Multicomponent fractional quantum Hall states with subband and spin degrees of freedom

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In two-dimensional electron systems confined to wide GaAs quantum wells we observe a remarkable sequence of transitions of the fractional quantum Hall states in the filling factor range $1 < \nu < 3$ near the crossing of two $N = 0$ Landau levels from different subbands and with opposite spins. The transitions attest to the interplay between the spin and subband degrees of freedom and can be explained as pseudospin polarization transitions where the pseudospins are the two crossing levels. The magnetic field positions of the transitions yield a new and quantitative measure of the transition energies or, equivalently, the composite Fermions' discrete energy level separations. Surprisingly, these energies are much larger than those reported for other systems with SU(2) symmetry; moreover, they are larger when the electron system is less spin polarized.

I. INTRODUCTION

Fractional quantum Hall states (FQHSs) are the hallmarks of an interacting two-dimensional electron system (2DES) at large perpendicular magnetic field ($B_\perp$) when a Landau level (LL) is partially occupied.[1,2] Adding extra electronic (pseudospin) degrees of freedom leads to additional sets of LLs which are separated by, e.g., the Zeeman energy (E_Z), subband separation (Δ_{SAS}), or valley splitting energy (E_V), in systems with spin, subband, or valley degree of freedom, respectively. When two such LLs cross at a particular LL filling factor (ν), the QHS at that filling typically weakens or disappears.[3] In very high quality samples, however, if two $N = 0$ LLs cross, the 2DES can form two-component FQHSs where the components are the crossing LLs (the pseudospins), and exhibit pseudospin polarization transitions as one tunes the pseudospin energy splitting, or the separation between the two LLs. Such transitions have been reported for 2DESs confined to AlAs quantum wells (QWs) where the energy separation between the two occupied valleys is tuned via the application of strain so that the two lowest ($N = 0$) LLs of the two valleys cross.[4–7] Multi-component FQHSs and transitions between their pseudospin configurations have also been studied in other 2DESs where two or more LLs with different spin or valley indices are close in energy.[8–17]

Here we present a study of 2DESs confined to wide GaAs QWs where E_Z and Δ_{SAS} are much smaller than the Coulomb energy at large $B_\perp$. Thus the lowest four LLs – the S0↑, S0↓, A0↑ and A0↓ levels – are close in energy so that, in principle, a full description of FQHSs would require the inclusion of all four LLs and using SU(4) symmetry (S and A refer to symmetric and antisymmetric subbands, ↑ and ↓ refer to up- and down-spin; see Fig. 1(a)). Via applying either a parallel magnetic field component ($B_{||}$, see Fig. 1(b)), or changing the electron density in the QW,[18] we reduce Δ_{SAS} and increase E_Z, so that the S0↓ and A0↑ levels cross when $\Delta_{SAS} = E_Z$; see Fig. 1(a). Near the crossing, we observe a remarkable pattern of appearing and disappearing FQHSs in the filling range $1 < \nu < 3$, revealing the formation of FQHSs with both spin and subband degrees of freedom simultaneously.[19] We can qualitatively describe the number of observed transitions and their positions in a simplified two-component picture with SU(2) symmetry, where the two pseudospins are the S0↓ and A0↑ levels. However, our quantitative analysis reveals several puzzling features. We find that at a fixed ν, the transition energies are much larger than reported previously in systems with SU(2) symmetry where the two pseudospins are either spin or valley levels. Also, the transition energies are about twice larger when the 2DES is less spin-polarized. We discuss possible origins of these unexpected features.

II. METHOD

Our sample, grown by molecular beam epitaxy, is a 65-nm-wide GaAs QW bounded on each side by undoped $Al_{0.24}Ga_{0.76}As$ spacer layers and Si δ-doped layers. We use a 4×4 mm^2 square piece with alloyed InSn contacts at four corners, and an evaporated Ti/Au front-gate and an In back-gate to change the 2DES density (n) while keeping the charge distribution symmetric [Fig. 1(b)]. We measure the transport coefficients in a dilution refrigerator with base temperature $T \approx 30$ mK, using low-frequency (< 30 Hz) lock-in technique and a rotatable sample platform to induce $B_{||}$. At $B_{||} = 0$, the Fourier transform of the Shubnikov-de Haas oscillations exhibits two peaks that correspond to the electron densities in the two subbands. The difference between these densities yields Δ_{SAS} which is in excellent agreement with the results of our $B = 0$ self-consistent calculations. To calculate Δ_{SAS} at finite $B_{||}$ and $B_\perp$ [Fig. 1(a)], we employ a perturbative simulation introduced in Ref. [20], where we assume $B_{||}$ only mixes LLs from different subbands but does not change the QW potential.

