Anisotropic magnetoresistance driven by surface spin-orbit scattering

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Anisotropic magnetoresistance driven by surface spin orbit scattering

Steven S.-L. Zhang[1,2], Giovanni Vignale[1] and Shufeng Zhang[2]
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In a bilayer consisting of an insulator (I) and a ferromagnetic metal (FM), interfacial spin orbit scattering leads to spin mixing of the two conducting channels of the FM, which results in an unconventional anisotropic magnetoresistance (AMR). We theoretically investigate the magnetotransport in such bilayer structures by solving the spinor Boltzmann transport equation with generalized Fuchs-Sondheimer boundary condition that takes into account the effect of spin orbit scattering at the interface. We find that the new AMR exhibits a peculiar angular dependence which can serve as a genuine experimental signature. We also determine the dependence of the AMR on film thickness as well as resistivity spin asymmetry of the FM.

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I. INTRODUCTION

Anisotropic magnetoresistance (AMR) is a generic magnetotransport property of ferromagnetic metals. In general, the longitudinal resistance of a bulk polycrystalline ferromagnetic metal only depends on the relative orientations of the magnetization vector and the current, which can be cast in the form[1],

$$\rho = \rho_0 + \Delta\rho_b(\hat{\mathbf{j}}_e \cdot \mathbf{m})^2 \tag{1}$$

where $\hat{\mathbf{j}}_e = \mathbf{j}_e/j_e$ is the unit vector in the direction of the current density, $\mathbf{m}$ is the unit vector in the direction of the magnetization, ρ_0 is the isotropic longitudinal resistivity and $\Delta\rho_b$ quantifies the magnitude of the bulk AMR effect (typically $\Delta\rho_b \sim 1\%$ for transition metals and their alloys). The effect has found many practical applications in magnetic recording and sensor devices[2].

Recently, the AMR effect has also played a key role in measurements of spin Hall angle[3–5] as well as spin torque generation[6–9] in FM/heavy-metal and FM/topological-insulator (TI) bilayers. The structural inversion asymmetry of these structures, combined with strong spin orbit coupling in the non-magnetic layers, generates a large Rashba-type spin-orbit coupling[10–12]

$$\hat{V}_{s.o.} = -\lambda_c^2 V'(z)\,\boldsymbol{\sigma}\cdot(\hat{\mathbf{p}}\times\mathbf{z}) \tag{2}$$

where $\boldsymbol{\sigma}$ is the Pauli spin matrix, $\hat{\mathbf{p}}$ is the momentum operator, λ_c is the effective Compton wave length, $\mathbf{z}$ is the unit vector normal to the interface, $V(z)$ is the potential in the vicinity of the interface (which only varies in the $z-$ direction), and $V'(z)$ is its derivative, which is large only in the interfacial region. A natural question arises: will the interfacial spin orbit interaction alter the AMR in the FM layer? At first glance, one might think the spin orbit interaction, commuting with the total in-plane momentum, p_x and p_y, cannot alter the in-plane resistivity. However, this argument fails for a ferromagnetic metal since the in-plane momenta of either spin component are not separately conserved, and the spin-orbit coupling transfers momentum from one spin channel into the other.

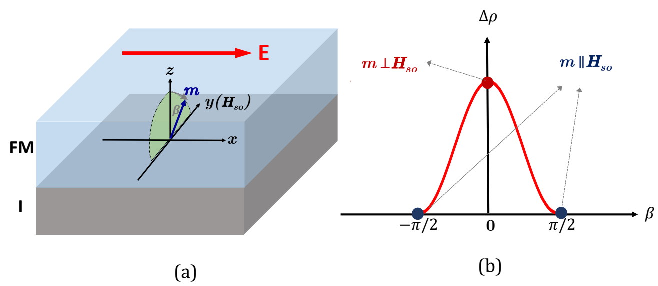

FIG. 1: Schematics of the transverse AMR effect induced by surface spin orbit scattering in a FM/I bilayer. The longitudinal resistivity changes when the magnetization $\mathbf{m}$ is rotated in the plane perpendicular to the electric field $\mathbf{E}$. Specifically, the resistivity is at a maximum when $\mathbf{m}$ is perpendicular to the interface (i.e., $\beta = 0$) and reaches a minimum when $\mathbf{m}$ lies in the plane of the interface (i.e., $\beta = \pm\pi/2$). This unusual AMR effect arises from spin mixing of the conducting channels of the FM, which depends on the relative directions of the magnetization $\mathbf{m}$ and the effective spin orbital field $\mathbf{H}_{so} \sim \mathbf{z}\times\mathbf{E}$.

