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Control of magnetism in singlet-triplet superconducting heterostructures

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We analyze the magnetization at the interface between singlet and triplet superconductors and show that its direction and dependence on the phase difference across the junction are strongly tied to the structure of the triplet order parameter as well as to the pairing interactions. We consider equal spin helical, opposite spin chiral, and mixed symmetry pairing on the triplet side and show that the magnetization vanishes at $\phi = 0$ only in the first case, follows approximately a $\cos\phi$ behavior for the second, and shows higher harmonics for the last configuration. We trace the origin of the magnetization to the magnetic structure of the Andreev bound states near the interface, and provide a symmetry-based explanation of the results. Our findings can be used to control the magnetization in superconducting heterostructures and to test symmetries of spin-triplet superconductors.

I. INTRODUCTION

Attempts to combine dissipationless transport in superconductors with the control of spin currents has driven many recent studies of superconducting heterostructures[1]. In the vast majority of superconductors the conduction electrons pair in a spin-singlet state, freezing out the spin degrees of freedom at low temperature. However, triplet correlations near interfaces with magnetic materials produce equal-spin Cooper pairs that can propagate through a magnet leading, for example, to the long-range proximity effect[2–4].

The alternative pathway to spin control by superconductivity is via utilizing compounds that support spin-triplet pairing. The number of known triplet superconductors (TSCs) has been growing steadily, and now includes UPt_3[5], ferromagnetic superconductors such as UGe_2, URhGe, UIr and UCoGe[6–8], quasi one-dimensional organic system $(TMTSF)_2X$ ($X=ClO_4$ and PF_6)[9,10]. Singlet and triplet states are mixed if the material lacks inversion symmetry[11,12], and among the non-centrosymmetric superconductors Li_2Pt_3B has the clearest indication of a significant triplet component[13,14]. The strongest evidence for triplet superconductivity has emerged for Sr_2RuO_4[15,16]. Existence of very pure single crystals with the perovskite structure made this material a testbed for studying heterostructures based on triplet superconductivity[17,18].

From energetic considerations most of the non-magnetic superconducting compounds that support triplet pairing should have a unitary order parameter[19], so that the Cooper pairs do not have a net average spin-derived magnetic moment[20]. Hence, triplet superconductors cannot by default be assumed to support dissipationless spin transport. However, in conjunction with superconducting orders of different symmetry, non-trivial spin aspects of triplet superconductivity appear. Andreev bound states in singlet-triplet Josephson heterostructures were analyzed[21–25], and spin-accumulation was found when a phase difference is established across the junction[26–28]. In parallel, it was shown that admixture of the subdominant order may lead to spin accumulation, spin and charge currents near a boundary of a chiral triplet superconductor[29] or anomalous flux response in mesoscopic loops[30].

In this paper we show that the magnetism in singlet-triplet superconducting heterostructures is fundamentally linked to the nature of the triplet pairing. We consider a microscopic model for a high-transparency interface between a singlet and a unitary triplet (or mixed parity, see below) superconductor, and self-consistently solve the corresponding Bogoliubov-de Gennes (BdG) equations to obtain the energy spectrum. In all cases we find spin-splitting of the Andreev bound states (ABS) near the interface, leading to a magnetization parallel to the spin-triplet $\boldsymbol{d}$-vector. However, the variation of the magnetization M with the phase difference ϕ across the junction is non-trivial, and depends on the nature of the pairing interaction on the triplet side. Our main results are summarized in Fig. 1, where we consider three distinct cases. If there is no singlet component of the pairing interaction on the triplet side, $M \propto \sin\phi$ only appears under a finite phase difference, in agreement with Refs. 26 and 27. This is realized, for example, for equal spin pairing states. Real space interactions leading to triplet $S^z = 0$ pairing often promote a (subdominant) singlet pairing, so that the singlet amplitude persists into TSC near the interface. The magnetization resulting from this mixed symmetry, $M \propto \cos\phi$, is finite already at $\phi = 0$. Finally, motivated by the results of Ref. 29, we consider a mixed parity superconductor in contact with a singlet counterpart, and show that the magnetization has a complex dependence on ϕ.

