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Multi-qubit gates protected by adiabaticity and dynamical decoupling applicable to donor qubits in silicon

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We present a strategy for producing multi-qubit gates that promise high fidelity with minimal tuning requirements. Our strategy combines gap protection from the adiabatic theorem with dynamical decoupling in a complementary manner. Energy-level transition errors are protected by adiabaticity and remaining phase errors are mitigated via dynamical decoupling. This is a powerful way to divide and conquer the various error channels. In order to accomplish this without violating a No-Go theorem regarding black-box dynamically corrected gates [Phys. Rev. A 80, 032314 (2009)], we require a robust operating point (sweet spot) in control space where the qubits interact with little sensitivity to noise. There are also energy gap requirements for effective adiabaticity. We apply our strategy to an architecture in Si with P donors where we assume we can shuttle electrons between different donors. Electron spins act as mobile ancillary qubits and P nuclear spins act as long-lived data qubits. This system can have a very robust operating point where the electron spin is bound to a donor in the quadratic Stark shift regime. High fidelity single qubit gates may be performed using well-established global magnetic resonance pulse sequences. Single electron spin preparation and measurement has also been demonstrated. Putting this all together, we present a robust universal gate set for quantum computation.

One of the main challenges in realizing a quantum information processor is the ability to implement high-fidelity entangling operations. It can be relatively easy to control well isolated qubits. Nuclear magnetic resonance (NMR) and electron spin resonance (ESR) are well developed for manipulating nuclear and electron spins with high fidelity [1, 2]. Turning interactions between qubits on and off in a controllable manner for a coherent quantum operation remains very challenging. The process of coupling different qubits is often accompanied by an enhanced sensitivity to the environment. When qubits are not isolated, they are vulnerable to noise.

The adiabatic theorem [3] provides remarkably robust operations in the sense that transitions between non-degenerate eigenstates are suppressed. If the Hamiltonian of a quantum system is varied slowly enough, instantaneous eigenstates will be tracked. Exploiting this phenomenon, dramatic improvements in single-qubit NMR operations have been observed [4] by combining the BIR-4 [5] pulse sequence with WURST-20 [6] adiabatic pulse shaping. In our proposal, we perform an adiabatic process involving two qubits and non-degenerate eigenstates (diabatic energy level crossings are allowed, however). Transition errors are suppressed by the adiabatic theorem, but phase errors must be mitigated using a different mechanism.

The Hahn echo [7] is simple and effective for canceling phase errors induced by low-frequency noise and uncertainty. An unknown but systematic \hat{Z} rotation on a qubit is reversed by flipping it with an \hat{X} gate. The Hahn echo, and a variety of more elaborate sequences or strategies [8–12] are very effective at prolonging coherence and storing quantum information. These are known as dynamical decoupling (DD) schemes because they decou-

ple the qubit system from its environment. There exist analogous strategies called dynamically corrected gates (DCGs) to cancel errors during nontrivial quantum gate operations [13–17]. However, a No-Go theorem forbids black-box DCGs [14], presenting a challenge relative to DD sequences. DCGs must assume there are relationships between the effects of noise induced under different control settings. In a two-qubit DCG, for example, you would need to vary the inter-qubit interaction but maintain consistent or correlated environmental interactions in order to cancel their effects. This presents a problem when interactions are varied by moving the qubits (such as localized electrons in a solid state material) and the environment varies at this length scale.

We demonstrate a way to circumvent this No-Go theorem when there exists a robust operating point (ROP), a “sweet spot” in control space where the qubits interact stably with respect to noise (already exploited in various semiconductor qubit settings [18–21]). We do not attempt to correct for errors induced during this ROP time, but we do correct for errors induced in transit (adiabatically) to and from this control space point. This is illustrated schematically in Fig. 1. We will show how this can be accomplished in a generic model in three nested components, prove that it provides a universal gate set when combined with single qubit operations, and then discuss the suitability of silicon donor qubits for implementing this scheme.