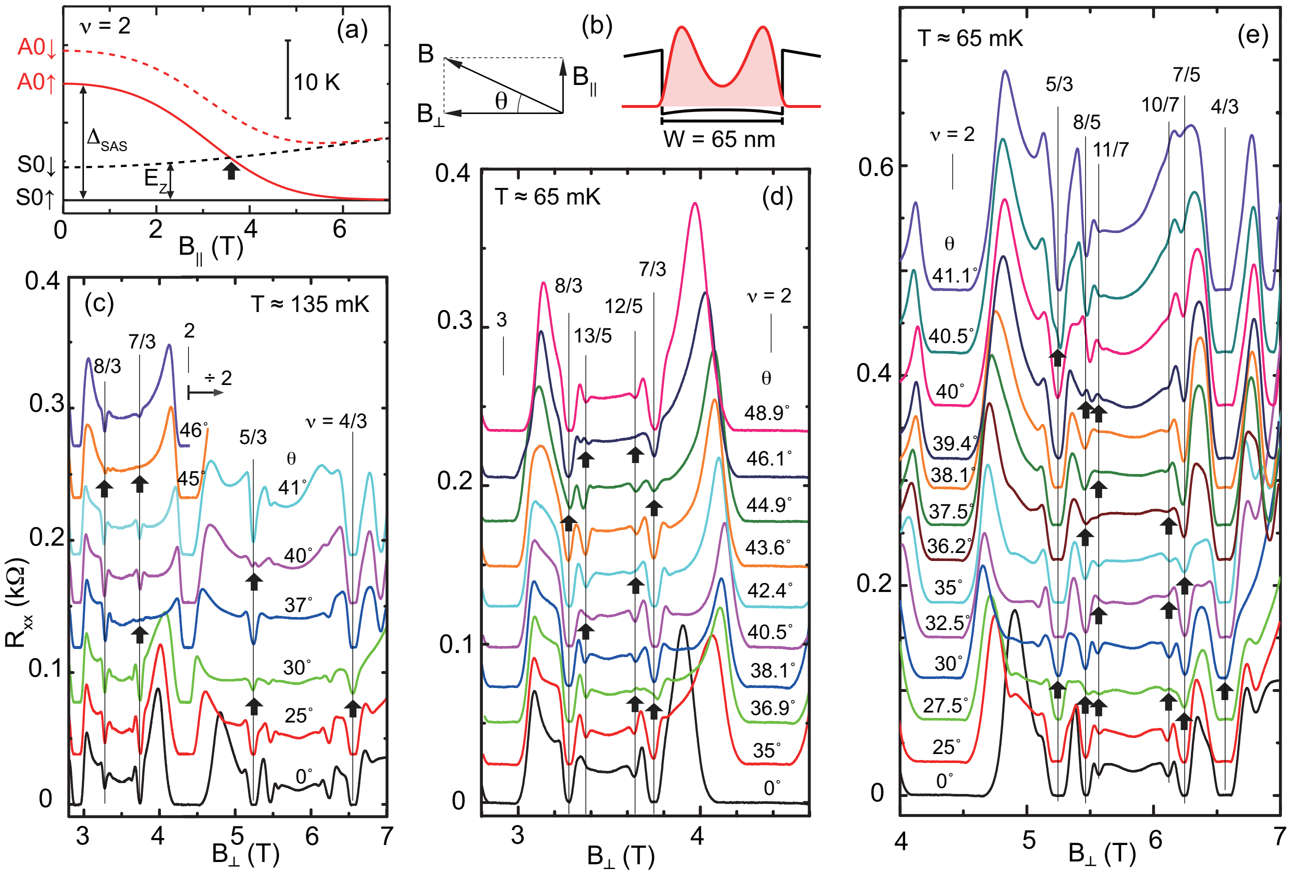

FIG. 1. (color online) (a) Relevant energy levels for $n = 2.12 \times 10^{11}$ cm^{-2} electrons confined to a 65-nm-wide GaAs QW near filling factor $\nu = 2$, calculated as a function of parallel magnetic field ($B_{||}$). (b) Charge distribution (red curve) and potential (black curve) calculated via solving Poisson-Schrödinger equations self-consistently at $B = 0$. The experimental geometry and the tilt angle θ are also shown. (c)-(e) Waterfall plots of R_{xx} vs $B_\perp$, measured at the indicated temperatures and different θ. Each trace is shifted vertically. The observed FQHS transitions are marked by the black arrows.

