This is the accepted manuscript made available via CHORUS. The article has been published as:

**Chiral plasmon in gapped Dirac systems**
Anshuman Kumar, Andrei Nemilentsau, Kin Hung Fung, George Hanson, Nicholas X. Fang, and Tony Low

Phys. Rev. B 93, 041413 — Published 19 January 2016
DOI: 10.1103/PhysRevB.93.041413
Chiral plasmon in gapped Dirac systems

Anshuman Kumar,1 Andrei Nemilentsau,2 Kin Hung Fung,3 George Hanson,2 Nicholas X. Fang,1,* and Tony Low4,†

1Mechanical Engineering Department, Massachusetts Institute of Technology, Cambridge, MA 02139, USA
2Department of Electrical Engineering & Computer Science, University of Wisconsin-Milwaukee, Milwaukee, WI 53211, USA
3Department of Applied Physics, The Hong Kong Polytechnic University, Hong Kong, China
4Department of Electrical & Computer Engineering, University of Minnesota, Minneapolis, MN 55455, USA
(Dated: December 22, 2015)

We study the electromagnetic response and surface electromagnetic modes in a generic gapped Dirac material under pumping with circularly polarized light. The valley imbalance due to pumping leads to a net Berry curvature, giving rise to a finite transverse conductivity. We discuss the appearance of nonreciprocal chiral edge modes, their hybridization and waveguiding in a nanoribbon geometry, and giant polarization rotation in nanoribbon arrays.

PACS numbers: 73.20.Mf, 78.20.-e, 78.67.-n

Introduction— The Berry curvature is a topological property of the Bloch energy band, and acts as an effective magnetic field in momentum space[1–3]. Hence, topological materials may exhibit anomalous Hall-like transverse currents in the presence of an applied electric field, in absence of a magnetic field. Examples includes topological insulators[4] with propagating surface states that are protected against backscattering from disorder and impurities, and transition metal dichalcogenides where the two valleys carry opposite Berry curvature giving rise to a bulk topological charge neutral valley currents[5, 6]. These bulk topological currents were also experimentally investigated in other Dirac materials, such as gapped graphene and bilayer graphene system[7, 8]. The electromagnetic response of these gapped Dirac systems, particularly that due to its surface electromagnetic modes (i.e plasmons) are relative unexplored.

In gapped graphene or transition metal dichalcogenides, electrons in the two valleys have opposite Berry curvature, ensured by time-reversal symmetry (TRS) of their chiral Hamiltonians[5]. Hence, far field light scattering properties of these system does not differentiate between circularly polarized light, i.e. zero circular dichroism in the classical sense. Optical pumping with circularly polarized light naturally breaks TRS, and a net chirality ensues. However, under typical experimental conditions, the transverse conductivity due to Berry curvature is less than the quantized conductivity $\frac{e^2}{\hbar}$, and the associated optical dichroism effect is not prominent. These effects, however, can potentially be amplified through enhanced light-matter interaction with plasmons[9–13].

In this letter, we discuss the emergence of new chiral electromagnetic plasmonic modes and their associated optical dichroism effect. We consider a gapped Dirac system under continuous pumping with circularly polarized light. We discuss the appearance of edge chiral plasmons, and how they can allow launching of one-way propagating edge plasmons in a semi-infinite geometry. We consider also the hybridization of these chiral edge modes in a nanoribbon geometry and the possibility of nonreciprocal waveguiding. Their far-field optical properties reveal resonant absorption accompanied by sizeable polarization rotation.

Model system— We consider the following Hamiltonian of a massive Dirac system (MDS),

$$\mathcal{H} = \hbar v_f \mathbf{k} \cdot \mathbf{\sigma} + \frac{\Delta}{2} \mathbf{\sigma}_z$$

where $\mathbf{\sigma} = (\sigma_x, \sigma_y)$, $\tau = \pm 1$ denotes the $K/K'$ valley, $\Delta$ is the energy gap and $v_f$ is the Fermi velocity. We denote the eigenenergy and wavefunctions of $\mathcal{H}$ as $E_{\tau,\nu}(\mathbf{k})$ and $\Psi_{\tau,\nu}(\mathbf{k})$, with $\nu = e, h$ denoting the electron and hole bands. We are interested in the dynamics of the electronic subsystem in an external electromagnetic field $\mathbf{E}$ as illustrated in Fig. 1a, which can be described with