In this paper, we show that, in the presence of surface spin orbit scattering, the AMR of a ferromagnet exhibits an angular dependence that is distinctly different from the conventional one described by Eq. (1). As shown in Fig. 1, when the magnetization vector $\mathbf{m}$ is swept in the plane perpendicular to the applied electric field $\mathbf{E}$, a variation in the longitudinal resistivity occurs, which has no analogue in Eq. (1): the resistivity has a maximum when $\mathbf{m}$ is along the $z-$axis (i.e., normal to the film plane) and reaches a minimum when $\mathbf{m}$ is along the $y-$axis (i.e., orthogonal to both $\mathbf{E}$ and $\mathbf{z}$), even though the angle between the magnetization and the current does not change. The physical origin of this unconventional angular dependence lies in the concerted actions of surface spin orbit scattering and spin asymmetry in the conductivity of the FM, which can be understood qualitatively within the two-current model[13,14]. The surface spin orbit scattering plays a crucial role in mixing the two parallel

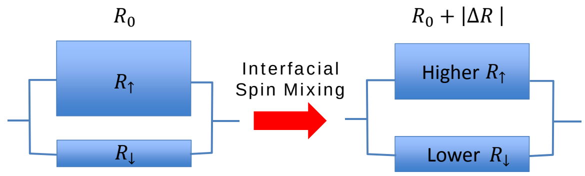

FIG. 2: The figure conceptually demonstrates, within a two-current model, that spin mixing induced by interfacial spin orbit scattering causes a redistribution of current in the two conducting channels by decreasing the resistance of the (spin-down) channel with higher resistance $R_\downarrow$ and increasing the resistance of the (spin-up) channel with lower resistance $R_\uparrow$. The overall effect of the spin mixing is an increase in the total resistance.

current channels of majority and minority spins; moreover, the degree of spin mixing depends on the relative orientation of the magnetization and the effective magnetic field $\mathbf{H}_{so} \sim \mathbf{j}_e \times \mathbf{z}$ seen by the electron spin. Specifically, spin mixing is strong when $\mathbf{m}$ is perpendicular to $\mathbf{H}_{so}$ while it is weak when $\mathbf{m}$ is aligned with $\mathbf{H}_{so}$. As shown in Fig. 2, the spin mixing causes, as long as the resistivities of the two current channels are *not* identical, a redistribution of current by decreasing the resistivity of the channel with higher resistivity and increasing the resistivity of the channel with lower resistivity: this results in an overall increase of the total resistivity[13,14]. The largest resistivity therefore coincides with the largest degree of spin mixing, which occurs when $\mathbf{m}$ is perpendicular to $\mathbf{H}_{so}$.

The remainder of this paper is organized as follows. In Sec. II, we present a quantitative theory of the spin-orbit driven AMR in a FM/I bilayer with strong interfacial spin orbit coupling. In Sec. III, we compare the predicted AMR with the recently interesting AMR found in heavy-metal/ferromagnetic-insulator (FI) (e.g. Pt/YIG bilayers and etc.). Material consideration for the experimental observation of our predicted AMR will also be discussed. Finally, we draw a conclusion in Sec. IV.

II. SPINOR BOLTZMANN EQUATION MODEL

Our theoretical analysis is based on the spinor form of the semiclassical Boltzmann equation: the non-equilibrium distribution function, $\hat{f}(\mathbf{k},z)$, is a 2×2 matrix in spin space[15,16]. At variance with previous studies[17–21] we exclude from our consideration heavy metals with strong spin-orbit coupling: in fact, we assume that the spin-orbit coupling is negligible in the bulk of the ferromagnet, so as to avoid contamination from the conventional bulk AMR effect. For simplicity, we also assume same Fermi wave vector k_F but different relaxation times for majority and minority electrons, just as in the seminal paper by Valet and Fert in calculating the CPP-GMR[22]. Such simplification is justified since the essence of our effect is in the difference of the relaxation times (and hence the resistivities) of majority and minority spins, while the exchange splitting is responsible for spin dephasing of the transverse spin component[15,23].

