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Magnetotransport signatures of a single nodal electron pocket constructed from Fermi arcs

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Abstract

Reconstruction of the Fermi surface into small electron pockets in the normal ground state of the underdoped high temperature superconducting cuprates has been related to charge ordering. It remains an open question, however, as to whether Fermi surface reconstruction occurs from a more conventional starting large hole-like Fermi surface yielding multiple pockets, or whether it occurs by connecting truncated ‘Fermi arcs’ in an unconventional pseudogap state to yield a single pocket per CuO_2 plane. Thus far, the observation of an in-plane magnetoresistance, and sign reversals and quantum oscillations in the Hall coefficient have been considered to provide support for the former conventional scenario. Here we show that in fact a single nodal diamond shaped electron pocket with concave sides produced by the latter unconventional scenario yields the observed experimental signatures in magnetotransport, negative Hall coefficient, and quantum oscillations in the negative Hall effect. Taken in conjunction with complementary signatures such as the experimentally observed low value of heat capacity at high magnetic fields and the experimental observation of a charge ordering wavevector that connects the tips of the truncated Fermi arcs, an origin of Fermi surface reconstruction from unconventional Fermi arcs is suggested.

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I. INTRODUCTION

A breadth of experimental evidence shows charge ordering to be an essential factor in reconstructing the Fermi surface in underdoped high temperature superconducting cuprates^{1–8}. Quantum oscillation^{9–12} and Hall effect^{13,14} experiments reveal a reconstructed Fermi surface consisting of a small electron pocket of area $\approx 2\%$ of the Brillouin zone. This pocket is present over the same range of hole dopings¹⁵ where nuclear magnetic resonance¹⁶ and x-ray scattering^{17–22} experiments detect charge density-wave order. The observed finite charge density-wave correlation length (≈ 100 Å) due to effects such as disorder^{23–25} is adequate for the observation of quantum oscillations characteristic of the small Fermi surface pocket^{26–28}, and further increases with the suppression of superconductivity by a magnetic field^{18,29}.

A major unresolved issue relates to whether the small Fermi surface pocket is created by charge order reconstruction of a starting conventional large Fermi surface, or a starting unconventional ground state comprising Fermi arcs, with models having been proposed for both scenarios^{2–6,19}. In a conventional metal, Fermi surface reconstruction at low temperatures is expected to involve the folding of the large hole-like Fermi surface sheet predicted by band structure calculations, yielding multiple pockets. In the alternative unconventional scenario that poses a challenge to our theoretical understanding of metals, however, quasi-particles are restricted to truncated nodal regions in momentum-space termed ‘Fermi arcs’ which become connected to form a single diamond-shaped electron pocket per CuO_2 plane.

Thus far, the in-plane magnetoresistance and sign reversals in the Hall coefficient observed in $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$, $\text{YBa}_2\text{Cu}_4\text{O}_8$ ¹³ and $\text{HgBa}_2\text{CuO}_{4+x}$ ¹⁴ and quantum oscillations in the Hall coefficient of $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ ⁹ have been widely interpreted as evidence for a Fermi surface comprising multiple Fermi surface pockets^{7,30,31}. The conventional scenario of a large starting Fermi surface has therefore largely been advocated, notwithstanding the low measured value of electronic specific heat at high magnetic fields, which appears to accommodate no more than a single Fermi surface pocket per CuO_2 plane^{1,32}, and the long length of the charge ordering wavevector connecting the Fermi arcs^{2,19}, suggestive of the unconventional ground state scenario.

We show here the surprising finding that signatures of magnetotransport previously associated with multiple Fermi surface pockets, including the magnetoresistance and sign reversals and magnetic quantum oscillations in the Hall coefficient, are in fact better explained

FIG. 1: (a), Schematic diamond-shaped electron pocket^{1–3,5–8,20}, with blue arrows indicating the direction of cyclotron motion and $\mathbf{v}_1$ and $\mathbf{v}_2$ indicating the Fermi velocity direction. (b), Schematic showing how the electron pocket is produced by connecting ‘arcs’ of a larger hole Fermi surface, with α being the angle subtended by the arc and the dotted lines indicating how they are connected.

by a single diamond-shaped nodal electron pocket with sufficiently concave sides, originally proposed in Ref.¹ (schematically depicted in Fig. 1), as a consequence of Fermi surface curvature effects³³. Taken in conjunction with complementary experiments, the unconventional scenario of a single diamond-shaped nodal electron pocket, yielded by connected Fermi arcs as in the pseudogap state^{2,19}, is supported by our findings.