---

**Fig. 1.** Optically induced valley polarization: (a) Polarization selective pumping leads to different populations in the $K$ and $K'$ valleys. (b) DC electronic carrier concentration in the two valleys as a function of the pump electric field. (c) DC $\sigma_{xy}$ in the two valleys as a function of the right circular polarized pump electric field.
the von Neumann equation, \(i\hbar \partial_t \hat{\rho} = \{\mathcal{H} + V, \hat{\rho}\}\), where \(\hat{\rho}\) is the statistical operator of the electron subsystem and \(V = -eE \cdot \mathbf{r}\) is the interaction term. In the \(\Psi_{\tau,\nu}(k)\) basis, the equation of motion is written explicitly as[14, 15],

\[
\frac{\partial \rho_{jj'}}{\partial t} + \frac{e}{\hbar} \mathbf{E} \cdot \frac{\partial \rho_{jj'}}{\partial k} = -\frac{i}{\hbar} \rho_{jj'}[\mathcal{E}_j(k) - \mathcal{E}_{j'}(k)] + \frac{i e}{\hbar} \mathbf{E} \cdot \sum_{\nu''} \mathbf{R}_{jj''}(k) \rho_{j''j'} - \rho_{jj'} \mathbf{R}_{j''j''}(k) \tag{2}
\]

where \([\rho(t,k)]_{j,j'}\) is the density matrix, \(\mathbf{R}_{jj'}(k) = (i/2) \int \Psi^*_j(k) \partial_\nu \Psi_{j'}(k) d\mathbf{k} + h c, \) with \(j = \{\nu, \tau\}\) designating quantum number of electrons. In Eq. (2) we neglect indirect interband optical transitions.

Here, we are interested in the interaction with a continuous (c.w.) monochromatic electromagnetic wave, \(\mathbf{E} = \mathbf{E}_0 e^{-i\omega t} + cc\). Using the rotating wave approximation and introducing relaxation phenomenologically within the relaxation time approximation, we obtain the steady-state solution as a system of four linear equations for diagonal components of the distribution function, \(\rho_{ jj} \equiv \rho_{jj}\).

\[
\rho_{\nu',\tau}(k) = \beta_{\tau} \left( \rho_{0,\nu',\tau}^0(k) + \gamma \rho_{\nu',\tau}(k) + \alpha_{\tau} \rho_{\nu',\nu}(k) \right) \tag{3}
\]

where \(\tau' \neq \tau\) and \(\nu' \neq \nu\). Here,

\[
\alpha_{\tau} = \frac{2e^2 |\mathbf{E}_0| \cdot \mathbf{R}_{ev}(k)|^2}{\hbar^2 (\omega - \omega_{ev})^2 + 1/\tau_0^2}, \tag{4}
\]

\[\omega_{ev} = (\mathcal{E}_{\tau,c} - \mathcal{E}_{\nu,v})/\hbar, \beta_{\tau} = 1/(1 + \gamma + \alpha_{\tau}), \gamma = \tau_0/\tau_1, \]

where \(\tau_0\) is the population relaxation time and \(\tau_1\) is the intervalley scattering time (see supplementary information). The equilibrium distribution function is given by the Fermi Dirac distribution, \(\rho_{0,\nu',\tau}^0(k) = [1 + \exp((\mathcal{E}_{\nu',\nu}(k) - \mu)/k_B T)]^{-1}\).

Let us consider positive or right circular polarized light, \(\mathbf{E}_0 = E_0 (e_x + i e_y)\), interacting with electrons at the top of valence band, \(\mathbf{R}_{ev}(0) = -(v_f / \hbar \Delta) (i\tau e_x + e_y)\). It can be clearly seen that \(\mathbf{E}_0 \cdot \mathbf{R}_{ev}(0) = -i(v_f / \hbar \Delta)(\tau + 1)\), and thus \(\alpha_{\tau}\) are zero at \(K\) valley while being finite at the \(K'\) valley. Hence, pumping with circularly polarized light would lead to carrier population imbalance between the two valleys.

**Net chirality with pumping**— The effective Hamiltonian in Eq.1 captures the valley physics in physical system such as monolayer graphene with staggered sublattice potential[16] and transition metal dichalcogenides[5], if the spin-orbit coupling term can be neglected. To proceed, we consider some reasonable numbers for our model gapped Dirac system: an energy gap \(\Delta = 0.5 eV\) and Fermi velocity \(v_f = 1 \times 10^6 m/s\). Our calculations assume temperature \(T = 300 K\), typical carrier lifetimes \(\tau_0 = 1 ps\) and that the system is undoped at equilibrium. With pumping, charge neutrality and electron-hole symmetry would require that the electron and hole carrier densities follow \(n_e^0 = n_h^0\). Fig.1b shows the increasing non-equilibrium electron densities as function of pump intensities \(E_0\), under continuous wave pumping with right circular polarized light. Finite transfers of electrons from the \(K\) to \(K'\) valley is determined by the inter-valley scattering rate described by \(\gamma\).

