more and more different, corresponding to a gradual breakdown of the ETH. Here the level statistics can be described by the effective plasma model. Although this model was proposed some time ago [38], it does not apply in the case of ALT, despite the presence of multifractality near single particle mobility edge. Hence, to the best of our knowledge, the present study is the first physical realization of the plasma model.

The second stage begins at $W \geq 2.5$, when $\gamma \geq 1$ so that interactions between levels are local. Although we cannot exclude the finite size effects, the numerical estimates for the MBL transition at $W_c \approx 3.6$ suggest that *at the MBL transition* interactions between levels are local. Thus, we conjecture that level statistics near and at the MBLT belongs to the same or similar "critical" family as the universal statistics at the ALT [39–41]. This also naturally explains why the average ratio of the level spacing $r = \min(\delta_n, \delta_{n+1})/\max(\delta_n, \delta_{n+1})$ at the MBLT, widely used in the literature [11–13, 18], is very close to the value expected from PS.

The semi-Poisson level statistics emerges at the same value of disorder where the boundary of the Griffiths phase was previously identified in the literature [18], $W \approx 2.5$ (Refs. [16, 17] report the onset of ergodicity breaking at the same location). The existing theories of the MBLT [14, 15] predict extensive entanglement and subdiffusive transport in the ergodic phase. The wide region of critical statistics near transition may be a manifestation of finite size effects (system sizes studied are smaller that diverging correlation length). Indeed the strong overlaps only between adjacent energy levels imply logarithmic transport [20], predicted at the MBLT [14, 15]. On the other hand, existence of thermodynamically stable Griffiths phase is another intriguing possibility.

In closing, we have found that Dyson's mapping of level statistics to Brownian motion allows one to understand the spectral statistics in the MBL transition at least as well as in the ALT for which it was introduced. There are basic differences between the two transitions, e.g., several quantities which are uniquely defined at the ALT allow inequivalent generalizations to the MBLT. There are two steps of the spectral statistics flow, one with long-range interactions (the plasma model) and one with local interactions, and the boundary between the two is found numerically to coincide with the onset of a Griffiths phase and subdiffusive transport. Since level statistics are known to be the simplest universal probe of the transition to quantum chaos in simpler problems, understanding the origin and universality of the two-step plasma model of level statistics is an important challenge for theories of the MBLT.

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[1] M. Srednicki, *Chaos and quantum thermalization,* Phys. Rev. E **50**, 888 (1994).
[2] M. Rigol, V. Dunjko, and M. Olshanii, *Thermalization and its mechanism for generic isolated quantum systems,* Nature **452**, 854 (2008).
[3] F. Haake, *Quantum Signatures of Chaos*, Springer Series in Synergetics (Springer Berlin Heidelberg, 2013).
[4] M. Mehta, *Random Matrices: Revised and Enlarged Second Edition*, Pure and Applied Mathematics (Elsevier Science, 2014).
[5] D. Basko, I. Aleiner, and B. Altshuler, *Metal–insulator transition in a weakly interacting many-electron system with localized single-particle states,* Annals of Physics **321**, 1126 (2006).
[6] I. V. Gornyi, A. D. Mirlin, and D. G. Polyakov, *Interacting electrons in disordered wires: Anderson localization and low-t transport,* Phys. Rev. Lett. **95**, 206603 (2005).
[7] D. A. Huse and V. Oganesyan, *A phenomenology of certain many-body localized systems,* ArXiv e-prints (2013), arXiv:1305.4915.
[8] M. Serbyn, Z. Papić, and D. A. Abanin, *Local conservation laws and the structure of the many-body localized states,* Phys. Rev. Lett. **111**, 127201 (2013).
[9] J. Z. Imbrie, Arxiv e-prints (2014), 1403.7837.
[10] V. Ros, M. Müller, and A. Scardicchio, *Integrals of motion in the many-body localized phase,* Nuclear Physics B **891**, 420 (2015).
[11] A. Pal and D. A. Huse, *Many-body localization phase transition,* Phys. Rev. B **82**, 174411 (2010).
[12] Y. Bar Lev, G. Cohen, and D. R. Reichman, *Absence of diffusion in an interacting system of spinless fermions on a one-dimensional disordered lattice,* Phys. Rev. Lett. **114**, 100601 (2015).
[13] D. J. Luitz, N. Laflorencie, and F. Alet, *Many-body localization edge in the random-field heisenberg chain,* Phys. Rev. B **91**, 081103 (2015).
[14] R. Vosk, D. A. Huse, and E. Altman, *Theory of the many-body localization transition in one dimensional systems,* ArXiv e-prints (2014), arXiv:1412.3117 [cond-mat.dis-nn].
[15] A. C. Potter, R. Vasseur, and S. A. Parameswaran, *Universal properties of many-body delocalization transitions,* ArXiv e-prints (2015), arXiv:1501.03501 [cond-mat.dis-nn].
[16] J. Goold, S. R. Clark, C. Gogolin, J. Eisert, A. Scardicchio, and A. Silva, *Total correlations of the diagonal ensemble herald the many-body localization transition,* ArXiv e-prints (2015), arXiv:1504.06872 [cond-mat.dis-nn].
[17] A. De Luca and A. Scardicchio, *Ergodicity breaking in a model showing many-body localization,* Europhys. Lett. **101**, 37003 (2013).
[18] K. Agarwal, S. Gopalakrishnan, M. Knap, M. Mueller, and E. Demler, *Anomalous diffusion and Griffiths effects near the many-body localization transition,* ArXiv e-prints (2014), arXiv:1408.3413 [cond-mat.dis-nn].
[19] E. J. Torres-Herrera and L. F. Santos, *Dynamics at the Many-Body Localization Transition,* ArXiv e-prints (2015), arXiv:1501.05662 [cond-mat.dis-nn].
[20] M. Serbyn, Z. Papic, and D. A. Abanin, *A criterion for many-body localization-delocalization phase transition,* ArXiv e-prints (2015), arXiv:1507.01635 [cond-mat.dis-nn].
[21] F. M. Izrailev, *Intermediate statistics of the quasi-energy spectrum and quantum localisation of classical chaos,* Journal of Physics A: Mathematical and General **22**, 865 (1989).
[22] G. Lenz and F. Haake, *Reliability of small matrices for large spectra with nonuniversal fluctuations,* Phys. Rev. Lett. **67**, 1 (1991).
[23] E. Caurier, B. Grammaticos, and A. Ramani, *Level repulsion near integrability: a random matrix analogy,* Journal of Physics A: Mathematical and General **23**, 4903 (1990).
[24] B. I. Shklovskii, B. Shapiro, B. R. Sears, P. Lambrianides, and H. B. Shore, *Statistics of spectra of disordered systems near the metal-insulator transition,* Phys. Rev. B **47**, 11487 (1993).
[25] F. Evers and A. D. Mirlin, *Anderson transitions,* Rev. Mod. Phys. **80**, 1355 (2008).
[26] A. D. Mirlin, *Statistics of energy levels and eigenfunctions in disordered systems,* Physics Reports **326**, 259 (2000).
[27] R. Modak, S. Mukerjee, and S. Ramaswamy, *Universal power law in crossover from integrability to quantum*