II. RESULTS

For modeling purposes, we consider the diamond-shaped nodal electron pocket to be constructed from the arcs of a circle subtended by the angle α , as depicted in Fig. 1, such that the magnitude for the Fermi velocity on the arc is constant. These arcs are considered to become connected at vertices upon translation by the charge density-wave ordering vector, resulting in a diamond-shaped pocket with concave sides. We adjust the k -space area of the pocket to match that corresponding to the dominant quantum oscillation frequency of ≈ 530 T observed in $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ ⁹. Below we show that hybridization between the translated arcs produces a small reduction in the Fermi velocity in the vicinity of the vertices that depends on α . Given that temperature-dependent quantum oscillation³⁴ and optical conductivity³⁵ measurements have shown the quasiparticles contributing to the reconstructed pocket to be Fermi liquid-like in behavior, we consider these quasiparticles to respond to

FIG. 2: (a), R_H for a single CuO_2 plane in the weak magnetic field limit (colored circles) according to Equation (6) and at arbitrary magnetic fields (colored lines) according to Equations (1) and (2). The different colors indicate different values of α , the corresponding pocket shapes for which are shown in the top-right-hand corner. The horizontal solid line indicates $R_H = 0$ while the horizontal dotted line indicates the high magnetic field limit of R_H . (b), $\rho_{xy} = \sigma_{xy}/(\sigma_{xy}^2 + \sigma_{xx}^2)$ and $\rho_{xx} = \sigma_{xx}/(\sigma_{xy}^2 + \sigma_{xx}^2)$ according to Equation (2) calculated for the same values of α and a single CuO_2 plane. For two CuO_2 planes per unit cell, bilayer-coupling produces bilayer-split electron pockets (e.g. Fig. 6b in Appendix B), which can (depending on the details of the model) have different cross-sectional areas or α values. In this case, their conductivity contributions must be summed, yielding saturated values of R_H and ρ_{xx} that are precisely a factor of 2 smaller than those shown here for a single CuO_2 plane.

electromagnetic fields in a similar way to electrons in conventional metals.

For a determination of the Hall coefficient of the diamond-shaped electron pocket

$$R_H = \frac{1}{B} \frac{\sigma_{xy}}{\sigma_{xy}^2 + \sigma_{xx}^2}, \quad (1)$$

in a magnetic field of arbitrary strength, we calculate the diagonal (σ_{xx}) and off-diagonal (σ_{xy}) conductivity components using

$$\sigma_{x\beta} = -\frac{e^3 B}{2\pi^2 \hbar^2} \int_0^T \left(\int_0^\infty v_x(t) e^{-\frac{t'}{\tau(t)}} v_\beta(t+t') dt' \right) dt, \quad (2)$$

which is the Shockley-Chambers tube integral^{36,37} expressed as a function of time t , with the subscript β referring to x or y . We neglect the dependence of the velocity $\mathbf{v}$ on energy, but consider the variation of its components around the diamond-shaped electron pocket orbit in time. These are given by $v_x = v_F \cos \left(((\omega t \bmod \frac{\pi}{2}) - \frac{\pi}{4}) \frac{2\alpha}{\pi} + \frac{\pi}{4} - \lfloor \frac{2\omega t}{\pi} \rfloor \pi \right)$ and $v_y = v_F \sin \left(((\omega t \bmod \frac{\pi}{2}) - \frac{\pi}{4}) \frac{2\alpha}{\pi} + \frac{\pi}{4} - \lfloor \frac{2\omega t}{\pi} \rfloor \pi \right)$, where $\omega = eB/m$ is the cyclotron frequency of the diamond-shaped orbit. Figure 2 shows R_H calculated for four different values of the angle α subtended by the concave sides of the pocket (shown schematically in Fig. 1b). For $\alpha = 90^\circ$, our results reproduce the findings of Banik and Overhauser³³.