These results show that the phase difference can be used to control interface magnetization. In addition, the interface magnetization probes the spin structure of the pairing interactions. This finding is especially relevant for Sr_2RuO_4. There is no consensus on the exact form of the triplet pairing in this system, and NMR Knight shift measurements[31], absence of edge currents[32], and the observation of half-quantum vortices[33] put in doubt strong pinning of the triplet order spin vector, $\boldsymbol{d}(\boldsymbol{k})$, to the crystalline c-axis. The chiral, $\boldsymbol{d}(\boldsymbol{k}) = \widehat{\boldsymbol{z}}(k_x + ik_y)$, and the

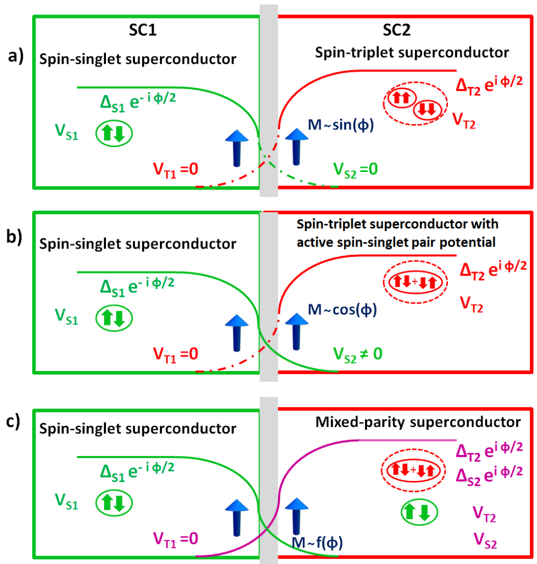

FIG. 1. (Color online). Schematics for the superconducting heterostructures between spin-singlet (SC1) and triplet-active (SC2) superconductors. Panel a): helical equal spin pairing state with no pairing interaction in the singlet channel. Panel b): chiral pairing with finite interaction in the singlet channel (see discussion in text). Panel c): mixed-parity order parameter near the interface. V_{Si} (V_{Ti}) are the couplings in the singlet (triplet) channel for sides $i = 1, 2$. We show the profile of the order parameters (full line), with dot-dashed line for the proximity-induced order. We also indicate the qualitative dependence of the interface magnetization on the phase ϕ.

helical, $\boldsymbol{d}(\boldsymbol{k}) = \widehat{\boldsymbol{x}}k_y + \widehat{\boldsymbol{y}}k_x$, states compete closely[34], and distinguishing between these two possibilities with similar bulk gap is an important application of our results. The remainder of the paper is organized as follows. We present in Sect. II the model and the formalism for the analysis of the singlet-triplet heterostructure. Sect. III is devoted to the presentation of the results and the discussion concerning the evolution of the order parameters and the magnetization at the inerface for the case of chiral, helical spin-triplets and mixed-parity superconductors. Finally, the Sect. IV is for the concluding remarks.

II. MODEL AND FORMALISM.

We consider a two-dimensional lateral heterostructure made of two superconductors, SC1 and SC2, having spin-singlet (SC1) and spin-triplet or mixed-parity (SC2) pairing. The x and y planar directions are perpendicular and parallel to the SC1/SC2 interface, respectively. The system is uniform along the y axis, so that the translational symmetry is broken only in the x direction. The Hamiltonian is then defined on a square lattice of size $L \times L$ (the lattice constant is unity), with periodic boundary conditions along y,

$$H = H_1 + H_2 + H_{12} \tag{1}$$

with H_m $(m = 1, 2)$ being the Hamiltonians of each superconducting side

$$H_m = -t_m \sum_{\langle \mathbf{i},\mathbf{j}\rangle,\,\sigma} (c^{\dagger}_{\mathbf{i}\,\sigma} c_{\mathbf{j}\,\sigma} + \text{h.c.}) - \mu_m \sum_{\mathbf{i},\sigma} n_{\mathbf{i}\sigma} \tag{2}$$
$$- \sum_{\langle \mathbf{i},\mathbf{j}\rangle} V_m^{\sigma\sigma'} n_{\mathbf{i}\sigma} n_{\mathbf{j}\sigma'} - U_m \sum_{\mathbf{i}} n_{\mathbf{i}\uparrow} n_{\mathbf{i}\downarrow}\,,$$

and the interface term,

$$H_{12} = \sum_{\delta=\pm\mathbf{1}} t_{\perp} (c^{\dagger}_{\mathbf{0}\,\sigma} c_{\delta\,\sigma} + \text{h.c.})\,. \tag{3}$$