III. EXPERIMENTAL RESULTS

Figure 1 (c) shows the longitudinal magnetoresistance (R_{xx}) traces for $1 < \nu < 3$, measured at $n = 2.12 \times 10^{11}$ cm^{-2}, $T \simeq 135$ mK, and different tilt angles (θ). As we increase θ, the $\nu = 2$ R_{xx} plateau narrows near $\theta \simeq 37°$ and then widens again at larger θ. The reduction of the R_{xx} plateau width signals a weakening of the integer QHS, which is widely used to identify LL crossings in high quality 2DESs and at low temperatures.[3,21,22] This weakening, but not disappearing, is similar to what is seen at certain LL crossings at other integral ν when interaction preserves the energy gap through QHS ferromagnetism.[3,23,24] In our samples, Δ_{SAS} is larger than E_Z at $B_{||} = 0$. As we increase $B_{||}$, Δ_{SAS} decreases and E_Z increases, so that the S0↓ and A0↑ levels cross when $\Delta_{SAS} \simeq E_Z$.[25] In Fig. 1(a), we show the calculated energies of the four $N = 0$ LLs relative to the S0↑ level as a function of $B_{||}$ at $\nu = 2$.[20] We use a three-fold enhanced E_Z to match the experimental observation that the $\nu = 2$ crossing occurs near $\theta \simeq 37°$ ($B_{||} \simeq 3.5$ T). This enhancement of E_Z is not surprising and has also been reported in previous studies.[3,17,21,22]

More interestingly, near $\theta \simeq 37°$, the FQHSs on both sides of $\nu = 2$ show a rich series of transitions, as marked with arrows in Figs. 1(c)-(e).[26] The $\nu = 4/3$ FQHS is strong at both small and large θ, but becomes weak at $\theta \simeq 30°$ [Figs. 1(c) and (e)]. The $\nu = 8/3$ FQHS also experiences one transition, at $\theta \simeq 45°$ [Figs. 1(c) and (d)]. Meanwhile, the $\nu = 5/3$ and $7/3$ FQHSs become weak twice: at $\theta \simeq 30°$ and $40°$ for $\nu = 5/3$, and at $\theta \simeq 37°$ and $45°$ for $\nu = 7/3$. Data taken at lower $T \simeq 65$ mK, shown in Figs. 1(d) and (e), reveal a more remarkable pattern of higher-order FQHS transitions. On the left side of $\nu = 2$ [Fig. 1(d)], the $\nu = 13/5$ FQHS weakens twice at $\theta \simeq 40.5°$ and $46.1°$. The $\nu = 12/5$ FQHS, on the other hand, becomes weak three times, at $\theta \simeq 36.9°$, $42.4°$ and $46.1°$. On the high-field side of $\nu = 2$, as seen in Fig. 1(e), the $\nu = 7/5$ FQHS weakens twice, at $\theta \simeq 27.5°$ and $35°$, and the $\nu = 8/5$ FQHS thrice, at $\theta \simeq 27.5°$, $36.2°$ and $39.4°$. We also observe in Fig. 2(e) three transitions at $\nu = 10/7$ and four transitions at $\nu = 11/7$.[18] We summarize in Fig. 2(a) the values of $B_{||}$ and $B_\perp$ for all the observed transitions.