As expected, R_H saturates at $-\frac{1}{en}$ (dotted line in Fig. 2a) in the strong magnetic field limit, where $n = \frac{A_k}{2\pi^2 c}$ is the carrier density and $c \approx 11.68 \text{ \AA}$ is the interlayer lattice constant. We use Onsager's relation $F = \frac{\hbar}{2\pi e} A_k$ ³⁹ to obtain the Fermi surface cross-sectional area A_k in momentum-space for a pocket of frequency $F \approx 530 \text{ T}$ ⁹.

To understand R_H in the weak magnetic field limit, we turn to the Jones-Zener solution to the Boltzmann transport equation³⁸. Ong⁴⁰ has shown that the Hall coefficient

$$R_H = (2\pi)^2 c A_l / e\pi (\bar{|\mathbf{l}|} S)^2 \quad (3)$$

can be re-expressed in terms of the Stokes area $A_l = (\frac{\mathbf{B}}{B}) \cdot \int \frac{1}{2} d\mathbf{l} \times \mathbf{l}$ swept out by the 'scattering path length' vector $\mathbf{l} = \tau \mathbf{v}$ on moving around the Fermi surface perimeter. Here, $\bar{|\mathbf{l}|}$ is the orbital average of the scattering path length, while B is the magnitude of the magnetic field $\mathbf{B}$. We define the perimeter of the diamond-shaped Fermi surface $S = 4\alpha k_F$ in terms of the effective radius k_F of the large hole-like circular Fermi surface from which the arcs originate in Fig. 1b.

Consistency between the Jones-Zener method for calculating the Hall coefficient and the Shockley-Chambers tube integral treatment^{36,37} requires an appropriate model for the angular variation of $\mathbf{v}$ at the Fermi surface in the vicinity of the vertices. Vertices occur when nearly free electrons (or holes in the case of the cuprates) are perturbed by a weak periodic potential. As a consequence, Fermi surface sections with velocities in different directions intersect owing to a translation of the Fermi surface by a new lattice vector $\mathbf{K}$. The origin

of such Fermi surface translation arises from a charge ordering potential in the underdoped cuprates^{17,18}, while in elemental systems like Al, it originates from the crystalline lattice potential³³.

We model such a vertex of the diamond-shaped pocket in Fig. 1a by making linear approximations to the electronic dispersion in the vicinity of a point of intersection between two arcs. Upon choosing the point of intersection to be located at the origin in our k -space coordinates, the untranslated and translated dispersions acquire the simple forms $\varepsilon_{\mathbf{k}} = \hbar v_F \left(-k_x \sin\left(\frac{\alpha}{2} + \frac{\pi}{4}\right) - k_y \cos\left(\frac{\alpha}{2} + \frac{\pi}{4}\right) \right)$ and $\varepsilon_{\mathbf{k}+\mathbf{K}} = \hbar v_F \left(k_x \sin\left(\frac{\alpha}{2} + \frac{\pi}{4}\right) - k_y \cos\left(\frac{\alpha}{2} + \frac{\pi}{4}\right) \right)$, respectively, where v_F is the constant magnitude of the Fermi velocity on the arcs. The Fermi surfaces at $\varepsilon_{\mathbf{k}} = 0$ and $\varepsilon_{\mathbf{k}+\mathbf{K}} = 0$ are represented by dotted lines in Fig. 3a. A finite amplitude periodic potential opens up a small hybridization gap of magnitude 2Δ between these dispersions, leading to reconstructed dispersions of the form

$$\varepsilon_{\mathbf{k}}^{\pm} = \hbar v_F \left(-k_y \cos\left(\frac{\alpha}{2} + \frac{\pi}{4}\right) \pm \sqrt{k_x^2 \sin^2\left(\frac{\alpha}{2} + \frac{\pi}{4}\right) + \frac{\Delta^2}{\hbar^2 v_F^2}} \right) \quad (4)$$

whose modified Fermi surfaces at $\varepsilon_{\mathbf{k}}^{\pm} = 0$ are indicated by solid lines in Fig. 3a. Using $\mathbf{v} = \frac{1}{\hbar} \nabla \varepsilon_{\mathbf{k}}^+$, we obtain

$$\mathbf{v} = v_F \left[\frac{k_x \sin^2\left(\frac{\alpha}{2} + \frac{\pi}{4}\right)}{\sqrt{k_x^2 \sin^2\left(\frac{\alpha}{2} + \frac{\pi}{4}\right) + \frac{\Delta^2}{\hbar^2 v_F^2}}}, -\cos\left(\frac{\alpha}{2} + \frac{\pi}{4}\right) \right] \quad (5)$$

for the velocity. The x component of the velocity changes from $-v_F \sin\left(\frac{\alpha}{2} + \frac{\pi}{4}\right)$ to $+v_F \sin\left(\frac{\alpha}{2} + \frac{\pi}{4}\right)$ on varying k_x from $-\infty$ to $+\infty$, while the y component remains at a constant value of $-v_F \cos\left(\frac{\alpha}{2} + \frac{\pi}{4}\right)$.