A. Bulk transport equations

The equation of motion for $\hat{f}(\mathbf{k},z)$ in the steady state is

$$v_z \frac{\partial \hat{f}(\mathbf{k},z)}{\partial z} - eEv_x \left(\frac{\partial f_0(k)}{\partial \varepsilon_k}\right)\hat{I} = -\frac{1}{2}\left\{\hat{\tau}^{-1}, \delta\hat{f}(\mathbf{k},z)\right\} \tag{3}$$

where $f_0(k)$ is the equilibrium distribution function, $\hat{\tau}^{-1} = (\tau')^{-1}\left(\hat{I} - p\boldsymbol{\sigma}\cdot\mathbf{m}\right)$ is the spin dependent relaxation rate with

$$(\tau')^{-1} = \left[(\tau_\uparrow)^{-1} + (\tau_\downarrow)^{-1}\right]/2 \tag{4}$$

and

$$p \equiv \left[(\tau_\downarrow)^{-1} - (\tau_\uparrow)^{-1}\right] / \left[(\tau_\uparrow)^{-1} + (\tau_\downarrow)^{-1}\right] \tag{5}$$

being, respectively, the average momentum relaxation rate and the spin asymmetry in resistivity, $\tau_\uparrow$ and $\tau_\downarrow$ being the momentum lifetimes of the two spin channels, and $\{\ ,\}$ standing for an anticommutator. Notice that in the collision term of Eq. (3) we require that the difference

$$\delta\hat{f}(\mathbf{k},z) \equiv \hat{f}(\mathbf{k},z) - \hat{f}_{l.e.}(k,z) \tag{6}$$

between the non-equilibrium distribution and a *local equilibrium distribution*[24–26]

$$\hat{f}_{l.e.}(k,z) = f_0(k)\hat{I} + \frac{\partial f_0}{\partial \varepsilon_k}\left[\mu_0(z)\,\hat{I} + \boldsymbol{\sigma}\cdot\boldsymbol{\mu}_s(z)\right] \tag{7}$$

tend to zero for long times. The parameters $\mu_0(z)$ and $\boldsymbol{\mu}_s(z)$ of the local distribution are fixed in such a way that the condition

$$\int d^3\mathbf{k}\left[\hat{f}(\mathbf{k},z) - \hat{f}_{l.e.}(k,z)\right] = 0 \tag{8}$$

is self-consistently satisfied. By doing this, we satisfy the physical requirements of particle and spin conservation in the collision processes, as well as the continuity equations that go with them.

To solve the Boltzmann equation, we further separate the distribution function into an equilibrium part and a small non-equilibrium perturbation, i.e.,

$$\hat{f}(\mathbf{k},z) = f_0(k)\hat{I} + \frac{\partial f_0}{\partial \varepsilon_k}\left[g(\mathbf{k},z)\,\hat{I} + \mathbf{h}(\mathbf{k},z)\cdot\boldsymbol{\sigma}\right] \tag{9}$$

where $g(\mathbf{k},z)\,\hat{I}$ and $\mathbf{h}(\mathbf{k},z)\cdot\boldsymbol{\sigma}$ are the spin-independent and spin-dependent components of the non-equilibrium distribution. By inserting Eq. (9) into Eq. (3), we obtain a set of coupled equations for the scalar and vector parts of the distribution function

$$v_z \frac{\partial g}{\partial z} - eE_x v_x = -\frac{g - \mu_0 - p\left(h_\parallel - \mu_{s\parallel}\right)}{\tau'} \tag{10}$$

$$v_z \frac{\partial h_\parallel}{\partial z} = -\frac{h_\parallel - \mu_{s\parallel} - p\left(g - \mu_0\right)}{\tau'} \tag{11}$$

and

$$v_z \frac{\partial \mathbf{h}_\perp}{\partial z} = -\frac{\mathbf{h}_\perp - \boldsymbol{\mu}_{s\perp}}{\tau'} \tag{12}$$

where $h_\parallel = \mathbf{m}\cdot\mathbf{h}(\mathbf{k},z)$, $\mu_{s\parallel} = \mathbf{m}\cdot\boldsymbol{\mu}_s(z)$, $\mathbf{h}_\perp = (\mathbf{m}\times\mathbf{h})\times\mathbf{m}$ and $\boldsymbol{\mu}_{s\perp} = (\mathbf{m}\times\boldsymbol{\mu}_s)\times\mathbf{m}$. The equations (10)-(12) have the general solutions