Here the lattice sites are labeled by $\mathbf{i} \equiv \{i_x, i_y\}$, with i_x and i_y integers between $-L/2$ and $L/2$, $\langle \mathbf{i}, \mathbf{j}\rangle$ denote nearest-neighbor sites, and μ is the chemical potential. Labels $\mathbf{0} = \{0, i_y\}$ and $\pm\mathbf{1} = \{\pm 1, i_y\}$ denote the sites at the interface and their nearest-neighbors, respectively. The attractive interaction $-V_m^{\sigma\sigma'}$ $(V_m > 0)$ can be chosen to be effective in the $S_z = 1, 0, -1$ projections for the the TSC and/or in the $\boldsymbol{S} = 0$ singlet sector. The local attractive term $-U_m$ $(U > 0)$ only promotes spin-singlet pairing. For simplicity we take $t_1 = t_2 = t_{\perp} = t$, and use the hopping parameter t as a unit of energy. Below, in the description of the results we discuss the qualitative consequences of relaxing this assumption.

To investigate the model of Eq. (1) we decouple the interaction term in the Hartree-Fock approximation by introducing the pairing amplitude on a bond, $\Delta_{\mathbf{ij}}^{\sigma\sigma'} = \langle c_{\mathbf{i}\,\sigma} c_{\mathbf{j}\,\sigma'}\rangle$, and on-site $\Delta_0 = \langle c_{\mathbf{i}\uparrow} c_{\mathbf{i}\downarrow}\rangle$, so that

$$V^{\sigma\sigma'} n_{\mathbf{i}\sigma} n_{\mathbf{j}\sigma'} \simeq V^{\sigma\sigma'} (\Delta_{\mathbf{ij}}^{\sigma\sigma'}\, c^{\dagger}_{\mathbf{j}\,\sigma'} c^{\dagger}_{\mathbf{i}\,\sigma} + \bar{\Delta}_{\mathbf{ij}}^{\sigma\sigma'}\, c_{\mathbf{i}\,\sigma} c_{\mathbf{j}\,\sigma'} - |\Delta_{\mathbf{ij}}^{\sigma\sigma'}|^2)\,.$$

These expressions yield the spin singlet (S) and triplet (T) components in the $S_z = 0$ sector, $\Delta^{S,T} = (\Delta_{\mathbf{ij}}^{\uparrow\downarrow} \pm \Delta_{\mathbf{ji}}^{\uparrow\downarrow})/2$, and the triplet pairing in the $S_z = \{1, -1\}$ sectors, $\Delta_T^{\sigma\sigma} = \Delta_{\mathbf{ij}}^{\sigma\sigma}$. They define in turn the superconducting pair amplitudes with s- or p-wave symmetry, i.e. $\Delta_s(\mathbf{i}) = (\Delta^S_{\mathbf{i},\mathbf{i}+\hat{x}} + \Delta^S_{\mathbf{i},\mathbf{i}-\hat{x}} + \Delta^S_{\mathbf{i},\mathbf{i}+\hat{y}} + \Delta^S_{\mathbf{i},\mathbf{i}-\hat{y}})/4$, $\Delta_{p_{x(y)}}(\mathbf{i}) = (\Delta^T_{\mathbf{i},\mathbf{i}+\hat{x}(\hat{y})} - \Delta^T_{\mathbf{i},\mathbf{i}-\hat{x}(\hat{y})})/2$ and $\Delta^{\sigma\sigma}_{p_{x(y)}}(\mathbf{i}) = \Delta^{\sigma\sigma}_{\mathbf{i},\mathbf{i}+\hat{x}(\hat{y})}$, which are then determined self-consistently[35]. It is important to note that, while the terms $V^{\uparrow\uparrow}$,$V^{\downarrow\downarrow}$ generate equal spin triplet pairing, the coupling $V^{\uparrow\downarrow}$ generically promotes both triplet $S^z = 0$ and non-local singlet (extended s-wave) pairing. Similar decoupling of the U term produces local singlet pairing only.

By suitable choices of the interactions $V^{\sigma\sigma'}$ and U we can therefore model interfaces between superconductors with different symmetries of the order parameter. We take the most conventional SC1 with $U \neq 0, V^{\sigma\sigma'} = 0$, and compare the results for three different choices of SC2.