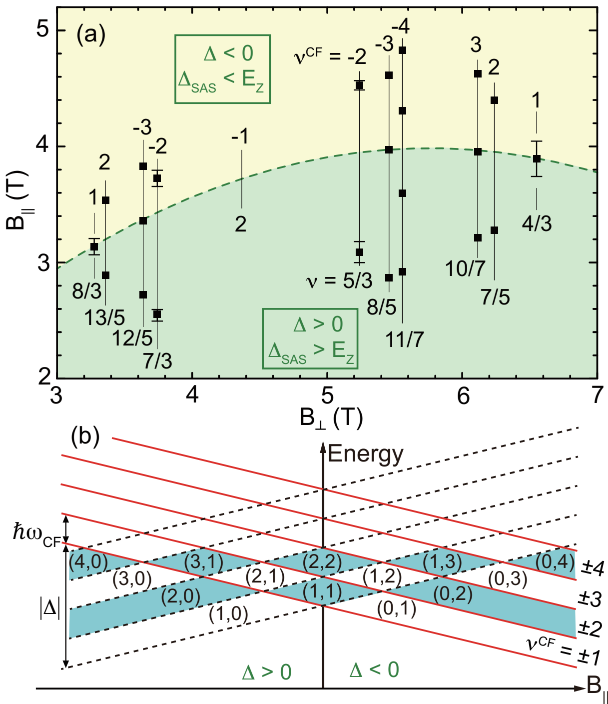

FIG. 2. (color online) (a) Summary of $B_{||}$ and $B_\perp$ where pseudospin transitions are seen in Fig. 1. The dashed line represenst zero pseudospin splitting $\Delta = 0$ (see text). $\Delta = \Delta_{SAS} - E_Z$ is positive in the green region, and becomes negative in the yellow region. Typical error bars are shown for the $q/3$ FQHS transitions. (b) Schematic depiction of the CF Λ-level energies in a simplified two-pseudospin picture. The dashed black and solid red lines represent the CF Λ-levels with up- and down-pseudospins (the electron S0↓ and A0↑ levels). Each configuration is labeled with its pseudospin Λ-level filling factors ($\nu_{\text{CF}\uparrow}$, $\nu_{\text{CF}\downarrow}$), where ↑ and ↓ stand for the up- and down-pseudospins.

IV. DISCUSSION

The fact that all these transtions occur near the crossing of the S0↓ and A0↑ levels when $\Delta_{SAS} \simeq E_Z$ suggests that these are FQHS pseudospin polarization transitions.[19] Such transitions can be readily understood in the framework of composite Fermions (CFs), quasi-particles formed by attaching two flux quanta to each electron.[2,27,28] The CFs form their own discrete energy levels, the so-called Λ-levels which are separated by the CF cyclotron energy $\hbar\omega_{\text{CF}}$, and the FQHS at $\nu = \nu_{\text{CF}}/(2\nu_{\text{CF}}+1)$ is the integer QHS of CFs with integral Λ-level filling factor ν_{CF}. Since the other two energy levels, S0↑ and A0↓, are reasonably far in energy and are always full or empty,[19] we neglect them and interpret our data in a simple (SU(2)) picture where the two crossing levels, S0↓ and A0↑, are the two pseudospins.[29] The pseudospin splitting energy $\Delta = \Delta_{SAS} - E_Z$ is then tuned from positive to negative via applying $B_{||}$ [Figs. 1(a) and 2(b)]. As discussed below, this simplified two-pseudospin model qualitatively explains our data.

In this model, the FQHS at $\nu = 4/3$ has only one occupied Λ level ($\nu_{\text{CF}} = 1$).[29] It therefore has two possible configurations with the Λ-level filling factors of the up- and down-pseudospin CFs being $(\nu_{\text{CF}\uparrow}, \nu_{\text{CF}\downarrow}) = (1,0)$ or (0,1), and should exhibit one transition at $\Delta = 0$ [see Fig. 2(b)], consistent with Fig. 2(a) data. The FQHS at $\nu = 7/5$, on the other hand, has two filled Λ-levels ($\nu_{\text{CF}} = 2$), three pseudospin configurations [$(\nu_{\text{CF}\downarrow}, \nu_{\text{CF}\downarrow}) = (2{,}0)$, (1,1) and (0,2)], and two transitions when $\Delta = \pm\hbar\omega_{\text{CF}}$ [Fig. 2(b)], also in agreement with Fig. 2(a) data. Using similar logic, we can explain the three transitions observed for the $\nu = 10/7$ FQHS which has $\nu_{\text{CF}} = 3$ and four configurations [Fig. 2(b)]. By invoking particle-hole symmetry, which links the FQHSs at ν to $(4-\nu)$ in our system (e.g., 4/3 to 8/3), and also utilizing negative CF fillings, e.g., $\nu_{\text{CF}} = -2$ for the $\nu = 5/3$ state, we can explain *all* the transitions summarized in Fig. 2(a). In general, a FQHS with ν_{CF} has $|\nu_{\text{CF}}|+1$ pseudospin configurations, and $|\nu_{\text{CF}}|$ pseudospin polarization transitions which occur whenever $|\Delta|$ equals a multiple integer of $\hbar\omega_{\text{CF}}$ [see Fig. 2(b)].