$$g^\pm(\mathbf{k},z) = e\tau v_x E_x + A^\pm_{\mathbf{k}} e^{\mp\frac{(1-p)z}{|v_z|\tau'}} + B^\pm_{\mathbf{k}} e^{\mp\frac{(1+p)z}{|v_z|\tau'}} + \sum_\alpha \int_0^z dt\left[\mu_0(t) + \alpha\mu_{s\parallel}(t)\right]\frac{\partial}{\partial t} e^{\mp\frac{(1-\alpha p)(z-t)}{|v_z|\tau'}}, \tag{13}$$

$$h^\pm_\parallel(\mathbf{k},z) = pe\tau v_x E_x + A^\pm_{\mathbf{k}} e^{\mp\frac{(1-p)z}{|v_z|\tau'}} - B^\pm_{\mathbf{k}} e^{\mp\frac{(1+p)z}{|v_z|\tau'}} + \sum_\alpha \alpha\int_0^z dt\left[\mu_0(t) + \alpha\mu_{s\parallel}(t)\right]\frac{\partial}{\partial t} e^{\mp\frac{(1-\alpha p)(z-t)}{|v_z|\tau'}}, \tag{14}$$

and

$$\mathbf{h}^\pm_\perp(\mathbf{k},z) = \mathbf{C}^\pm_{\mathbf{k}} e^{\mp\frac{z}{|v_z|\tau'}} + \int_0^z dt \boldsymbol{\mu}_{s\perp}(t)\frac{\partial}{\partial t} e^{\mp\frac{(z-t)}{|v_z|\tau'}}, \tag{15}$$

where the superscript + labels the solution for $v_z > 0$ and the subscript − for $v_z < 0$. The sum over α runs over the values $\alpha = \pm 1$. The four unknown constants $A_{\mathbf{k}}$, $B_{\mathbf{k}}$, and $\mathbf{C}_{\mathbf{k}}$ (where $\mathbf{C}_{\mathbf{k}}$ is a vector orthogonal to $\mathbf{m}$, hence with only two components) will now be determined from the boundary conditions.

B. Boundary conditions

Up to this point, the interfacial spin-orbit interaction has not appeared in our calculations. In particular, the collision term in Eq. (3) did not contain it, and therefore conserved spin. The spin-orbit coupling appears in the boundary condition that connects the distribution function for electrons impinging on the interface (label −) to the distribution function for electrons that are scattered off the interface (label +). Specifically, in order to take into account the rotation of spin upon scattering off the interface with the potential

$$\hat{V}_{scat.} = V_b\Theta(-z) - \left(V_b\lambda_c^2\right)\delta(z)\,\boldsymbol{\sigma}\cdot(\hat{\mathbf{p}}\times\mathbf{z}) \tag{16}$$

(where V_b is the barrier height of the insulator, $\Theta(z)$ is the unit step function, $\mathbf{z}$ is the unit vector normal to the interface and the delta function confines the SO coupling to the interface at $z = 0$), we introduce the following spinor generalization of the Fuchs-Sondheimer boundary condition[27–29]:

$$\hat{f}^+(\mathbf{k}; z=0) = s_I \hat{R}^\dagger \hat{f}^-(\mathbf{k}; z=0)\hat{R} + (1 - s_I) f_0 \hat{I} \tag{17}$$

where the superscripts + and − correspond to the distribution functions with $v_z > 0$ and $v_x < 0$ respectively, the parameter s_I varies between 0 and 1, characterizing the fraction of electrons being specularly reflected[29] ($s_I = 1$ when the interface is perfectly smooth and $s_I = 0$ when the interface is extremely rough) and $\hat{R}$ is a 2×2 reflection amplitude matrix in spin space which captures the spin rotation of the reflected electrons. The physical assumption underlying the boundary condition (17) is that only the electrons that are specularly reflected (with probability s_I) have their spins rotated by a definite angle. Electrons that are diffusively reflected return to the metal with randomly oriented momentum and spin direction, which are determined by the random orientation of the normal to the rough surface and the associated Rashba field. For these electrons we assume that the distribution coincides with the original equilibrium distribution.