Case a): equal spin triplet pairing, $V^{\uparrow\uparrow}$=$V^{\downarrow\downarrow} \neq 0$; $V^{\uparrow\downarrow} = U = 0$. In the bulk this corresponds to the helical state, $\widehat{\boldsymbol{\Delta}}(\boldsymbol{k}) = i(\boldsymbol{d}(\boldsymbol{k}) \cdot \boldsymbol{\sigma})\sigma^y$ with $\boldsymbol{d}(\boldsymbol{k}) =$

$\widehat{\boldsymbol{x}}k_y + \widehat{\boldsymbol{y}}k_x$, where $\widehat{\boldsymbol{\Delta}}(\boldsymbol{k}) = V^{-1/2}\int d\boldsymbol{r}_{\mathbf{ij}}\widehat{\boldsymbol{\Delta}}_{\mathbf{ij}}\exp(i\boldsymbol{k}\cdot\boldsymbol{r}_{\mathbf{ij}})$ and $\widehat{\boldsymbol{\Delta}}_{\mathbf{ij}}$ is the order parameter in spin space defined above.

Case b): a TSC with $S^z = 0$ and a subdominant extended s-wave pairing, $V^{\uparrow\uparrow}{=}V^{\downarrow\downarrow} = U = 0$; $V^{\uparrow\downarrow} \neq 0$.

Case c): a TSC with $S^z = 0$, $V^{\uparrow\downarrow} \neq 0$, but possible local s-wave pairing, $U \neq 0$, while $V^{\uparrow\uparrow}{=}V^{\downarrow\downarrow} = 0$.

For the last two situations the most favorable pairing state due to the $V^{\uparrow\downarrow}$ depends on the electron density, n, and the chiral $k_x + ik_y$ order is stabilized in the region between low doping $\mu \simeq 1.2$ and high (low) density ($|\mu| \simeq 2.25$)[36]. Hence we choose $|\mu| \simeq 1.8$. All the numerical results below have been obtained for nonvanishing components of the pairing interaction $V = 2.5$, and $U = 2.5$, and system size $L = 120$. Greater values of L and modification of the couplings leave the results qualitatively unchanged. To investigate the effects of the phase difference between the two superconductors, as is commonly done for the Josephson junctions, we transform the pairing wave-function in the SC1 and SC2 by the phase factors $\exp[-i\phi/2]$ and $\exp[i\phi/2]$, respectively. Self-consistent solution of the BdG equations gives the spin-resolved energy spectrum of the system, including both bulk and the surface (Andreev) states. The spectrum sets the surface magnetization, and we compute M by summing the spin expectation values over all the occupied states at zero temperature.

The computational procedure used to determine the self-consistent pairing amplitude at any given phase difference across the junction follows the general approaches for the solution of the Bogoliubov-de Gennes equations[35–38]. The decoupling of the quartic term on the singlet and triplet superconducting sides of the junction introduces a set of N variational parameters $\Lambda_{\mathbf{i}}$ given by the set of amplitudes $\left\{\Lambda_{\mathbf{i}}^{\uparrow\downarrow}, \Lambda_{\mathbf{i}}^{\uparrow\uparrow}, \Lambda_{\mathbf{i}}^{\downarrow\downarrow}, \Delta_{0,\mathbf{i}}\right\}$, where $\Lambda_{\mathbf{i}}^{\uparrow\downarrow} = \{\Delta^{\uparrow\downarrow}_{\mathbf{i},\mathbf{i}+\hat{\mathrm{x}}}, \Delta^{\uparrow\downarrow}_{\mathbf{i},\mathbf{i}-\hat{\mathrm{x}}}, \Delta^{\uparrow\downarrow}_{\mathbf{i},\mathbf{i}+\hat{\mathrm{y}}}, \Delta^{\uparrow\downarrow}_{\mathbf{i},\mathbf{i}-\hat{\mathrm{y}}}\}$, $\Lambda_{\mathbf{i}}^{\sigma\sigma} = \{\Delta^{\sigma\sigma}_{\mathbf{i},\mathbf{i}+\hat{\mathrm{x}}}, \Delta^{\sigma\sigma}_{\mathbf{i},\mathbf{i}+\hat{\mathrm{y}}}\}$, with $\sigma = (\uparrow, \downarrow)$ and $\Delta_{0,\mathbf{i}}$ are the local singlet order parameters. Here, $\hat{\mathrm{x}}(\hat{\mathrm{y}})$ are the unit vectors connecting the nearest neighbor sites along the direction perpendicular (parallel) to the interface. Due to the translational invariance along the y direction, the number of variational parameters can be further reduced by performing a Fourier transform. The mean-field Hamiltonian H_{MF} is, then, a function of the variational parameters Λ_{i_x}.

For a given set of variational Λ_{i_x}, and for fixed values of the microscopic parameters $\{t, V^{\sigma\sigma'}, U, \mu\}$ in the singlet and triplet sides of the heterostructures, the spectrum of the bilinear mean-field Hamiltonian is obtained by standard diagonalization routines.