chaos, Phys. Rev. B **90**, 075152 (2014).
[28] V. Oganesyan and D. A. Huse, *Localization of interacting fermions at high temperature,* Phys. Rev. B **75**, 155111 (2007).
[29] Y. Avishai, J. Richert, and R. Berkovits, *Level statistics in a heisenberg chain with random magnetic field,* Phys. Rev. B **66**, 052416 (2002).
[30] R. Modak and S. Mukerjee, *Finite size scaling in crossover among different random matrix ensembles in microscopic lattice models,* New Journal of Physics **16**, 093016 (2014).
[31] F. J. Dyson, *A Brownian-Motion Model for the Eigenvalues of a Random Matrix,* Journal of Mathematical Physics **3**, 1191 (1962).
[32] J. T. Chalker, I. V. Lerner, and R. A. Smith, *Random walks through the ensemble: Linking spectral statistics with wave-function correlations in disordered metals,* Phys. Rev. Lett. **77**, 554 (1996).
[33] R. Nandkishore, S. Gopalakrishnan, and D. A. Huse, *Spectral features of a many-body-localized system weakly coupled to a bath,* Phys. Rev. B **90**, 064203 (2014).
[34] E. B. Bogomolny, U. Gerland, and C. Schmit, *Models of intermediate spectral statistics,* Phys. Rev. E **59**, R1315 (1999).
[35] J. T. Chalker and G. J. Daniell, *Scaling, diffusion, and the integer quantized hall effect,* Phys. Rev. Lett. **61**, 593 (1988).
[36] See Supplemental Material.
[37] E. Cuevas and V. E. Kravtsov, *Two-eigenfunction correlation in a multifractal metal and insulator,* Phys. Rev. B **76**, 235119 (2007).
[38] V. E. Kravtsov and I. V. Lerner, *Effective plasma model for the level correlations at the mobility edge,* Journal of Physics A: Mathematical and General **28**, 3623 (1995).
[39] V. E. Kravtsov and K. A. Muttalib, *New class of random matrix ensembles with multifractal eigenvectors,* Phys. Rev. Lett. **79**, 1913 (1997).
[40] K. A. Muttalib, Y. Chen, M. E. H. Ismail, and V. N. Nicopoulos, *New family of unitary random matrices,* Phys. Rev. Lett. **71**, 471 (1993).
[41] A. M. García-García and J. J. M. Verbaarschot, *Critical statistics in quantum chaos and calogero-sutherland model at finite temperature,* Phys. Rev. E **67**, 046104 (2003).
[42] J. M. G. Gómez, R. A. Molina, A. Relaño, and J. Retamosa, *Misleading signatures of quantum chaos,* Phys. Rev. E **66**, 036209 (2002).