The explicit form of the spin rotation matrix for specularly reflected electrons is

$$\hat{R} = \frac{\left[-k_b^2 + (\lambda_c k_b)^4 q^2\right]\hat{I} + 2i(\lambda_c k_b)^2 k_z \boldsymbol{\sigma}\cdot(\mathbf{q}\times\mathbf{z})}{(\kappa - ik_z)^2 - (\lambda_c k_b)^4 q^2} \tag{18}$$

where $\kappa \equiv \sqrt{k_b^2 - k_z^2}$ with $k_b \equiv \sqrt{2m_e^* V_b/\hbar^2}$. The derivation of $\hat{R}$ is presented in the Appendix. We note that in Eqs. (17) and (18) we have assumed that the Fermi wave vector of the electrons in the FM is spin independent. This assumption allows us to focus on the spin rotation due to the interfacial spin orbit coupling, while neglecting the well-studied spin dependent scattering based on the exchange band splitting.[30,31] In addition, our treatment neglects the interference between the incident and the reflected electron waves at the interface. This is justified when the correlation length of the surface roughness is comparable to the Fermi wavelength in which case the phase coherence between incident and reflected waves is destroyed by surface roughness.[32]

For simplicity, we assume spin independent specular reflection only at the outer surface of the FM layer ($z =$

d), i.e.,

$$\hat{f}^{+}(\mathbf{k};z=d)=\hat{f}^{-}(\mathbf{k};z=d) \tag{19}$$

Neglecting spin-dependent scattering from the other surface simplifies the calculation without altering any qualitative features of the results. By inserting Eq. (9) into the boundary conditions as well as Eq. (8) , we can find unique solutions for $\hat{f}(\mathbf{k},z)$ and the charge current density can be calculated as

$$\mathbf{j}_e\left(z\right)=\frac{-e}{\left(2\pi\right)^3}Tr\int d\mathbf{k}\hat{f}(\mathbf{k},z)\mathbf{v} \tag{20}$$

C. Anisotropic magnetoresistance

After some algebra, we find the charge current density up to second order in the spin orbit coupling, i.e., $O\left(\left(\lambda_c k_b\right)^4\right)$

$$\mathbf{j}_e\left(z\right)=c_0E_x\left\{\left[1-\alpha^{(1)}_{xx}\left(z\right)-\alpha^{(2)}_{xx}\left(z\right)\right]\hat{\mathbf{x}}+\alpha_{yx}\left(z\right)\hat{\mathbf{y}}\right\} \tag{21}$$

where $c_0=e^2\tau k_F^3/3\pi m_e^*$ is Drude conductivity and two position dependent coefficients read

$$\alpha^{(1)}_{xx}\left(z\right)=\left(1-s_I\right)\sum_{\sigma}\left(1+\sigma p\right)F_{p\sigma}\left(z\right) \tag{22}$$

$$\alpha^{(2)}_{xx}\left(z\right)=s_Ip\left(\lambda_c k_F\right)^4\left[4-\left(m_x^2+3m_y^2\right)\right]\sum_{\sigma}\sigma G_{p\sigma}\left(z\right), \tag{23}$$

and

$$\alpha_{yx}\left(z\right)=s_Ip\left(\lambda_c k_F\right)^4 m_x m_y\sum_{\sigma}\sigma G_{p\sigma}\left(z\right), \tag{24}$$

where

$$F_{p\sigma}\left(z\right)\equiv\frac{3}{4}\int_0^1 d\xi\frac{(1-\xi^2)\cosh\left[\frac{(1-\sigma p)(d-z)}{\xi\lambda_0(1-p^2)}\right]}{\exp\left[\frac{(1-\sigma p)d}{\xi\lambda_0(1-p^2)}\right]-s_I\exp\left[-\frac{(1-\sigma p)d}{\xi\lambda_0(1-p^2)}\right]} \tag{25}$$

and

$$G_{p\sigma}\left(z\right)\equiv\frac{3}{2}\int_0^1 d\xi\frac{\xi(1-\xi^2)^{3/2}\cosh\left[\frac{(1-\sigma p)(d-z)}{\xi\lambda_0(1-p^2)}\right]}{\exp\left[\frac{(1-\sigma p)d}{\xi\lambda_0(1-p^2)}\right]-s_I\exp\left[-\frac{(1-\sigma p)d}{\xi\lambda_0(1-p^2)}\right]}, \tag{26}$$

and $\lambda_0\equiv v_F(\tau_\uparrow+\tau_\downarrow)/2$ is the average electron mean free path. The first term, $\alpha^{(1)}_{xx}$, is independent of the magnetization direction: it is the resistivity due to interfacial roughness[32]. The third term, α_{yx}, corresponds to the well-known planar Hall effect[1,13]. Here p is the spin asymmetry in the resistivity, defined in Eq. (5).