In the presence of an external electric field \(\mathbf{E}\), the carrier velocity acquires a non-classical transverse term due to Berry curvature, \(\Omega_\nu(k)\), given by \(-\frac{e}{\hbar} \mathbf{E} \times \mathbf{\Omega}(k)\). For a MDS, the form of the Berry curvature is well known[5]. Within the semiclassical Boltzmann transport theory, this would give rise to a transverse conductivity, which in the charge neutral case we are considering here is simply given by \(\sigma_{xy} = 2e^2/\hbar \int [dk] \rho_{e,\tau}(k) \mathbf{\Omega}(k)\). The factor of 2 accounts for contributions from both electrons and holes. It can further be shown that \(\sigma_{xy} = -\sigma_{yx}\). Since TRS requires \(\Omega_K(k) = -\Omega_{K'}(k), \sigma_{xy} = -\sigma_{yx}\) at equilibrium. However, under continuous wave pumping, the asymmetric carrier populations in the two valleys would produce a net transverse conductivity, as shown in Fig. 1c. We note that over the frequency range that we are interested in, i.e. \(\hbar \omega \ll \Delta, \sigma_{xy}\) is real and frequency independent[17]. Our calculations suggest that \(\sigma_{xy}\) an order smaller than \(e^2/\hbar\) is obtainable with pump intensities \(E_0\) routinely used in pump-probe experiments. The non-equilibrium longitudinal components of the conductivity, \(\sigma_{xx} = \sigma_{yy}\), are computed with the Kubo formula[17]. For all the subsequent results of this paper, we use a pump intensity of \(E_0 = 10^8 V/m\) and \(\gamma = 0.01\).

**Valley induced bulk and edge chiral plasmon**— Armed with the conductivity sum of the two valleys, \(\sigma_{ij}\) we discuss general results for the plasmon modes in this system. Plasmon dispersion in a continuous sheet of the MDS is given by[18–20]:

\[
\frac{\epsilon_1}{\kappa_1} + \frac{\epsilon_2}{\kappa_2} + \frac{i\sigma_{xx}}{\omega \epsilon_0} \left[ \kappa_1 + \kappa_2 - \frac{\sigma_{yy}}{\epsilon_0 \kappa_0} \right] = 0
\]

where \(\kappa_{1,2} = \sqrt{\eta^2 - \epsilon_1 \epsilon_2 \beta^2}\) are the evanescent decay constants on either side of the 2D sheet. As shown in Fig. 2, this “bulk plasmon” dispersion is symmetric with respect to the wavevector \(q\), since it appears quadratically in Eq. 5.

Edges can also accommodate plasmon modes[21]. Symmetry arguments show that although bulk plasmon dispersion respects \(\omega(q) = \omega(-q)\) with nonsymmetric conductivity tensor, the presence of an edge can break this degeneracy[22]. Here we consider the case of semi-infinite MDS. Within the quasi-static picture, the edge plasmon dispersion is approximately given by[21, 23]:

\[
\eta^2 - \chi^2 - 3\eta + 2\sqrt{2} \chi \text{sngn}(q) = 0, \quad \eta = |q| \sigma_{xx}/(\epsilon_0 \omega)\]