The first of our nested components is the adiabatic cycle of moving isolated qubits to a ROP, where they interact, and then back. In an ideal limit, adiabatic operations are, by definition, diagonal with respect to instantaneous eigenstate bases. Up to an irrelevant global phase such

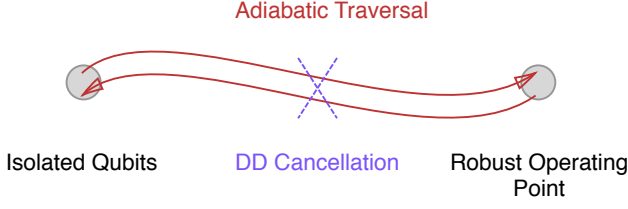


FIG. 1. Control space schematic showing an adiabatic path between isolated qubits and a ROP where the qubits interact. We will cancel phase accumulated during this traversal using dynamical decoupling.

an operation for two qubits is generically

$$\begin{bmatrix} e^{i\alpha} & 0 & 0 & 0 \\ 0 & e^{i\beta} & 0 & 0 \\ 0 & 0 & e^{i\gamma} & 0 \\ 0 & 0 & 0 & e^{-i(\alpha+\beta+\gamma)} \end{bmatrix} \equiv \begin{array}{c} \text{---} [Z_a] \text{---} \bullet \text{---} \\ \text{---} [Z_b] \text{---} [Z_c] \text{---} \end{array} \quad (1)$$

in the eigenstate basis using a matrix representation (left) or a circuit-model representation (right). We assume that there is good energy gap protection throughout, including when the qubits are isolated, so that low frequency noise will only induce phase errors. That is, we assume we are not T_1 limited. Consider the path illustrated in Fig. 1 consisting of three stages: 1) traversing from isolated qubits to the ROP; 2) waiting at the ROP; 3) traversing back to isolated qubits. Each is an operation as in Eq. (1) and these operations commute. Generically, we may write the circuit model representation as

$$\begin{array}{c} \text{---} [Z_{a_1}] \text{---} \bullet \text{---} [Z_{a_2}] \text{---} \bullet \text{---} [Z_{a_3}] \text{---} \bullet \text{---} \\ \text{---} [Z_{b_1}] \text{---} [Z_{c_1}] \text{---} [Z_{b_2}] \text{---} [Z_{c_2}] \text{---} [Z_{b_3}] \text{---} [Z_{c_3}] \text{---} \\ \\ \text{---} [Z_a] \text{---} \bullet \text{---} [Z_{d(\tau)}] \text{---} \bullet \text{---} \\ \text{---} [Z_b] \text{---} [Z_c] \text{---} [Z_{e(\tau)}] \text{---} [Z_{f(\tau)}] \text{---} \end{array} \quad (2)$$

where $a = a_1 + a_3$, $b = b_1 + b_3$, $c = c_1 + c_3$, $d(\tau) = a_2$, $e(\tau) = b_2$, and $f(\tau) = c_2$. We parameterize this operation with τ , the amount of time spent at the ROP. Somewhat arbitrarily, we will refer to the top rail of Eq. (2) as the ancilla and the bottom rail as the data.

The $d(\tau)$, $e(\tau)$, and $f(\tau)$ phases are incurred at the ROP and therefore presumed to have little sensitivity to noise. In contrast, the a , b , and c phases are sensitive to noise and uncertainty during transit. The second and third components of our procedure are designed to cancel these dependences. In the second component, we cancel out the a dependence by applying DD to the ancilla qubit in what we will call an ‘‘ancilla-refocused double-cycle’’. This component has three stages. First, we perform an adiabatic cycle where we set τ such that $f(\tau) = \pi$. Next, we apply a refocusing X gate on the ancilla qubit. Finally, we perform another adiabatic cycle that is the same

as the first except we take $\tau = 0$. It is only important that the difference in the two adiabatic cycles amounts to an extra controlled- Z operation (it is unnecessary to literally spending zero time at the ROP for one of the cycles). To the extent these operations are not ideally realized, there will be noise that is not canceled, but it is instructive to assume idealism in the initial analysis. The net operation of this component is then

$$\begin{array}{c} \text{---} [Z_a] \text{---} \bullet \text{---} [Z_d] \text{---} \bullet \text{---} [X] \text{---} [Z_a] \text{---} \bullet \text{---} \\ \text{---} [Z_b] \text{---} [Z_c] \text{---} [Z_e] \text{---} \bullet \text{---} [Z_b] \text{---} [Z_c] \text{---} \end{array} = \begin{array}{c} \text{---} [Z_d] \text{---} [X] \text{---} \\ \text{---} [Z_g] \text{---} \end{array} \quad (3)$$

where $g = 2b + c + e$. Notice that, in addition to canceling the a dependence, we have also made the c rotation on the data qubit deterministic rather than dependent upon the ancilla state. As far as the ancilla qubit is concerned, it has performed a CPhase operation with the data qubit and the uncertainty has been canceled.