At finite temperature the functional to be used for determining the variational parameters is the Gibbs energy F. It can be directly computed from the spectrum of the mean-field Hamiltonian by performing the trace over all the eigenstates as

$$F(\Lambda_{i_x}, \phi) = -\frac{1}{L_x L_y \beta}\ln\left(\mathrm{Tr}\left\{\exp[-\beta H_{MF}]\right\}\right) .$$

The variational parameters are then determined by solving the coupled set of gap equations

$$\frac{\partial F(\Lambda_{i_x})}{\partial \Lambda_{i_x}} = 0 .$$

To evaluate the phase response of the heterostructure, we have carried out two different approaches. First, we evaluate the response of the system to the application of a phase drop with a step function profile at the interface. Specifically, we first minimize the Gibbs energy at $\phi = 0$ to obtain the pairing amplitudes $\bar{\Lambda}_{i_x}(\phi = 0)$. The pairing amplitudes are calculated iteratively until the difference between successive iterations is smaller than the desired accuracy. Then, from $\bar{\Lambda}_{i_x}(\phi = 0)$ we obtain the pairing amplitude $\bar{\Lambda}_{i_x}(\phi)$ at a given phase differences ϕ by first multiplying the pairing amplitudes in the two superconductors forming the heterostructure by the phase factors $\exp[-i\phi/2]$ and $\exp[i\phi/2]$, and then evaluating the expectation value of the superconducting order parameter in the ground state or at finite temperature by means of the thermal density matrix. This approach is commonly employed in the study of Josephson junctions.

In the second approach we determine the response to a phase drop by searching for the optimal spatial configuration of the pairing amplitude that minimizes the Gibbs potential for a fixed phase profile. Such solution is obtained by iteratively calculating the pairing amplitudes $\bar{\Lambda}_{i_x}$ in the presence of the constraint of the given phase difference at the interface of the heterostructure until one achieves the convergence to the desired accuracy. In the next section of the results we present the phase dependent magnetization for the cases with and without performing the optimization of the pairing amplitudes. We stress that the phase difference is to be treated as a parameter like an effective external field, not as a minimization variable, in order to obtain the phase dependence of the observables.

III. RESULTS AND DISCUSSION

In this section we first present the case of a phase response with no optimization of the amplitude of the order parameters. Then we discuss the results obtained following the iterative approach previously explained where the order parameters that minimize the Gibbs potential are determined under the constraint of a fixed phase drop at the junction interface.

On general grounds, the interface breaks the translational symmetry in the x direction, and is therefore pair-breaking for the k_x component of the triplet pairing. The differences between the three cases listed in previous section stem from the consequences of this suppression for the interface magnetization. Case a), equal

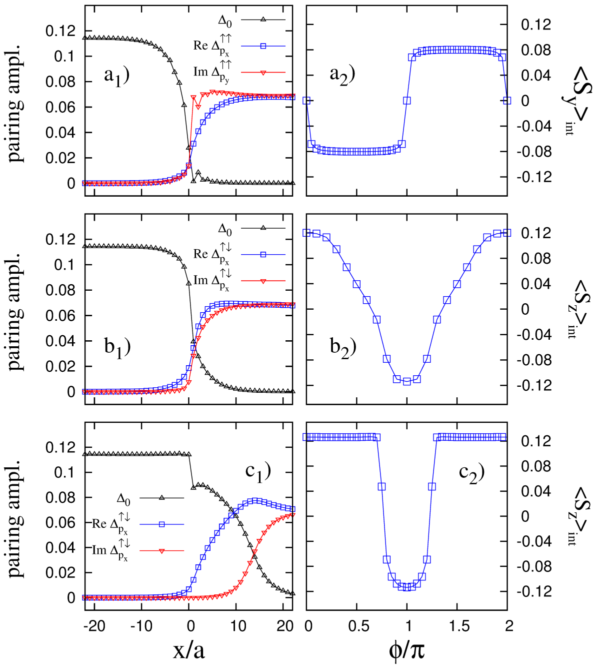

FIG. 2. (Color online). Order parameter and magnetization for heterostructures based on a spin-singlet (left side of the junction) interfaced to different spin-triplet superconductors. Panels (a) are for helical equal spin pairing, (b) for chiral $S^z = 0$pairing, and (c) for the mixed symmetry case. Left column shows the evolution of the singlet and triplet pairing amplitudes, while the right column shows the net magnetization along the direction of the $\boldsymbol{d}$-vector component for which the surface is pairbreaking as a function of the phase difference, ϕ.