The second term, $\alpha^{(2)}_{xx}$, describes the new AMR effect. We note that this effect is of second order in the spin-orbit coupling and vanishes when the resistivity spin asymmetry $p=0$. The experimentally relevant quantity is the spatially averaged longitudinal resistivity, which is obtained from the formula $\rho^{-1}_{xx}\left(d\right)=\left(1/d\right)\int_0^d dz\ \ j_{e,x}\left(z\right)/E_x$. As we discussed earlier, the bulk spin-orbit coupling (not included in our calculation) produces a conventional AMR with angular dependence shown in Eq. (1). Therefore in general, the longitudinal resistivity of the FM thin film should take the form

$$\rho_{xx}\left(d\right)=\rho_0+\Delta\rho^{(b+s)}_{so}m_x^2-\Delta\rho^{(s)}_{so}m_y^2 \tag{27}$$

where the first term on the rhs of Eq. (27) is the isotropic resistivity and the second term is the AMR with conventional angular dependence of $(\hat{\mathbf{j}}_e\cdot\mathbf{m})^2$ to which both bulk and surface spin orbit coupling may contribute[57]. The most interesting term is the third term which is solely due to the surface spin orbit scattering and can be distinguished from the bulk AMR based on the different angular dependence.

In Fig. 2, we show $\Delta\rho^{(s)}_{so}$ (normalized by ρ_0) as a function of the resistivity spin asymmetry p. This figure delivers two main messages. First, $\Delta\rho^{(s)}_{so}$ is positive when

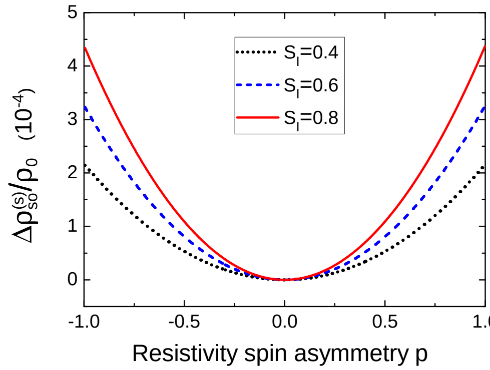

FIG. 3: The transverse AMR $\Delta\rho^{(s)}_{so}$ as a function of the resistivity spin asymmetry p for several values of s_I. Parameters: $\left(\lambda_c k_F\right)^2=0.05$, $d/\lambda_0=1$.

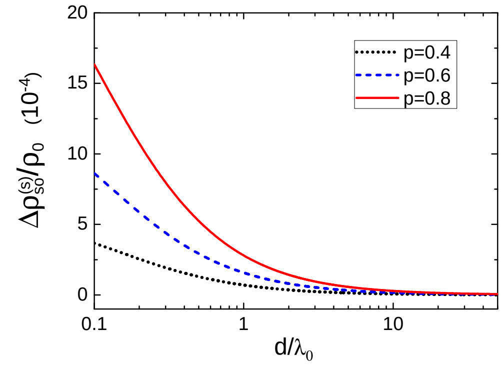

FIG. 4: The transverse AMR $\Delta\rho^{(s)}_{so}$ as a function of d/λ_0 for several values of p. Parameters: $\left(\lambda_c k_F\right)^2=0.05$ and $s_I=0.8$.

$p \neq 0$. This confirms the angular dependence of the AMR we sketched in Fig. 1(b). The second message is that $\Delta\rho_{so}^{(s)}$ is an even function of the resistivity spin asymmetry p. This is consistent with our two-current model: the spin mixing resistivity only relies on the absolute value of the difference between the two conduction channels but not on the sign of that difference. We point out that the resistivity spin asymmetry in transition metal alloys depends strongly on the alloy type, as nicely demonstrated experimentally by Fert and Campbell[33]. For example, they showed $p = 0.83$ for FeMn while $p \sim 0$ for FeCo. This suggests the possibility of verifying the theory in detail by studying the AMR in different transition metal alloys.

In Fig. 3, we show the thickness dependence of $\Delta\rho_{so}^{(s)}$ for several values of the resistivity spin asymmetry. When the FM layer thickness is much larger than the mean free path, i.e., $d \gg \lambda_0$, $\Delta\rho_{so}^{(s)}$ exhibits a standard $1/d$ thickness dependence as can be analytically worked out via Eq. (26). As a note of caution, we point out that when spin dependent scattering is present at both surfaces of the FM layer and the thickness of the FM is comparable to the mean free path, one may need to take into account the quantum interference between electrons reflected from both interfaces and thus a quantum approach may be desirable[34–39]. However, when the thickness is much larger than the mean free path, the contributions from the two interfaces add up constructively and hence the angular and thickness dependence of the AMR should remain qualitatively the same.