The final, top-level component applies DD in a three stage process as before, but it will involve two different ancilla qubits. In the first stage, we do an ancilla-refocused double-cycle with one ancilla. Next, we perform a refocusing X gate on the data qubit. Finally, we do another ancilla-refocused double-cycle but with a different ancilla. The circuit-model representation is

$$\begin{array}{c} \text{---} [Z_d] \text{---} [X] \text{---} \\ \text{---} [Z_d] \text{---} \bullet \text{---} [X] \text{---} \\ \text{---} [Z_g] \text{---} [X] \text{---} [Z_g] \text{---} \end{array} = \begin{array}{c} \text{---} [Z_d] \text{---} [X] \text{---} \\ \text{---} [Z_d] \text{---} \bullet \text{---} [X] \text{---} \\ \text{---} [X] \text{---} \end{array} \Rightarrow \begin{array}{c} \bullet \text{---} \\ \bullet \text{---} \\ \bullet \text{---} \end{array} \quad (4)$$

where the bottom rail is the data qubit. We permuted commuting operators on the left side of Eq. (4) for compactness. The right side of the equation shows how the g dependence (which is a function of the uncertain b and c parameters) is canceled. On the right of the arrow is the equivalent operation up to single qubit operations that are perfectly known in the ideal limit with respect to being adiabatic, repeatable, insensitive to noise at the ROP, and with ideal single-qubit operations. We can understand non-ideal operations as evolving states into superpositions of ideal and errant parts, limiting the overall gate fidelity.

The robust multi-qubit gate that we have produced, shown at the right of Eq. (4), is not standard. We produce two CPhase gates between one data qubit two different ancilla. To prove that this is a sufficient for universality, we show how to produce a single CPhase gate between two data qubits, mediated by ancilla. First, it is straightforward to produce a CPhase between an ancilla and data by discarding one of the ancilla:

$$|0\rangle \begin{array}{c} \bullet \text{---} \\ \bullet \text{---} \\ \bullet \text{---} \end{array} \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} = \begin{array}{c} \bullet \text{---} \\ \bullet \text{---} \end{array} \quad (5)$$

This is wasteful, but suffices for a proof. With this data-

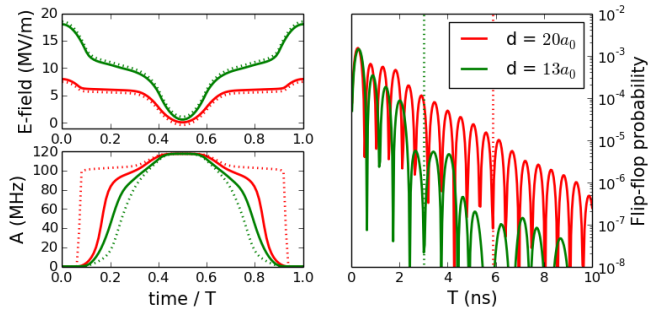


FIG. 3. Left: Heuristic electron shuttling schedule with respect to E-field (top) and HF interaction A (bottom) for neutralizing then ionizing a donor near an interface corresponding to Fig. 2. Dotted curves show hypothetical E-field shifts of $\Delta E = \pm 0.3$ MV/m as considered in Fig. 4. Right: Flip-flop failure probability as a function of shuttling time in a 100 mT B-field, showing the timescale required to be adiabatic. The vertical dotted lines indicate shuttle times used in Fig. 4.