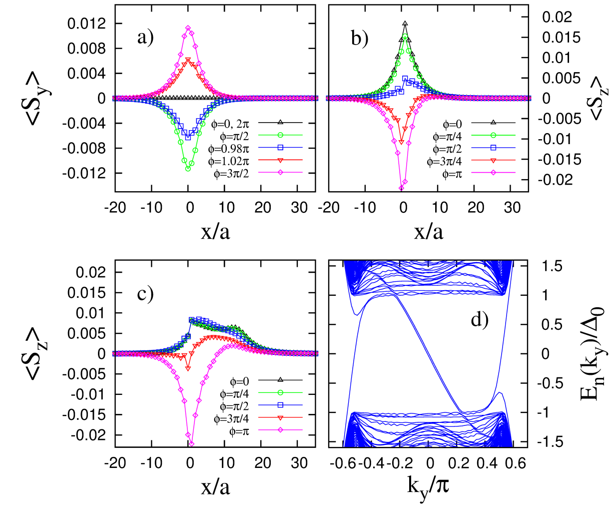

FIG. 3. (Color online). Spatial dependence of the spin-polarization. Panels (a)-(c) correspond to the cases of helical, chiral, and mixed symmetry order parameters as discussed in the text. Panel (d) shows a typical energy spectrum (for mixed symmetry case and $\phi = 0$). The thick line through the gap is the chiral edge state away from the interface. A significant contribution to the magnetization is due to the Andreev state just within the gap at large values of k_y.

spin pairing, corresponds to the usual proximity coupling with step-like change of the symmetry of the pairing interaction across the junction, where the reduction of the amplitude of the p_x component simply changes the effective coupling between the two sides, see Figs. 2(a). The magnetism of the Andreev states localized at the interface is due to the splitting of the energy levels for opposite spins. This splitting appears because the phase shift between the singlet and the triplet enters the continuity condition for the wave function at the boundary with opposite signs for the two spin orientations. Hence the magnetization vanishes at $\phi = 0$, and is nearly constant away from this point, see Fig. 2(a2). The situation is analogous to that discussed in Ref. 27, where a step-like change was found without the self-consistency on the order parameters, and assuming a delta-function barrier at the interface (BTK approximation).

The origin of this dependence is easy to understand from the Ginzburg-Landau (GL) expansion of the free energy density, with the relevant terms $f = am^2 + i\, b\, m(\psi^\star \eta_x - \psi \eta_x^\star)$. Here $a > 0$ is the inverse susceptibility, ψ (η_x) are the amplitudes of the singlet (triplet k_x) order, and m is the component of the magnetization along the $\boldsymbol{d}$-vector coupled to k_x, in our case $\widehat{\boldsymbol{y}}$. Near the interface where both $\psi = |\psi| e^{-i\phi/2}$ and $\eta_x = |\eta_x| e^{i\phi/2}$ coexist, linear coupling to the magnetization ensures that the mimimum of the free energy is at $m \propto \sin\phi$. The GL analysis is valid near T_c where indeed a $\sin\phi$-like shape develops from the step-like behavior of the same symmetry.[27] The spatial profile of the magnetization, shown in Fig. 3(a), confirms that it is constrained to the region of the order of the coherence length around the junction's interface.

In general the magnetization linearly couples to the component of the $\boldsymbol{d}$-vector for which the surface is pair breaking, and hence will be along the $\widehat{\boldsymbol{z}}$ axis for the remaining two situations. The phase dependence also becomes more complex if there is pairing in the subdominant s-wave channel on the triplet side, as in cases b) and c). Recall that we have a chiral triplet order stabilized in the bulk. Reduction in the triplet k_x component near the interface generically implies $\Delta^{\uparrow\downarrow}_{\mathbf{i},\mathbf{i}+\widehat{\mathbf{x}}} \neq -\Delta^{\uparrow\downarrow}_{\mathbf{i},\mathbf{i}-\widehat{\mathbf{x}}}$, and allows for admixture of singlet pairing. The difference from the well-known case of subdominant pairing near the surfaces in d-wave superconductors[39] is that it is the same coupling constant $V^{\uparrow\downarrow}$ that promotes coupling in both channels, and therefore the admixture is at least parametrically stronger than in the case of subdominant coupling. Indeed, it was found that even small variations in the surface barrier at the boundary of a triplet superconductor generated a substantial admixture of a singlet component and emergence of mixed par-