Next we proceed to a quantitative analysis of Fig. 2(a) data. Figure 2(b) implies that, when ν_{CF} is odd, one transition occurs exactly at $\Delta = 0$. In Fig. 2(a), we first fit the dashed line through these transition points. We then focus on a particular ν, e.g. 8/5, and calculate Δ_{SAS} as a function of $B_{||}$ at this filling, as shown in Fig. 3(a). Using the value of $B_{||}$ at which the $\Delta = 0$ transition for $\nu = 8/5$ occurs ($B_{||} = 3.96$ T), we determine a value for g^* ($\simeq -1.34$) so that $\Delta_{SAS} = E_Z$ at $B_{||} = 3.96$ T. We then plot E_Z ($= g^*\mu_B B$) at $\nu = 8/5$ as a function of $B_{||}$ in Fig. 3(a), and determine Δ for the other two 8/5 transitions. (See Fig. 3(b) for the four pseudospin configurations of the $\nu = 8/5$ FQHS.) Since these transitions are expected to occur when $\Delta = \pm 2\hbar\omega_{\text{CF}}$ [see Fig. 2(b)], we find $\hbar\omega_{\text{CF}} = 5.5$ K for $\Delta > 0$ and $\hbar\omega_{\text{CF}} = 2.5$ K for $\Delta < 0$. Using this procedure we can deduce $\hbar\omega_{\text{CF}}$ for FQHSs at $\nu = 8/5$, 12/5, and 10/7 which have a $\Delta = 0$ transition on the dashed line in Fig. 3(a). For the FQHSs at $\nu = 13/5$, 7/3, 5/3 and 7/5, we assume $\Delta = 0$ at the intersection of the dashed line and the vertical lines that mark these fillings in Fig. 3(a) and, following a similar procedure, find $\hbar\omega_{\text{CF}}$ from the transitions' $B_{||}$ values.

In Fig. 3(c) we plot as a function of $1/(2\nu_{\text{CF}}+1)$ all the deduced values of $\hbar\omega_{\text{CF}}$, normalized to the Coulomb energy ($V_C = e^2/4\pi\epsilon l_B$, where $l_B = \sqrt{\hbar/eB_\perp}$ is the magnetic length and ϵ is the dielectric constant), for different FQHSs. Figure 3(c) data allow us to make a quantitative comparison of the deduced $\hbar\omega_{\text{CF}}/V_C$ to previous experimental reports. The dashed lines in Fig. 3(c) represent $\hbar\omega_{\text{CF}}/V_C$, measured near $\nu = 1/2$ and 3/2, from FQHS *spin* transitions in QWs with same well-width (65 nm) but much lower density where the S0↑ and S0↓ levels are closer in energy and the A0↑ level is well above S0↓.[13,18,19] Figure 3(c) plot clearly shows that our measured $\hbar\omega_{\text{CF}}/V_C$ is much larger than the dashed lines.

It is possible that the transition energies we deduce are somewhat exaggerated because of the inaccuracy of the