To compute the total scattering path area (A_l) swept out, contributions are added from both the vertex regions, and the arc regions. The vertex contribution to the scattering path area is identified as the triangular area swept out between $\mathbf{v}_1$ and $\mathbf{v}_2$ in Fig. 3b multiplied by τ^2 , yielding $A_v = -\frac{1}{2}(\tau v_F)^2 \cos \alpha$ in the limit of small Δ . The arc contribution to the scattering path area (e.g. Fig. 3c) is identified as the sector of a circle swept out between $\mathbf{v}_2$ and $\mathbf{v}_3$ in Fig. 3d, yielding $A_a = \frac{1}{2}(\tau v_F)^2 \alpha$. On summing the contributions to A_l from 4 vertices and 4 arcs, we arrive at

$$R_H = \frac{1}{en} \frac{(\alpha - \cos \alpha)(\sin \alpha + 1 - \cos \alpha - \alpha)}{2\alpha^2}, \quad (6)$$

where $n = 2Nk_F^2(\sin \alpha + 1 - \cos \alpha - \alpha)/\pi^2 c$ is the carrier density of the diamond-shaped pocket. In Fig. 2, the weak magnetic field limit Hall coefficient given by Equation (6) is seen to yield identical results to those obtained using the Shockley-Chambers tube integral when τ is taken to be uniform, thereby showing that we have appropriately modeled the variation of the Fermi velocity in the vicinity of vertices.

The sign of the Hall coefficient is determined by the net of the opposing vertex and arc contributions, which is reflected in the first term within parenthesis in the numerator of Equation (6). For uniform τ , the condition for obtaining a net positive Hall coefficient in this weak magnetic field limit is $|A_a| > |A_v|$, corresponding to $\alpha > \cos \alpha$ (i.e. $\alpha > 42.3^\circ$) in Fig. 1. This therefore sets the condition for a sign reversal to occur on increasing $\omega\tau$ in Fig. 2.

III. DISCUSSION

On considering only the simplest case of a uniform τ , key features of the Hall coefficient and magnetoresistance data in the underdoped systems $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$, $\text{YBa}_2\text{Cu}_4\text{O}_8$ and $\text{HgBa}_2\text{CuO}_{4+\delta}$ ^{13,14} naturally arise from a diamond-shaped pocket with $\alpha > 42.3^\circ$ in Fig. 1. *First*, the calculated R_H in Fig. 2a changes sign from positive to negative on increasing $\omega\tau$ from 0 to ≈ 1 (upper axis) in a manner that is qualitatively similar to that seen experimentally in the non-superconducting regime of magnetic field and temperature. The high magnetic field saturation of R_H at a value corresponding to one electron pocket per CuO_2 plane is quantitatively consistent with the most recent measurements of the Hall coefficient (see Fig. S1 of Ref.⁴¹ and the Appendix). Note that the saturated R_H for two CuO_2 planes is half that for a single CuO_2 plane. The lower axis in Fig. 2a represents the corresponding magnetic field for $\tau = 0.5 \times 10^{-12} \text{ s}^3$ and an effective mass of $m = 1.6 m_e$ (where m_e is the free electron mass), which are the parameters obtained from $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ quantum oscillation measurements³. *Second*, the Hall sign reversal is accompanied by magnetoresistance in ρ_{xx} in Fig. 2b, which resembles the observed magnetoresistance in transport experiments^{13,14}. In the calculations (see Fig. 2b), the magnetoresistance is found to increase with α . *Third*, the calculated form of R_H as a function of τ^{-1} in constant magnetic field in Fig. 4a, is found to be qualitatively consistent with temperature-dependent measurements of R_H ^{13,14}. The upper axis shows the corresponding ‘Dingle temperature’³⁹, which approximates the true tempera-