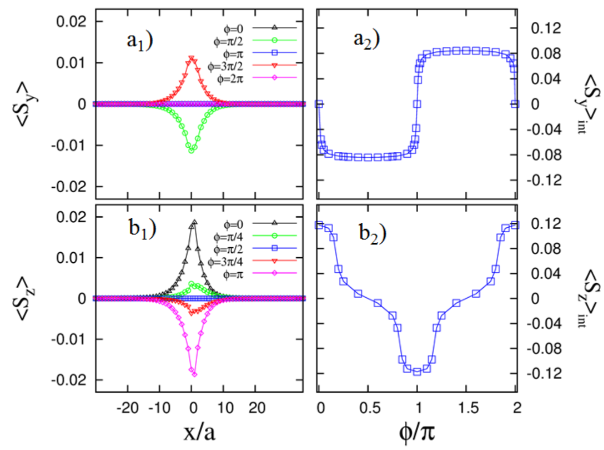

FIG. 4. (Color online). Spatial dependence of the spin-polarization and integrated interface magnetization vs applied phase difference for the cases of helical (a), and chiral (b) symmetry order parameters. The pairing amplitudes are computed self-consistently with the constraint of a given phase drop at the interface.

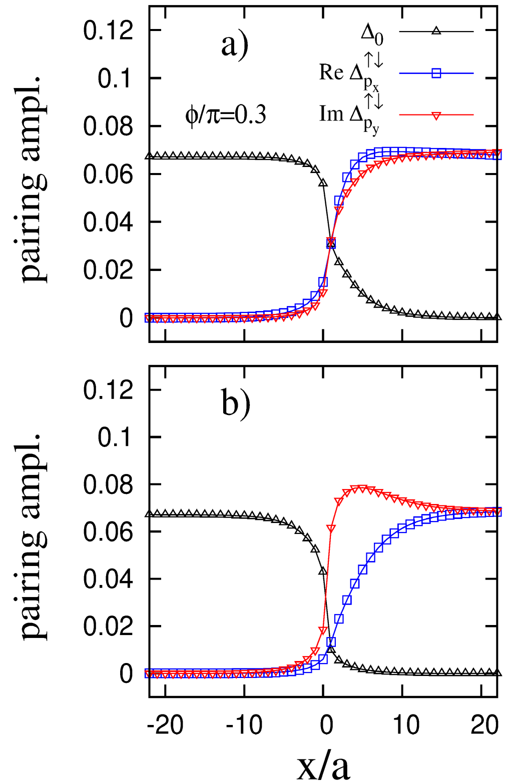

FIG. 5. (Color online). Comparison of the order parameters for a chiral spin-triplet heterostructure at a representative phase difference $\phi/\pi = 0.3$ as obtained by a non optimized procedure for the pairing amplitudes (a), and by minimizing the Gibbs potential in the presence of the phase drop constraint (b).

ity superconductivity near the edge[29]. Assuming that it is the subdominant extended s-wave that is coupled to the triplet component near the interface, the linear coupling terms in the GL expansion take the form $f_m = b'm[\psi(x)^*\partial_x\boldsymbol{\eta}(x) + \psi(x)\partial_x\boldsymbol{\eta}(x)^*]$, and promote $m \propto \cos\phi$ phase dependence of the magnetization (a similar term was found in the microscopic theory[40]). This is illustrated in Figs. 2(b_1) and 2(c_1), where the k_y component of the chiral order is suppressed concomitantly with the k_x component (in contrast to Fig. 2(a_1)), and the magnetization already exists without the phase difference across the junction.

In principle both $\sin\phi$ and $\cos\phi$ contributions should be present in such a junction, leading to a non-trivial phase dependence. However, we believe that in case b) the competition with the subdominant pairing on the SC2 side causes the triplet components to decay more rapidly towards the interface and save energy via singlet pairing (compare Fig. 2(a_1) and Fig. 2(c_1)), enhancing the latter term, and exhibiting dominant $\cos\phi$ behavior. Indirectly this is also supported by the magnetization that is more sharply localized near the interface, compare Fig. 3(a) and Fig. 3(b).