and \(\chi = |q| \sigma_{xy}/(\epsilon_0 \omega)\). Fig. 2a indeed shows that the right moving edge plasmon has a different dispersion compared to the left moving one. A simple realization of this nonreciprocity effect consists of placing a dipole near the edge of the material. Finite element simulation of near field dipole emission was performed using COMSOL. As shown in Fig. 2b and c, the linear dipole
reciprocity with respect to edge localization as discussed spatial symmetry[22]. However, these modes show non-
these plasmon modes is symmetric due to the presence of 
gation direction being coupled to the ribbon edge. It 
how the chirality of the plasmon leads to the propa-
gation direction will involve a
nonreciprocal devices. For instance, we can break the spa-
tial symmetry between left and right by introducing an-
other medium on one side of the ribbon. In the most 
nonreciproal, selection of the magnitude of the plasmon
momentum can be selected either by use of a grating near 
the edge[26–28]. Since the edge plasmon dispersion is
miniaturization.
In terms of experiment, the appropriate plasmon mo-
momentum can be selected either by use of a grating near 
the edge[25], by adjusting the distance between the tip of 
a near field microscope and the edge of the MDS or the 
tip radius[26–28]. Since the edge plasmon dispersion is
nonreciproal, selection of the magnitude of the plasmon
momentum will also lead to selectivity in the propaga-
tion direction. In addition to the different intensities of 
the two, the different wavelengths for the left and right
moving edge modes in this configuration might be used 
for nonreciprocal phase shifters[29].
Waveguiding in nanoribbons– Any practical realization 
of the semi-infinite case discussed above will involve a
stripe or waveguide geometry. Waveguides are an im-
portant component of plasmonic circuitry[30] and ribbon
waveguides based on the plasmon modes in graphene
have been proposed[31, 32]. In this section, we show 
how the chirality of the plasmon leads to the propaga-
gation direction being coupled to the ribbon edge. It
should be noted that unlike the semi-infinite case for 
ribbons placed in homogeneous space, the dispersion of 
these plasmon modes is symmetric due to the presence of 
spatial symmetry[22]. However, these modes show non-
reciprocity with respect to edge localization as discussed 
below.
As shown in Fig. 3, qualitatively the typical profile 
of plasmons in ribbon[31, 32] or nanowire[33] geometries 
is observed: there an acoustic branch arising from a 
monopole like mode and a discrete set of higher order
 guided modes which show a cutoff. The high frequency
field profiles of the two lowest order modes (for exam-
ple, 1’ and 2’ for \( k_z > 0 \)) in Fig. 3 reveal that these
modes have the character of edge modes in semi-infinite
MDS. In fact, these edge modes of the ribbon lie outside 
the “cone” of the bulk plasmon mode for the continuous
MDS. As we approach lower frequencies, the edge locali-
zation of these two modes becomes weaker and they start 
hybridizing.
All the other modes are guided modes with field max-
ima in the bulk. These lie inside the cone formed by 
the dispersion of the plasmon in continuous MDS. Thus 
these modes are analogous to the guided modes in slab 
waveguides. The cutoff frequencies for all except the low-
est mode are consistent with the Fabry-Perot condition,
\( k_B w + \phi_R = n \pi \), where \( k_B \) is the bulk plasmon 
momentum in the MDS, as given by Eq. 5 and \( \phi_R \approx -3 \pi/4 \)
is the approximate phase acquired by the plasmon upon 
reflection from the ribbon edge[34].
The chirality of the plasmon mode in our case, gives 
rise to the coupling between the propagation direction 
and the edge. For instance, for positive \( k_z \), at higher
frequencies, we observe that the field is only confined 
to the left edge for the lowest mode. Such a coupling is 
useful for enhancing the lifetime of the mode propagating 
in a given direction.
This special coupling between the edge mode direction 
and the ribbon edge can be utilized to produce explicit 
nonreciproal devices. For instance, we can break the sp-
tial symmetry between left and right by introducing an-
other medium on one side of the ribbon. In the most
more extreme case, a perfect conductor can be used to short
the edge mode on one side[29].
Valley induced giant polarization rotation– Polariza-
rotation rotation is usually discussed in the context of 
magneto-optical materials (also called Faraday effect),
where the plane of polarization of the incident wave is
rotated upon passage through such a material[37].
Cyclotron resonances in various two dimensional electron 
gases[38] were employed to produce this effect, with 
graphene being the most promising candidate[39]. Op-
tically induced valley polarization in a MDS presents a
promising route to achieve a similar effect without the
application of a static magnetic field, which can be cum-
bersome in the context of on-chip photonic components
miniaturization.
We first consider polarization rotation in a continu-
ous sheet of MDS. The polarization rotation angle is
given by[40]: \( \theta_F \approx \Im \langle \sigma_{xy} \rangle / 2 \epsilon_0 \) and the transmission
by \( T(\omega) \approx 1 - \Re \langle \sigma_{xx} \rangle / \epsilon_0 \). It should be noted that as
opposed to optical activity[41] which is reciprocal, the
polarization rotation in our case is analogous to Farad-
day rotation which is a purely nonreciprocal effect[42].