As a final point of this section, we provide an order of magnitude estimate of our new AMR for some real material and experimental condition. Let us consider a Fe thin film at room temperature with $|p| \sim 0.5$[40], $s_I = 0.8$, $\lambda_c^2 \sim 0.05$ $\mathring{A}^2$, $k_F \sim 1.7 \times 10^8$ cm^{-1}[41] and $d = \lambda_0 \sim 3.7$ nm[42], we find the transverse AMR ratio (i.e.,$\Delta\rho_{so}^{(s)}/\rho_0$) is about $\sim 1 \times 10^{-3}$, which is comparable to its bulk value ($\sim 0.2\%$[1]) and is at least an order of magnitude larger than the AMR found in heavy-metal/FI bilayers such as Pt/YIG[43–50].

III. DISCUSSION

It is instructive to compare the AMR that we predict in this paper for FM/I bilayers with the intriguing AMR recently observed in NM/FI bilayers.[43–50] Although the angular dependences of these AMR's turn out to be similar, their physical origins are remarkably different. The AMR discussed in this paper arises from the combined effect of the intrinsic resistivity spin asymmetry of the FM and the extrinsic interfacial spin orbital scattering. Whereas, the AMR observed in NM/FI bilayers remains controversial and has been attributed to several different mechanisms such as (i) the bulk spin Hall effect (also known as the spin Hall magnetoresistance)[18,46] (ii) surface states with spin orbit coupling,[19,20] and (iii) magnetic proximity effect.[44,45,50]

While the magnetic proximity effect is no longer operative in FM/I bilayers, the spin Hall and surface state mechanisms would in principle contribute to the AMR in such bilayer structures. However, these mechanisms should be distinguishable from the one proposed in this paper for the following reasons. In FM/I bilayers, the bulk spin orbit coupling is expected to be much weaker than that in heavy metals (such as Pt, Ta, Pd, etc.), resulting in a much weaker spin Hall magnetoresistance. Similarly, the surface state contribution to the AMR in a NI/FI bilayer system has been claimed to scale as $\lambda_c^2 k_F \sim 1$ $\mathring{A}$[20] which is much shorter than the electron mean free path of the metal layer. We therefore expect our interfacial scattering mechanism to dominate the AMR of the system.

Lastly, let us consider the choice of materials for the observation of our novel AMR. In order to obtain a sizable transverse AMR, it is essential to have a FM/I bilayer with a large difference between the conductivities of majority and minority spin carriers in the ferromagnetic metal, and a strong spin-orbit interaction at the FM/I interface. A very promising system in this respect is FM grown on top of a TI such as Bi_2Se_3, $Bi_{1.5}Sb_{0.5}Te_{1.7}Se_{1.3}$ or Sn-doped $Bi_2Te_{1.7}Se_{1.3}$[51–53]. Recently, large spin transfer torque[9] and spin-charge conversion[54] effects were observed in these Py/TI bilayers, indicating the presence of strong spin orbit coupling at the interface. Non-topological-oxide/ferromagnetic interfaces may also provide large spin-orbit interaction, as implied by tunneling AMR studies in Fe/MgO/Fe junctions[55,56]

IV. CONCLUSION

In conclusion, we have predicted an unconventional AMR in FM/I bilayers. This new AMR arises from the concerted actions of the surface spin orbit scattering and the spin asymmetry in the conductivity of the FM layer. Furthermore, we found this new AMR exhibits an angular dependence that is distinct from that of the conventional bulk effect: the resistance changes when the magnetization is rotated around the current, even if the angle between these two vectors does not change. Also, the thickness dependence of the AMR scales with electron mean free path of the FM layer, which can be experimentally distinguished from other possible surface induced magnetoresistance.