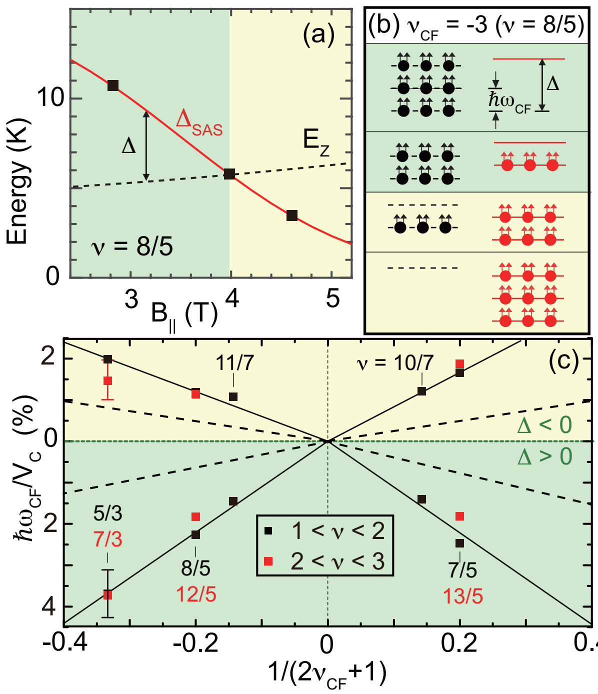

FIG. 3. (color online) (a) Calculated Δ_{SAS} and E_Z as a function of $B_{||}$ at $\nu = 8/5$. (b) Λ-level diagram showing the four different pseudospin configurations for $\nu = 8/5$ ($\nu_{\rm CF} = -3$) in our two-component picture. The pseudospin polarization transitions are expected when $\Delta = 0, \pm 2\hbar\omega_{\rm CF}$ [see Fig. 2(b)]. (c) The Λ-level separation in units of $V_C = e^2/(4\pi\epsilon l_B)$ as a function of $1/(2\nu_{\rm CF}+1)$, deduced from Δ at which the FQHS transitions are observed in Fig. 2(a). Data are shown for both $1 < \nu < 2$ (black) and $2 < \nu < 3$ (red). The dashed lines represent data for 2DESs with only spin degree of freedom[13]; results from transitions near $\nu = 1/2$ and 3/2 are shown in the $\Delta < 0$ and $\Delta > 0$ regions, respectively.

perturbative calculations we use to determine the $B_{||}$-dependence of Δ_{SAS}. To test this possibility, we made measurements on another QW with slightly narrower well width (55 nm).[18] In this sample, we induce the crossing of the S0↓ and A0↑ levels via applying $B_{||}$ and also by tuning n at $B_{||} = 0$. The $\hbar\omega_{\rm CF}$ deduced from the tilting data matches Fig. 3 data, but the n-tuning results are somewhat smaller,[18] suggesting the inaccuracy of our perturbative calculations.[30] However, we emphasize that even the n-tuning data which do not rely on calculations of Δ_{SAS}, yield $\hbar\omega_{CF}/V_C$ values that are about a factor of two larger than the dashed lines in Fig. 3(c).[18] We therefore conclude that $\hbar\omega_{\rm CF}$ in our 2DES with both spin and subband degrees of freedom is larger than in a 2DES with only spin degree of freedom. The reason for this is not entirely clear. Perhaps the nature of pseudospins plays a role in determining the polarization energies and $\hbar\omega_{CF}$. It is also possible that in our system the close proximity of the other two LLs (S0↑ and A0↓; see Fig. 1(a)) is important. Both of these interesting possibilities deserve further theoretical examination.

We highlight two additional noteworthy features of Fig. 3(c). First, the results for $1 < \nu < 2$ approximately match those for $2 < \nu < 3$, suggesting that $\hbar\omega_{\rm CF}$ does not depend on which LL (S0↓ or A0↑) hosts the CFs. Second, in our 2DES we tune the crossing of two $N = 0$ LLs with opposite spins while a lower level (S0↑) is fully occupied. This allows us to observe FQHS pseudospin transitions at a given ν for both $\Delta < 0$ and $\Delta > 0$, when the net spin orientation of the CFs is aligned or anti-aligned with the spin (↑) of majority electrons, respectively. Data of Fig. 3(c) indicate that $\hbar\omega_{\rm CF}/V_C$ is about twice larger for the case $\Delta > 0$ compared to the $\Delta < 0$ case. This is an important observation as it sheds light on the mysterious asymmetry between $\hbar\omega_{\rm CF}/V_C$ for CFs near $\nu = 3/2$ and 1/2, deduced from CF spin-polarization transitions in GaAs 2DESs,[13] or CF valley-polarization transitions in AlAs 2DESs.[6] In those 2DESs the polarization transition energies of FQHSs near 3/2, where electrons are partially spin- (or valley-) polarized, are much larger than those near 1/2 where they are fully polarized.[6,13] Our data, afforded by the tunability of the subband/spin degrees of freedom, provide clear evidence that the spin polarization of the total electron system can affect the CF properties.

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