FIG. 3: **(a)**, Solid lines showing the reconstructed Fermi surface in the vicinity of a vertex according to Equation (4). Dotted lines indicate the Fermi surface in the absence of hybridization. $\mathbf{v}_1$ and $\mathbf{v}_2$ are velocities before and after a quasiparticle traverses the vertex. **(b)** Schematic of the triangular area swept out by the velocity vector at a vertex, bounded by the arrows and the magenta dotted line. **(c)**, Schematic showing the change in velocity along the concave arc-like portion of a Fermi surface orbit. **(d)**, Schematic of the area (in this case a sector of a circle) swept out by the velocity vector along the Fermi surface arc, bounded by the arrows and the cyan dotted line.

ture in the case where τ^{-1} linearly increases with temperature. *Finally*, the diamond-shaped Fermi surface pocket provides a natural explanation for the observation of quantum oscillations in the Hall coefficient of $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ ^{9,13}, which has previously been assumed to require multiple pockets^{2,7,30}. Quantum oscillations in the electronic density-of-states lead directly via Fermi's golden rule^{42–44} to an oscillatory transport scattering rate $\tilde{\tau}^{-1}$. Whereas $\tilde{\tau}$ factorizes out of the semiclassical expression for R_H in the case of a circular Fermi surface pocket⁴⁴, this is no longer the case for a Fermi surface consisting of concave sections³³. On substituting $\tilde{\tau}^{-1}$ in place of τ^{-1} in the calculations, we find quantum oscillations in R_H that

FIG. 4: (a), R_H versus τ^{-1} (lower axis) at a constant magnetic field of 50 T. The upper axis shows the corresponding ‘Dingle temperature’ $T = 1/2\pi\tau k_B$ ³⁹. (b), R_H versus B calculated using a quantum oscillatory $\tilde{\tau}^{-1} \approx (1 + 2\cos(\frac{2\pi F}{B} - \pi)e^{-\frac{\pi}{\omega\tau}})\tau^{-1}$ ⁴², using $\tau = 0.5 \times 10^{-12}$ s (estimated from global fits to experimental data).

are a significant fraction of R_H in Fig. 4b when α is large.

Our findings remain intact if τ^{-1} ^{45,46} (or alternatively m^8) increases gradually on moving from the middle of the arcs (i.e. the nodal direction) towards the direction of the antinodes, as reported for the large unreconstructed Fermi surfaces of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ and $\text{Tl}_2\text{Ba}_2\text{CuO}_{6+\delta}$ ^{45,46}. The resulting reduction in the negative contribution to R_H from the vertices relative to the positive one from the arc regions, yields a sign reversal for smaller values of α than in the uniform τ^{-1} scenario (provided τ^{-1} continues to remain uniform for the small interval of time spent at each vertex).

Our findings of a sign change in R_H also remain intact should τ^{-1} vary as a function of magnetic field. Such a change in τ^{-1} with magnetic field may occur, for example, due to an evolution in the charge-density wave correlation length with magnetic field^{26,28}. While a field-dependent τ^{-1} can distort the shape of the curves in Figs. 2 and 4, it leaves the low and high magnetic fields limits of R_H and ρ_{xx} (in which $\omega_c\tau \rightarrow 0$ and $\omega_c\tau \gg 1$, respectively) unchanged.

IV. CONCLUSION

At the heart of the interpretation of the observed small Fermi pocket in the underdoped cuprates is the question of whether it is constructed from the folding of a conventional large hole-like band-structure Fermi surface, despite the unconventional pseudogap regime in which these materials are positioned. Thus far, the more conventional multiple pocket scenario has been heavily favored^{4,5,7,30,31} given that multiple pockets were considered a prerequisite to explain the magnetoresistance, the negative Hall coefficient, and quantum oscillations in the negative Hall effect. For instance, in the proposal of Ref.⁵, multiple hole and electron pockets are invoked, and a low quantum oscillation frequency reported in Shubnikov-de Haas oscillations^{3,7} is associated with the calculated small hole pocket. Multiple unresolved issues, however, raise difficulties with the multiple pocket scenario. We show in Appendices A and B that experimental evidence strongly supports the observed low frequency quantum oscillations originating from quantum interference between bilayer split sheets of a single nodal pocket, rather than a separate hole pocket as previously argued within the multiple pocket scenario⁵. The multiple pocket scenario is also called into question by the experimental observation of a low value of specific heat capacity³², which is only large enough to support a single Fermi surface pocket^{1,2}.