FIG. 2. (a) Chiral plasmon dispersion in bulk and semi-
infinite MDS. (b) Selective excitation of edge modes using 
circular and linear polarized dipoles placed at the origin: Line 
plots of electric field \( E_x \) in the plane and perpendicular to the 
edge. The vertical offset is \( 6 \times 10^{18} \text{V/m} \). (c) \( |E| \) field (nor-
malized to max) profile for a \( \hat{z} \)-polarized emitter located near 
the edge of semi infinite Dirac material \( (z < 0) \) at \( \omega = 0.1 \text{eV} \).
Both of these field profiles show nonreciprocal emission into 
the edge mode. The dipoles are placed 10 nm above the MDS.
preferentially emits into the left propagating edge state.
Taking a cue from [24], we can also use a circular dipole
to couple emission into left or right edge state, depend-
ing on dipole helicity. The results for circular dipoles are 
presented in Fig. 2b.
In terms of experiment, the appropriate plasmon mo-
momentum can be selected either by use of a grating near 
the edge[25], by adjusting the distance between the tip of 
a near field microscope and the edge of the MDS or the 
tip radius[26–28]. Since the edge plasmon dispersion is
nonreciproal, selection of the magnitude of the plasmon
momentum will also lead to selectivity in the propaga-
tion direction. In addition to the different intensities of 
the two, the different wavelengths for the left and right
moving edge modes in this configuration might be used 
for nonreciprocal phase shifters[29].
FIG. 3. Guided modes in freestanding MDS ribbons. Ribbon width is assumed to be \( w = 100 \) nm. The grey dashed lines represents solutions of \( k_B w + \phi_R = n\pi \), where \( k_B \) is the bulk plasmon momentum in MDS and \( \phi_R \approx -3\pi/4 \)[34], which explains the cutoff for all the guided modes (except the edge mode). The black solid lines represent the bulk plasmon in a continuous sheet of MDS (same as Fig. 2). The color plots below represent the real part of the electric field along the ribbon at the indicated \( q \).

These equations suggest that the polarization rotation values in the continuous 2D sheet is only dependent on the \( \sigma_{xy} \) which can be tuned by adjusting the intensity and polarization of the pump. Even with a pump intensity of the order of \( 10^8 \) V/m, rotation angle of only about 0.1 degrees is obtained.

However, it is possible to use localized plasmon resonances[43, 44] of nanoribbons[45] to enhance the polarization rotation values. In Fig. 4, we present the simulation results for transmission and polarization rotation in nanoribbons. We obtain significant enhancement of polarization rotation by more than an order of magnitude upon using nanoribbons as opposed to a continuous 2D sheet. Moreover, at the frequency of the resonant enhancement, transmitted intensity is still about 10–20%. The spectral location of the resonance is strongly tunable as a function of the ribbon width. These frequencies correspond to the solutions of \( k_B w + \phi_R = n\pi \), as described earlier but with the constraint that \( n \) is an even integer[34]. Odd \( n \) solutions are non-dipolar modes, hence do not couple with normally incident plane waves. The largest polarization rotation occurs for smaller ribbon sizes. This is because smaller ribbons, correspond to larger in-plane wavevectors, thus providing a higher field confinement. The polarization rotation we obtained with nanoribbons was found to even surpass Faraday rotation angles in monolayer graphene under a magnetic field of 7 T[39].

FIG. 4. Transmission and polarization rotation in freestanding MDS ribbons: (a) Transmission (vertical offset is one unit). Inset: Schematic of the configuration. Note that in general the transmitted wave is expected to be elliptically polarized as opposed to linear as shown here. (b) polarization rotation spectrum for different ribbon sizes \( w \) (vertical offset is 6 degrees). For ribbon arrays, a filling factor of 50% has been assumed.

7 Conclusion and summary— In summary, we have shown how polarization selective pumping in a generic gapped Dirac material can impart chirality to bulk and edge plasmons without the need for an external magnetic field. Experimentally testable predictions in the context of near field imaging, giant valley induced polarization rotation as well as nonreciprocal waveguiding were presented. Our theoretical approach can be applied to a general class of two dimensional materials with broken inversion symmetry. A rich array of nonreciprocal phenomenon can be potentially explored, from the point of view of applications to isolators, circulators, etc. Finally, since unlike a magnetic field, the field profile of the optical pump can be easily manipulated on the subwavelength scale by the use of nanostructures[46], our work might pave the way for chip scale nonreciprocal photonics and optically tunable metasurfaces[47, 48].

During the preparation of our manuscript, we became aware of a related preprint[49].

Acknowledgement. A.K. and N.X.F. acknowledge the financial support by the NSF (grant CMMI-1120724) and AFOSR MURI (Award No. FA9550-12-1-0488). K.H.F. acknowledges financial support from Hong Kong RGC grant 15300315. T.L. acknowledges support from the MRSEC Program of the National Science Foundation under Award Number DMR-1420013.

* nicfang@mit.edu
† tlow@umn.edu
[1] M. V. Berry, in Proceedings of the Royal Society of Lon-