Finally, inclusion of the on-site s-wave pairing for case c) dramatically extends the range of coexistence of triplet and singlet pairing in SC2, see Fig. 2(c_1). This implies a more pronounced competition between the non-gradient and gradient couplings of the superconducting orders to the magnetization, but there is an additional complexity because the "local" s-wave component suppresses the k_y component of the pairing faster than the k_x component, Fig. 2(c_1). The induced s-wave order on the SC2 side is phase-locked to the triplet component, yielding a finite magnetization that only weakly depends on the phase for $\phi \in (0, \pi/2)$, but changes sign as the phase difference approaches π. Presumably this occurs because of additional states appearing due to the singlet component changing sign across the junction. This view is supported by considering the spatial profile of the magnetization, Fig. 3(c), which shows a two-peak structure for small ϕ, including the contribution away from the interface, but only exhibits a single peak at the interface for $\phi = \pi$. The resulting phase dependence of the magnetization contains many Fourier components.

Analysis of the energy spectrum, Fig. 3(d), supports these conclusions. If the structure of the order parameter near interface plays the major role in the emergence of the magnetization, nearly grazing trajectories must have a significant contribution. Indeed, the spin-split branches of Andreev states crossing the Fermi surface are moved to large values of $k_y/\pi \approx 0.5$. In addition, there is a secondary branch for the occupied states just below the gap edge at large k_y, and the peak in the energy dispersion of this state implies a van-Hove-like singularity contributing to the net M.

At this stage, it is useful to compare the obtained results with the phase dependent magnetization profiles for the same singlet-triplet junctions found by by searching for solutions where the pairing amplitudes are optimized under the constraint of a step-function profile for the phase ϕ across the interface. Such a comparison allows to evaluate how the phase dependent interface magnetization is affected by allowing the singlet and triplet order parameters to further adjust in order to minimize the Gibbs potential in the presence of a constrained phase drop at the interface. In Fig. 4 we present the resulting spin-polarization for the cases of helical and chiral symmetry of the order parameters in the triplet side of the heterostructure. As one can observe the helical case is substantially unaffected by allowing further optimization of the order parameters with respect to the case in Fig. 2. Hence, the result indicates that the response to a single-step phase twist of the order parameter is very close to the ground state configuration and further optimization does not produce significant changes in the order parameter amplitudes. On the other hand, slight differences in the overall magnetization profile are obtained for the case of the chiral symmetry spin-triplet superconductor. Though the qualitative behavior of the spin polarization is not modified, confirming a $\cos[\phi]$ like functional behavior, differences emerge close to the point where the magnetization changes sign. The magnetization exhibits rapid variations close to $\phi = \pi/4$ with a further slope change close to $\phi \sim \pi/2$. Such behavior indicates that in the phase window $[\pi/4 \pm 2\pi, \pi/2 \pm 2\pi]$ the system optimizes the energy by reducing the mixing of the order parameters close to interface, and renormalizing down the integrated magnetization accordingly. This is evident from Fig. 5 where we compare the order parameter profiles for the two approaches. Finally, for the heterostructure between the singlet and a mixed-parity singlet-triplet superconductor, we find that the magnetization exhibits largely the same behavior as that already shown in Fig. 2(c_2). However, notably, in the region where the magnetization has an abrupt jump ($\phi \sim \pi/4$) the analysis shows a tendency to an instability towards solutions with an inhomogeneous phase gradient across the interface. We find that there are many competing states which are very close in energy and thus the system exhibits a highly non trivial phase response which was already anticipated by the first-order like jump in the magnetization response (Fig. 2(c_2)). For the phase differences very close to that value we may expect slight differences in the magnetization values from those found above, but the overall $M(\phi)$ dependence, and our qualitative results, remains unchanged. A variation of the phase away from the interface may reduce the phase drop at the junction interface thus constraining the phase space of the possible magnetic configurations that can be achieved in the heterostructure. Further investigation along this direction is left for future work.

IV. CONCLUSIONS

We showed that the magnetic properties of singlet-triplet heterostructures are extraordinary sensitive to the type of triplet pairing, and to subdominant pairing channels. The dependence of the magnetization on the phase distinguishes between helical and chiral pairing. Vice versa, our results allow control of the interface magnetization by the phase difference. We considered an interface of good transparency and identical band structure on both sides. Relaxing this assumption will reduce the amplitude of the leaking superconducting component. In the first two cases we expect a reduction in the amplitude of $M(\phi)$, while in the last example the magnetization is weakly sensitive to the interface mismatch except near $\phi \sim \pi$. Finally, our results suggest that such junctions act as possible spin pumps. A voltage bias across the heterostructure would make the phase difference time-dependent, $\phi = 2eVt$, and the dynamics of the magnetization is determined by this driving force as well as damping. If damping is small, for our last case of mixed-parity superconductor in contact with a conventional s-wave system, the magnetization oscillations do not average to zero over a period of π/eV. Detailed investigation of this effect is left for future work.

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