In contrast, the scenario of a single nodal pocket formed by connecting Fermi arcs in an unconventional ground state is supported by (i) its excellent agreement with the experimentally observed magnetotransport signatures and quantum oscillations in the Hall effect as shown in this work, (ii) the low value of experimentally measured heat capacity³² consistent with a single Fermi surface pocket^{1,3}, and (iii) a long charge ordering wave vector measured by x-ray diffraction that connects the tips of Fermi arcs^{3,19} characteristic of an unconventional ground state. In particular, previous difficulties in explaining the experimental magnetotransport and Hall effect signatures by a single Fermi pocket are overcome by our current work, which shows for the first time that in fact a single nodal diamond pocket with sufficiently concave Fermi surface curvature yields better agreement with the experimentally observed magnetoresistance and negative Hall effect and quantum oscillations in the Hall coefficient, than the multiple pocket model.

The most recent extended set of angle-resolved quantum oscillation measurements³, also provides support for a single nodal electron pocket per CuO_2 plane. In such a scenario, the

validity of Luttinger’s sum rule⁵⁰ is called into question. It is therefore intriguing to consider the challenging problem of the correct theoretical description of such an unusual metallic state.

V. ACKNOWLEDGEMENTS

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VI. APPENDIX A: HALL COEFFICIENT WITH AND WITHOUT ADDITIONAL SMALL HOLE POCKETS

In Fig. 5 we compare calculations of R_H with (dotted and dashed lines) and without (solid line) the presence of additional hole pockets. In all cases, R_H changes sign with magnetic field. The calculated R_H without small hole pockets quickly saturates at a value that is comparable to recent quantitative measurements on R_H in $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ in high magnetic fields (see Fig. S1 of Ref.⁴¹). By contrast, the calculated R_H with small hole pockets yields a Hall coefficient that significantly exceeds the latest experimental value⁴¹ and fails to saturate below ≈ 100 T. A non saturating R_H has also been obtained by Chakravarty and Kee for larger hole pockets³⁰ (using a scattering rate for the hole pocket that exceeds that of the electron pocket). The calculated R_H without hole pockets is therefore in better agreement with the experimentally measured R_H .

VII. APPENDIX B: SMALL HOLE POCKETS VERSUS STARK QUANTUM INTERFERENCE

In addition to the main quantum oscillation frequency of $F \approx 530$ T, recent measurements of the magnetoresistance and thermopower have identified a second frequency of ≈ 90 T^{3,7}, which is proposed to originate either from small hole pockets⁷ (see Fig. 6a) or

FIG. 5: (a), A comparison of R_H calculated for a diamond-shaped electron pocket residing on two CuO_2 planes per unit cell and $\alpha = 90^\circ$ with (dotted and dashed lines) and without (solid line) additional small hole pockets present⁷. We consider two hole pockets per CuO_2 plane of frequency $F = 90$ T and effective mass $m = 0.45 m_e$. The electron pocket parameters are the same as those in Fig. 2. The dotted line is calculated using $\tau = 0.5 \times 10^{-12}$ s for the hole pockets (such that the scattering time is equal for the electron and hole pockets) while the dashed line is calculated using $\tau = 0.14 \times 10^{-12}$ s for the hole pockets (such that the mean free path is equal for the two types of pocket).

from Stark quantum interference effects³ (see Fig. 6b). The hole pockets are proposed to result from folding of the large Fermi surface predicted by band structure calculations^{5,7}, while quantum interference is suggested to result from bilayer splitting and magnetic breakdown tunneling^{3,51}.