V. ACKNOWLEDGEMENTS

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Appendix A: Spinor form reflection amplitude with interfacial Rashba spin orbit coupling

In this appendix, we derive the spinor form reflection amplitude given by Eq. (18) in the main text. First, we write down the following piece-wise scattering wave functions corresponding to the interfacial potential described by Eq. (16) in the main text

$$\psi_{\mathbf{k}}\left(\mathbf{r}_{\|},z>0\right)=\frac{1}{\sqrt{2}}e^{-ik_z z}e^{i\mathbf{q}\cdot\mathbf{r}_{\|}}\chi+\frac{1}{\sqrt{2}}e^{ik_z z}e^{i\mathbf{q}\cdot\mathbf{r}_{\|}}\hat{R}\chi \tag{A1}$$

and

$$\psi_{\mathbf{k}}\left(\mathbf{r},z<0\right)=\frac{1}{\sqrt{2}}e^{\kappa z}e^{i\mathbf{q}\cdot\mathbf{r}_{\|}}\hat{T}\chi \tag{A2}$$

where $\mathbf{r}_{\|}$ is the in-plane position vector, $\mathbf{q}$ and k_z are the in-plane and perpendicular-to-plane wave vectors respectively, $\kappa=\sqrt{k_b^2-k_z^2}$ with $k_b\equiv\sqrt{2m_e^*V_b/\hbar^2}$, $\hat{R}$ and $\hat{T}$ are the 2×2 reflection and transmission amplitude matrices in spin space, and χ is an arbitrary spinor.

Now we are ready to determine $\hat{R}$ and $\hat{T}$ matrices by the following quantum mechanical matching conditions

$$\psi_{\mathbf{k}}\left(\mathbf{r}_{\|},0^+\right)=\psi_{\mathbf{k}}\left(\mathbf{r}_{\|},0^-\right) \tag{A3}$$

and

$$\psi'_{\mathbf{k}}\left(\mathbf{r}_{\|},0^+\right)-\psi'_{\mathbf{k}}\left(\mathbf{r}_{\|},0^-\right)=\left[\left(k_b\lambda_c\right)^2\boldsymbol{\sigma}\cdot\left(\hat{\mathbf{p}}\times\mathbf{z}\right)\right]\psi_{\mathbf{k}}\left(\mathbf{r}_{\|},0\right) \tag{A4}$$

By placing the scattering wave functions into the above two equations, we find

$$\left(\hat{I}+\hat{R}\right)\chi=\hat{T}\chi \tag{A5}$$

and

$$\left(-\kappa\hat{T}-ik_z\hat{I}+ik_z\hat{R}\right)\chi=\left[\left(k_b\lambda_c\right)^2\boldsymbol{\sigma}\cdot\left(\mathbf{q}\times\mathbf{z}\right)\right]\hat{T}\chi \tag{A6}$$

Combining the two equations, we find an equation for $\hat{R}$ only,

$$\left\{\left[\left(\kappa-ik_z\right)\hat{I}+\left(k_b\lambda_c\right)^2\boldsymbol{\sigma}\cdot\left(\mathbf{q}\times\mathbf{z}\right)\right]\hat{R}+\left(ik_z+\kappa\right)\hat{I}+\left(k_b\lambda_c\right)^2\boldsymbol{\sigma}\cdot\left(\mathbf{q}\times\mathbf{z}\right)\right\}\chi=0 \tag{A7}$$

For any χ, the equation is valid if

$$\left[\left(\kappa-ik_z\right)\hat{I}+\left(k_b\lambda_c\right)^2\boldsymbol{\sigma}\cdot\left(\mathbf{q}\times\mathbf{z}\right)\right]\hat{R}+\left(ik_z+\kappa\right)\hat{I}+\left(k_b\lambda_c\right)^2\boldsymbol{\sigma}\cdot\left(\mathbf{q}\times\mathbf{z}\right)=0 \tag{A8}$$

From Eq. (A8), we find the reflection amplitude matrix

$$\hat{R}=\frac{\left[-k_b^2+\left(k_b\lambda_c\right)^4q^2\right]\hat{I}+2i\left(k_b\lambda_c\right)^2k_z\boldsymbol{\sigma}\cdot\left(\mathbf{q}\times\mathbf{z}\right)}{\left(ik_z-\kappa\right)^2-\left(k_b\lambda_c\right)^4q^2} \tag{A9}$$

. Note that $\hat{R}$ is a unitary matrix as can be easily checked via Eq. (A9). This unitarity is required by flux conservation.

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[57] Note that the conventional bulk AMR is dominated by the sd interband scattering[1], while the new AMR that we predicted here arises from the spin-dependent surface scattering of s electrons. Therefore, according to Matthienssen's rule, the total resistivity can be considered as the sum of the two contributions.