To test adiabaticity, we used a heuristic control schedule, shown on the left of Fig. 3, that limits the first and second order time derivatives of the E-field and A (motivated by findings of Ref. 34 that adiabaticity improves by setting time derivatives of the initial and final Hamiltonian to zero). While the schedule could be optimized further, our simple heuristic already performs very well as shown on the right of Fig. 3 based upon simulations using QuTiP [35, 36]. In a 100 mT field (or more), the probability of non-adiabatic electro-nuclear flip-flops is very low for shuttle times of a few nanoseconds.

To illustrate the great benefit from the DD that our scheme employs, Fig. 4 compares sensitivity to static versus dynamic E-field shifts for the various error channels. The error probability for each channel in Fig. 4 (and right of Fig. 3) is the Born rule probability for the worst-case initial state (orthogonal to the error channel). Static shifts are the limit of low-frequency noise and well tolerated because of DD. Flip-flop error probabilities (top) increase with abrupt transitions in the shuttling schedule that result from static E-field shifts. The remaining error channels are sensitive to static shifts away from the ROP. Alternating shifts are the worst-case higher frequency noise, flipping the sign of the shift at the time of the refocusing pulse. This causes differences of the time integration of A , increasing error rates for the bottom three error channels of Fig. 4. The contrast between the solid and dotted curves illustrates the tremendous benefit of DD. We calculate very high entangling gate operations, well below error correction thresholds, from estimates of fast and slow voltage fluctuations/drift based upon actual device observations [Refs. 37 and 38].

In conclusion, we present a procedure for making robust, universal multi-qubit operations even if qubit-environment interactions change non-trivially when inter-qubit interactions turn on and off. Our composite

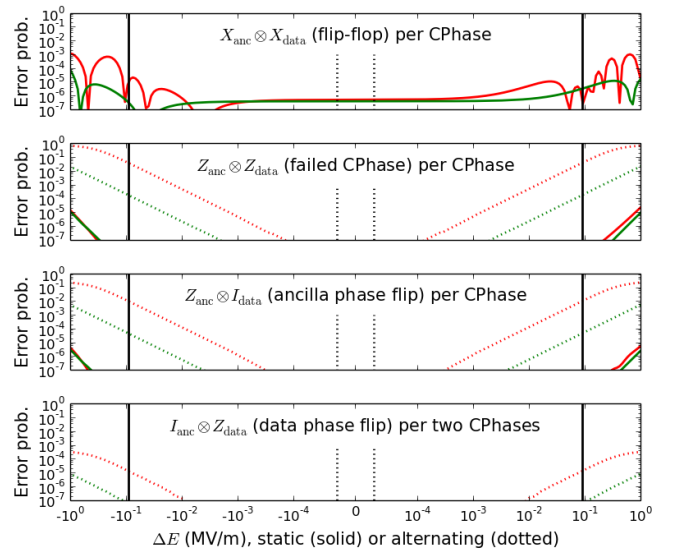


FIG. 4. (Color online) Sensitivity to static (solid) or alternating (dotted) shifts in the E-field, ΔE , corresponding to schedules indicated in Fig. 3 for a donor $20a_0$ (red) or $13a_0$ (green) from an interface. Each panel is for a different noise channel. Static shifts probe the low-frequency noise limit. Alternating shifts probe the worst-case sensitivity to E-field fluctuations with a $\pm \Delta E/2$ shift before/after the relevant DD refocusing pulse. Vertical lines indicate magnitude estimates of slow (solid), 0.1 day scale, and fast (dotted), millisecond scale (to be conservative), fluctuations. These are based upon charge drift observations reported in Ref. 37 (0.15×22 mV = 3.3 mV), E-field versus voltage for a Si qubit device in Ref. 38 (0.026 MV/m per mV), and an assumption that the noise behaves in a diffusive manner like a random walk).

sequence is extremely efficient relative to other DD [10] and DCG [15, 39] strategies, making it less vulnerable to control noise, because we exploit the adiabatic theorem to eliminate most error channels. In order to circumvent a No-Go theorem that prohibits black-box DCGs [14], we require a sweet spot (ROP) for pairwise interactions that may be turned on and off adiabatically. The requirements are well met for a system of P donors in Si, using electron and nuclear spins as two species of qubits. Our calculations estimate remarkable insensitivity to expected low frequency E-field noise.

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