Stark quantum interference refers to oscillations similar in form to Landau quantum oscillations, but that are unique to transport phenomena such as the Shubnikov-de Haas (SdH) effect³⁹. They occur wherever there exists a relative Onsager phase between two different quasiparticle trajectories, such as that readily occurring in a magnetic breakdown network in momentum-space. Because the effective mass associated with Stark quantum interference oscillations is given by the difference in mass between the two interfering trajectories, its value tends to be significantly smaller than those associated with Landau quantized orbits^{39,51}. Also, the origin of Stark quantum interference, which is distinct from Landau quantization³⁹, means that the area enclosed between the interfering paths does not contribute to R_H , nor do Stark quantum interference oscillations contribute to static thermodynamic

FIG. 6: (a), Schematic reconstructed Fermi surface resulting from charge order, similar to that of Allais *et al.*⁵ (neglecting bilayer coupling). Electron and hole pockets are indicated in red and blue respectively. The latter has been proposed to be responsible for the observed ≈ 90 T frequency⁷. Using $m = \frac{\hbar}{2\pi}(\frac{\partial A}{\partial \epsilon})$ to estimate the band masses, we obtain $\approx 0.25 m_e$ and $\approx 0.19 m_e$ for the electron and hole pockets, respectively, yielding $m_{90 \text{ T}}/m_{530 \text{ T}} \approx 0.75$ in Table I. (b), Possible schematic reconstructed Fermi surface consisting of an electron pocket after Sebastian *et al.*^{3,52}, resulting from charge order, but where bilayer coupling leads to different bonding and antibonding cross-sections. Stark quantum interference occurs when quasiparticles can take two different trajectories between two magnetic breakdown junctions (e.g. ‘A’ and ‘B’). The relative Onsager phase between these trajectories has been proposed to give rise to the observed ≈ 90 T frequency in transport measurements^{3,51}. From the Fermi surface in Ref.⁵² (reproduced here), we obtain $\approx 2.4 m_e$ and $\approx 1.0 m_e$ for the bonding and antibonding band masses, respectively. The mass of the dominant ≈ 530 T quantum oscillation frequency is given by their average $\approx 1.7 m_e$, while the mass of the quantum interference frequency corresponds to the difference in area between paths connecting ‘A’ and ‘B,’ yielding $\approx 0.36 m_e$. Taking the ratio, we obtain $m_{90 \text{ T}}/m_{530 \text{ T}} \approx 0.21$ in Table I.

quantities such as the de Haas-van Alphen (dHvA) effect³⁹ and the electronic heat capacity γT .

Very different values of the effective mass ratio $m_{90\text{T}}/m_{530\text{T}}$, R_H (see Appendix B) and γ are therefore expected, depending on whether the ≈ 90 T oscillations originate from small hole pockets or Stark quantum interference. In Table I, we compare estimates of these quantities with experiment. We find that closer agreement with experiment is obtained when the low frequency oscillations are considered to originate from Stark quantum interference, such that the entire reconstructed Fermi surface consists of a single electron pocket per CuO_2 plane (e.g. as depicted in Fig. 6b). A compelling argument for Stark quantum interference being the origin of the $F \approx 90$ T oscillations is the absence of such oscillations measured in the dHvA effect^{53–55}. Were the $F = 90$ T oscillations to originate from the proposed hole pockets, their very small effective mass⁷ and numerical supremacy in the Brillouin zone⁷ would produce dHvA oscillations of greater amplitude than those from the electron pocket.

quantity	hole pocket	quantum interference	experiment
ratio $m_{90\text{ T}}/m_{530\text{ T}}$	0.75	0.21	0.28 ⁷
saturated R_H (mm^3C^{-1})	-21	-14	-11.5 ⁴¹
γ ($\text{mJmol}^{-1}\text{K}^{-2}$)	7.6 ⁷	4.7 ²	5 ³²
$F \approx 90$ T present in dHvA	yes	no	no ^{53–55}
$F \approx 90$ T present in SdH	yes	yes	yes ^{3,7}

TABLE I: A comparison of the effective mass ratio, saturated Hall coefficient and high magnetic field electronic heat capacity values expected for situations in which the ≈ 90 T oscillations originate from small hole pockets or from Stark quantum interference with experimental values. Here we assume two CuO_2 planes. The theoretical band mass ratios for the two situations are given in Fig. 6. For the experimental mass ratio, we take the measured quantities from Ref.⁷. For the heat capacity, we use $\gamma = 1.46 \times \sum \frac{m^*}{m_e} \text{ mJmol}^{-1}\text{K}^{-22}$, where the sum is made over the number of pockets contributing in each situation. In the last two rows, we show in which experimental quantities the low frequency quantum oscillations are present.

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